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Investor's Information Sharing with Firms in Oligopoly*

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We study the incentives for an investor to transmit information to its invested firms in an oligopoly. The investor has more information on market conditions than the firms and reveals it publicly or privately before the firms produce the goods. When the investor uses a public channel to transmit information, the investor does not reveal any of its information to the firms. When the investor uses a private channel to transmit information, it partially reveals such a private information to the firm. Indeed, this is possible only when the investor invests relatively more in one firm than in another firm.

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I. Introduction

Institutional investors have recently played an important role in the economy. The main role of institutional investors is to invest in several firms. In addition, they provide various information on the market. Firms are often more knowledgeable of their markets than their investors. However, we frequently observe that investors also have some information that firms do not have and provide such an information to improve the market reliability and performance of their invested firms. For example, many investment banks, such as J.P. Morgan, Goldman Sachs, and Morgan Stanley, operate economic research teams that provide analyses of

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economic environments and market trends. Venture capitals provide funding for early-stage firms and a variety of consulting services for their invested firms. These consulting services include providing market information to develop business strategies. Moreover, institutional investors with a global network may be better aware of the situation in the overseas market than domestic companies.

In this paper, we are interested in the investor's incentive in providing information to its invested firms in an oligopoly market. Although an investor investing in several firms is interested in the performance of its portfolio, which is the (weighted) joint profit of its invested firms, an individual firm is interested only in its own profit. Thus, an investor may have an incentive to behave strategically in providing information to firms to induce the profit-maximizing firms to make a decision to improve the joint profit. When the investor provides information to its invested firms, it may use a public or private channel. When an investor provides information that utilizes a public channel, the same information is provided to all firms. For example, investment banks produce and publicly disclose a report on economic or market trends. When an investor provides information that utilizes a private channel, only a specific firm receives this information. For example, an institutional investor provides a consultancy service for a specific firm.

The aim of this paper is to investigate how an investor provides its private information to firms when it utilizes a public or private channel. We consider a model with two firms producing homogeneous goods and one investor owning shares of these firms. To focus on the investor's incentive to transmit its private information, the investor is assumed to have aquired private information on the market conditions. That is, the investor has some information on the market conditions that firms do not have. After acquiring private information on the market conditions, the investor decides how to provide such an information to the firms by utilizing either a public or private channel. The information provided by the investor cannot be verified, thereby enabling the investor to send any messages at no cost regardless of what it knows. After the investor delivers information on market conditions, the firms engage in a Cournot competition and choose output to maximize profit on the basis of the information provided by the investor. The profit of each firm is determined by the output chosen by the firms and shared by the investor and the relevant firm in accordance with their share ratio.

We first find that when an investor utilizes a public channel, no information is provided to the firms in any equilibrium. Given that each firm's output that is optimum to the investor is different from the output that is optimum to the firm, the investor may be reluctant to provide its private information to the firms. On the other hand, when the investor provides information to a specific firm utilizing a private channel, it is possible in an equilibrium for the investor to partially reveal information on market conditions to the relevant firm. Indeed, when the investor invests relatively more in one firm than in another firm, the investor's concern for the relevant firm's profit is high enough for the investor to partially provide its information to the firm to improve its performance.

There are a number of studies that investigate information sharing in an oligopoly. Examples are Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Fried (1984), Li (1985), Gal-Or (1985, 1986), Shapiro (1986), Kirby (1988), Ziv (1993), Raith (1996), and Jansen (2008). Most of these studies focus on information sharing between firms in an oligopoly. A variety of results are provided in these studies depending on the particular specifications of a model. Indeed, the results depend on the type of market-based competition (Cournot or Bertrand competition), source of uncertainty (demand or cost side), type of uncertainty (common or private value), and verifiability of the revealed information. Our model departs from these studies by introducing the investor and focusing on the investor's incentive to transmit information to oligopoly firms.

The model in Eliaz and Forges (2015) has a common feature with our model, in that the planner concerning the joint profit of firms is more informed on the market condition than the firms and transmits its information to the firms in a Cournot oligopoly. Eliaz and Forges (2015) show that the optimal information transmission policy to the planner is to completely inform one firm and disclose no information to the other firm. In their model, the information sent by the planner can be verified. Hence, the planner has to tell the truth but not necessarily the entire truth as in Milgrom (1981). Moreover, the planner can commit to a transmission rule before gathering information.[1] In this situation, the planner can get the firm's credibility on the information and providing complete information to the firms is more advantageous for the planner than hiding some of the information whether the planner uses the private or public channel to transmit such an information.[2]

Our model differs from that of Eliaz and Forges (2015) in that the investor's information is unverifiable. We consider that an unverifiable investor's information is considerably realistic. For example, investment banks are not pressured to provide "hard facts" on their reports and not sanctioned for their incorrect economic outlooks. We assume that the investor's information is unverifiable and show the impossibility of the investor providing complete information to the firms whether the private or public channel is used to transmit information. For the investor to provide complete information, firms should completely trust the investor's information. Given the discrepancy in the optimum level of outputs between the investor and firms, the former has an incentive to lie to induce the latter to choose

[1] The inability to commit to a transmission rule eliminates all the informative equilibria in Eliaz and Forges (2015).

[2] In addition, the firm's actions in a Cournot competition are strategic substitutes and so, as Angeletos and Pavan (2007) point out, the effect of private information on joint profit is positive, whereas the effect of public information is ambiguous. Thus, the planner's payoff increases when it uses a private channel instead of a public channel.

the outputs favorable to the investor. Moreover, the firms recognize the investor's incentive to lie. Thus, if information is unverifiable, then the investor cannot obtain complete trust from the firms on the information it provides.

This paper is also related to studies on the strategic transmission of private information. Many previous studies consider the model in which individuals play the role of either a sender who has private information and makes a decision on how to reveal it or a receiver who chooses an action that affects her own and the sender's payoffs. Crawford and Sobel (1982) consider a model with one sender and one receiver and study the possibility for the sender to partially disclose its private information. Melumad and Shibano (1991) borrow this approach to compare the equilibria in models with and without commitment on decision rule. Farrell and Gibbons (1989) and Goltsman and Pavlov (2011) also extend the model of Crawford and Sobel (1982) by considering multiple receivers, whose payoffs do not depend on each other's actions. Compared with these studies, we also consider a model with one sender and multiple receivers, but the receivers in our model are strategically interdependent through the Cournot competition. Baliga and Sjöström (2012) also consider a model with one sender and multiple receivers. In their model, the receivers are strategically dependent through a game with finite actions.[3]

II. Model

There are one investor and two firms (firm 1 and firm 2). The investor owns a share of each firm. Let $\beta_i \in (0,1)$ be the investor's share of firm i. The firms produce a homogeneous good with a marginal cost $c > 0$ and are involved in a Cournot competition. Let q_i be the output of firm i. The market demand for the good is given by

$$p = a + \theta - q_1 - q_2, \tag{1}$$

where $a > 0$ is constant and θ is randomly drawn from a distribution. The distribution of θ has a continuous density f with a support of $[0, \hat{\theta}]$. That is, $\text{supp}(f) = [0, \hat{\theta}]$, which is publicly known to the investor and the firms. A realization of θ is private information to the investor. This can be interpreted as the investor being more informed on the market condition than the firms.[4] For

[3] Many previous studies have also analyzed information transmission with verifiable messages. Examples of these studies are Milgrom (1981), Milgrom and Roberts (1986), Okuno-Fujiwara et al. (1990), and Eliaz and Serrano (2014).

[4] Given this assumption, we do not insist that the investor is more knowledgeable of the market conditions than the firms in general. This assumption attempts to capture that the investor has some information that the firms cannot access. This allows us to focus on the investor's incentives to

simplicity, let $\bar{\theta} = \mathbb{E}[\theta]$ be the expectation of θ. We assume that $\hat{\theta}$ is sufficiently small to satisfy

$$a - c > 3\hat{\theta}. \tag{2}$$

This condition can be interpreted as the uncertainty in the demand side is not so great.

Given an output (q_1, q_2) of the firms, each firm i's (ex-post) profit is

$$\pi_i(q_i, q_j; \theta) = (a - c + \theta - q_i - q_j)q_i, \tag{3}$$

which is shared with the investor in accordance with their share ratio. Thus, given firm i's profit $\pi_i(q_i, q_j; \theta)$, firm i's (ex-post) payoff is

$$u_i(q_i, q_j; \theta) = (1 - \beta_i)\pi_i(q_i, q_j; \theta) \tag{4}$$

and the investor's (ex-post) payoff is

$$v(q_i, q_j; \theta) = \beta_i \pi_i(q_i, q_j; \theta) + \beta_j \pi_j(q_i, q_j; \theta). \tag{5}$$

In this environment, we consider a two-stage decision procedure. In Stage 1, the investor observes a realization of θ and sends a message about θ to the firms using a public or private channel. When the investor uses a public channel, it sends a message $m \in [0, \hat{\theta}]$ to both firms. When the investor uses a private channel, it sends a message $m \in [0, \hat{\theta}]$ to only one firm, say firm i. Given that the firms cannot verify the investor's messages, the investor may be able to hide or partially reveal its information by sending a message m that is different from θ. In addition, we allow the investor to send a random message. In Stage 2, each firm i forms a belief on θ and chooses its output $q_i \in \mathbb{R}_+$ to maximize its expected payoff.

Given that the investors are allowed to send a random message, the investor's strategy, which is denoted as μ, can be represented as a function that assigns a distribution on $[0, \hat{\theta}]$ to each $\theta \in [0, \hat{\theta}]$.[5] Thus, $\mu(\theta)$ represents a random message, the distribution of which is generated by the investor's strategy μ given that θ is realized. A message m belonging to the support of $\mu(\theta)$ means that the investor observing θ can send m under its strategy μ. If the investor constantly sends a non-random message, the strategy μ can be represented as a

transmit its information.

[5] This means that the investor is allowed to use behavioral strategies.

function that assigns a message $m \in [0, \hat{\theta}]$ to each θ, where $\mu(\theta)$ means a message that the investor sends when it observes θ.

A message m that the investor playing strategy μ can send is said to be *observable under* μ. This means that m belongs to the support of $\mu(\theta)$ (i.e., $m \in \text{supp}(\mu(\theta))$) for some θ. In addition, $\bar{M}(\mu)$ denotes the set of messages that are observable under strategy μ. That is, $\bar{M}(\mu) = \bigcup_{\theta \in [0,\hat{\theta}]} \text{supp}(\mu(\theta))$.

When the investor sends a message to firm i (using a public or a private channel), firm i's strategy σ_i is a function that assigns an output to each message $m \in [0, \hat{\theta}]$. Thus, $\sigma_i(m)$ represents the output that firm i chooses when it receives a message m from the investor. When the investor does not send a message to firm i (using a private channel), firm i's strategy σ_i is a choice of output in $\mathbb{R}_+$.[6]

In the analysis, we focus on the *perfect Bayesian equilibrium* $(\mu^*, \sigma_1^*, \sigma_2^*)$, which has to satisfy two conditions, namely, *consistency* and *sequential rationality*. In general, consistency requires that each firm i forms a belief using Bayes' rule from μ^* as long as it observes a message m observable under μ^*. If firm i has a belief consistent with μ, then for any message $m \in \bar{M}(\mu)$, firm i has to form an expectation on θ as $\mathbb{E}[\theta \mid \mu(\theta) = m]$. Sequential rationality requires that the investor observing θ maximizes its payoff under the belief that the firms play (σ_1^*, σ_2^*) and each firm i maximizes its expected payoff under its belief.

III. Public Information Transmission Channel

This section considers the situation, in which the investor uses a public channel to transmit its private information. Accordingly, the investor has to send the same message to both firms. Thus, the firms have the same expectation on θ. We use backward induction to find an equilibrium $(\mu^\dagger, \sigma_1^\dagger, \sigma_2^\dagger)$.

Consider firm i's problem in Stage 2, in which the investor plays an equilibrium strategy $\mu^\dagger$ and the firms observe a message $m \in \bar{M}(\mu^\dagger)$ that can be sent by the investor under $\mu^\dagger$. Given that the firms have a belief consistent with $\mu^\dagger$, each firm i's choice $\sigma_i^\dagger(m)$ in the equilibrium has to maximize

$$\begin{aligned} &\mathbb{E}[u_i(q_i, \sigma_j^\dagger(m); \theta) \mid \mu^\dagger(\theta) = m] \\ &= \mathbb{E}[(1-\beta_i)(a - c + \theta - q_i - \sigma_j^\dagger(m)) q_i \mid \mu^\dagger(\theta) = m] \end{aligned} \tag{6}$$

[6] Here, we do not allow the firms to choose randomized outputs by adopting behavioral strategies. Indeed, given a message m, the firm's optimal choice on outputs is uniquely determined, and so the firms will not take a randomized action on the outputs in any equilibrium. Thus, we can restrict our attention to pure strategies for the firms.

with respect to $q_i \in \mathbb{R}_+$.[7] From the first-order necessary condition, we have:

$$\sigma_i^\dagger(m) = \frac{1}{2}(a-c) + \frac{1}{2}\mathbb{E}[\theta \mid \mu^\dagger(\theta) = m] - \frac{1}{2}\mathbb{E}[\sigma_j^\dagger(m) \mid \mu^\dagger(\theta) = m]. \tag{7}$$

Taking expectations conditional on $\mu^\dagger(\theta) = m$ to both sides of (7), we have:

$$\begin{aligned}&\mathbb{E}[\sigma_i^\dagger(m) \mid \mu^\dagger(\theta) = m]\\ &= \frac{1}{2}(a-c) + \frac{1}{2}\mathbb{E}[\theta \mid \mu^\dagger(\theta) = m] - \frac{1}{2}\mathbb{E}[\sigma_j^\dagger(m) \mid \mu^\dagger(\theta) = m].\end{aligned} \tag{8}$$

Solving the equations in (8) for i and j, we obtain:

$$\mathbb{E}[\sigma_i^\dagger(m) \mid \mu^\dagger(\theta) = m] = \mathbb{E}[\sigma_j^\dagger(m) \mid \mu^\dagger(\theta) = m] = \frac{1}{3}(a-c) + \frac{1}{3}E_\theta(m), \tag{9}$$

where $E_\theta(m) = \mathbb{E}[\theta \mid \mu^\dagger(\theta) = m]$. Substituting (9) into (7), each firm i 's equilibrium strategy $\sigma_i^\dagger$ should satisfy that for each $m \in \bar{M}(\mu^\dagger)$,

$$\sigma_i^\dagger(m) = \frac{1}{3}(a-c) + \frac{1}{3}E_\theta(m). \tag{10}$$

We next move to Stage 1, in which the investor has to choose a message. Suppose that the investor observes $\theta \in [0, \hat{\theta}]$ being realized. Given that firm i plays an equilibrium strategy $\sigma_i^\dagger$ in (10), firm i receiving an observable message m under $\mu^\dagger$ achieves the (ex-post) profit[8] as follows:

$$\begin{aligned}\Pi_i^\dagger(m) &= (a-c+\theta-\sigma_i^\dagger(m)-\sigma_j^\dagger(m))\sigma_i^\dagger(m)\\ &= \frac{1}{9}(a-c+E_\theta(m))(a-c+3\theta-2E_\theta(m)).\end{aligned} \tag{11}$$

Thus, each firm i and the investor's (ex-post) payoffs are, respectively,

$$U_i^\dagger(m) = (1-\beta_i)\Pi_i^\dagger(m)$$

[7] Since $u_i(q_i, \sigma_j^\dagger(m); \theta)$ is concave in q_i, the first-order necessary conditions are sufficient for the solution of the problem of maximizing $u_i(q_i, \sigma_j^\dagger(m); \theta)$. This is true for the other maximization problems considered in this paper.

[8] Note that the condition in (2) ensures that $\Pi_i^\dagger(m) > 0$ holds. This implies that $U_i^\dagger(m) > 0$ and $V^\dagger(m) > 0$.

$$=\frac{1}{9}(1-\beta_i)(a-c+E_\theta(m))(a-c+3\theta-2E_\theta(m)), \tag{12}$$

$$V^\dagger(m)=\beta_i\Pi_i^\dagger(m)+\beta_j\Pi_j^\dagger(m)$$
$$=\frac{1}{9}(\beta_i+\beta_j)(a-c+E_\theta(m))(a-c+3\theta-2E_\theta(m)). \tag{13}$$

The investor cannot improve its payoff by sending other messages in the equilibrium. Thus, for any message m that the investor observing θ can send in the equilibrium, $V^\dagger(m)\geq V^\dagger(m')$ should be satisfied for any observable message m' under $\mu^\dagger$.

Lemma 1 states that the firms have the same expectation on θ regardless of messages along the equilibrium path.

Lemma 1 *In any equilibrium* $(\mu^\dagger,\sigma_1^\dagger,\sigma_2^\dagger)$, $E_\theta(m)=\mathbb{E}[\theta\mid\mu^\dagger(\theta)=m]$ *is uniquely determined as* $E_\theta(m)=\overline{\theta}$ *for any* $m\in\overline{M}(\mu^\dagger)$.

Proof. Suppose that $E_\theta(m)>E_\theta(m')$ for some $m\in\overline{M}(\mu^\dagger)$ and $m'\in\overline{M}(\mu^\dagger)$. Consider the investor who observes θ such that $m\in\operatorname{supp}(\mu^\dagger(\theta))$. For m and m', define the investor's payoffs $V^\dagger(m)$ and $V^\dagger(m')$ as in (13), respectively. Then, (2) implies that

$$V^\dagger(m)-V^\dagger(m')$$
$$=\frac{1}{9}(\beta_i+\beta_j)(E_\theta(m')-E_\theta(m))(2E_\theta(m')+2E_\theta(m)+a-c-3\theta)<0. \tag{14}$$

That is, the investor observing θ has an incentive to send message m', which contradicts that $\mu^\dagger$ is an equilibrium strategy. Thus, $E_\theta(m)=E_\theta(m')$ holds for any $m\in\overline{M}(\mu^\dagger)$ and $m'\in\overline{M}(\mu^\dagger)$. In addition, since

$$\overline{\theta}=\mathbb{E}[\mathbb{E}[\theta\mid\mu^\dagger(\theta)=m]\mid m\in\overline{M}(\mu^\dagger)]=\mathbb{E}[E_\theta(m)\mid m\in\overline{M}(\mu^\dagger)] \tag{15}$$

holds, we have $E_\theta(m)=\overline{\theta}$ for any $m\in\overline{M}(\mu^\dagger)$. ■

Lemma 1 implies that the investor constantly sends an uninformative message in any equilibrium. The investor cares about the (weighted) joint profit of the firms, while each firm cares about its own profit. In addition, the production level in a Cournot competition is higher than the production level maximizing the joint profit of the firms. Thus, the investor may have an incentive to lie to make the firms believe that the market condition is bad. Indeed, this is possible if the firms change their belief on market condition depending on the investor's messages. However, once the firms recognize the investor's incentive to lie, they will not completely trust the messages sent by the investor. If the uncertainty in market demand is sufficiently large, then sending the firms an overly negative message on the market condition has a negative effect on the investors and firms. Hence, the investor may send a message from which the firms are partially informed of market conditions. However, the condition in (2) rules out this possibility.

The condition in (2) used in the proof of Lemma 1 is stronger than required. The condition for Lemma 1 to hold is that there does not exist $\boldsymbol{\theta} = (\theta_1, \dots, \theta_{K-1}) \in \mathbb{R}_+^{K-1}$ satisfying $0 \equiv \theta_0 < \theta_1 < \dots < \theta_{K-1} \le \theta_K \equiv \hat{\theta}$ and, for each $k = 1, \dots, K-1$,

$$\theta_k = \frac{2(\overline{\theta}_k + \overline{\theta}_{k+1})}{3} + \frac{a-c}{3}, \tag{16}$$

where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \le \theta \le \theta_k]$. Similar arguments in Section 4 can be applied to verify this condition. Note that (2) ensures the non-existence of $\boldsymbol{\theta} \in \mathbb{R}_+^{K-1}$ satisfying (16). In addition, many distributions on $[0, \hat{\theta}]$ for which there does not exist a partition $\boldsymbol{\theta} = (\theta_1, \dots, \theta_{K-1}) \in \mathbb{R}_+^{K-1}$ of $[0, \hat{\theta}]$ satisfying (16) even if (2) is not satisfied. The uniform distribution on $[0, \hat{\theta}]$ is an example of such distributions.

Proposition 1 shows the existence of equilibrium that has the property in Lemma 1 and the uniqueness of such an equilibrium in terms of the payoffs.[9]

Proposition 1 *There exists an equilibrium* $(\mu^\dagger, \sigma_1^\dagger, \sigma_2^\dagger)$ *such that, for any* $m \in \overline{M}(\mu^\dagger)$,

$$\mathbb{E}[\theta \mid \mu^\dagger(\theta) = m] = \overline{\theta} \quad \text{and} \tag{17}$$

$$\sigma_1^\dagger(m) = \sigma_2^\dagger(m) = \frac{1}{3}(a-c) + \frac{1}{3}\overline{\theta}. \tag{18}$$

In addition, every equilibrium $(\mu^\dagger, \sigma_1^\dagger, \sigma_2^\dagger)$ *satisfies (17) and (18).*

Proof. Consider the investor's strategy $\mu^\dagger$, such that for all θ, $\mu^\dagger(\theta)$ is uniformly distributed on $[0, \hat{\theta}]$. Note that $\overline{M}(\mu^\dagger) = [0, \hat{\theta}]$. Since, for any $m \in [0, \hat{\theta}]$, $\mathbb{E}[\theta \mid \mu^\dagger(\theta) = m] = \overline{\theta}$ holds, the investor does not have an incentive to deviate from $\mu^\dagger$. Given that $\mathbb{E}[\theta \mid \mu^\dagger(\theta) = m] = \overline{\theta}$ for any $m \in [0, \hat{\theta}]$, (10) implies that $\sigma_i^\dagger(m)$ in (18) is an equilibrium between the firms after they observe any message $m \in [0, \hat{\theta}]$. This proves the first assertion. The second assertion follows from Lemma 1 and (10). ■

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[9] Since Lemma 1 uses the condition in (2), Proposition 1 also relies on the condition in (2) to be established.

Note that the equilibrium payoffs depend on the firms' outputs and not on the investor's messages. Thus, if the output is uniquely determined in equilibria, then the equilibrium payoffs are uniquely determined. In the proof of Proposition 1, we construct an investor's equilibrium strategy $\mu^{\dagger}$, such that the investor constantly sends a random message that is uniformly distributed on $[0,\hat{\theta}]$. We note that $\mu^{\dagger}$ is not the only equilibrium strategy for the investor. For example, a strategy μ^{o} in which the investor consistently sends the same message regardless of the realization of θ constitutes an equilibrium.

From Proposition 1, we can obtain each firm i and the investor's (ex-post) equilibrium payoffs as follows:

$$U_i^{\dagger} = \frac{1}{9}(1-\beta_i)(a-c+\overline{\theta})(a-c+3\theta-2\overline{\theta}) \tag{19}$$

$$V^{\dagger} = \frac{1}{9}(\beta_i+\beta_j)(a-c+\overline{\theta})(a-c+3\theta-2\overline{\theta}). \tag{20}$$

From (19) and (20), we also obtain each firm i and the investor's ex-ante equilibrium payoffs as follows:

$$\mathbb{E}[U_i^{\dagger}] = \frac{1}{9}(1-\beta_i)(a-c+\overline{\theta})^2 \tag{21}$$

$$\mathbb{E}[V^{\dagger}] = \frac{1}{9}(\beta_i+\beta_j)(a-c+\overline{\theta})^2. \tag{22}$$

IV. Private Information Transmission Channel

This section considers the situation, in which the investor uses a private channel to transmit its private information. Private channel means that the investor sends a message to only one firm, say firm i. Given that only firm i receives the investor's messages, firm i's strategy is a function $\sigma_i : [0,\hat{\theta}] \to \mathbb{R}_+$, while firm j's strategy is a scalar $\sigma_j \in \mathbb{R}_+$. Backward induction is used to find a perfect Bayesian equilibrium $(\mu^{\ddagger},\sigma_1^{\ddagger},\sigma_2^{\ddagger})$.

Consider firm i's problem in Stage 2, in which the investor plays an equilibrium strategy $\mu^{\ddagger}$ and firm i observes message $m \in \overline{M}(\mu^{\ddagger})$. Given that firm i has a belief consistent with $\mu^{\ddagger}$, firm i's equilibrium strategy $\sigma_i^{\ddagger}(m)$ has to maximize

$$\mathbb{E}[u_i(q_i,\sigma_j^{\ddagger};\theta) \mid \mu^{\ddagger}(\theta)=m] = \mathbb{E}[(1-\beta_i)(a-c+\theta-q_i-\sigma_j^{\ddagger})q_i \mid \mu^{\ddagger}(\theta)=m] \tag{23}$$

with respect to $q_i \in \mathbb{R}_+$. In addition, given that firm j does not receive any message from the investor, firm j's equilibrium strategy $\sigma_j^{\ddagger}$ has to maximize

$$\mathbb{E}[u_j(\sigma_i^{\ddagger}(m), q_j; \theta)] = \mathbb{E}[(1-\beta_j)(a-c+\theta-\sigma_i^{\ddagger}(m)-q_j)q_j] \tag{24}$$

with respect to $q_j \in \mathbb{R}_+$. The first-order necessary conditions for problems (23) and (24) imply that

$$\sigma_i^{\ddagger}(m) = \frac{1}{2}(a-c)+\frac{1}{2}\mathbb{E}[\theta \mid \mu^{\ddagger}(\theta)=m]-\frac{1}{2}\sigma_j^{\ddagger} \tag{25}$$

$$\sigma_j^{\ddagger} = \frac{1}{2}(a-c)+\frac{1}{2}\overline{\theta}-\frac{1}{2}\mathbb{E}[\sigma_i^{\ddagger}(m)]. \tag{26}$$

Taking an expectation on both sides of (25), we have

$$\mathbb{E}[\sigma_i^{\ddagger}(m)] = \frac{1}{2}(a-c)+\frac{1}{2}\overline{\theta}-\frac{1}{2}\sigma_j^{\ddagger}. \tag{27}$$

From (25), (26), and (27), we obtain each firm's equilibrium strategy $(\sigma_i^{\ddagger}, \sigma_j^{\ddagger})$ as for each $m \in \overline{M}(\mu^{\ddagger})$,

$$\sigma_i^{\ddagger}(m) = \frac{1}{3}(a-c)-\frac{1}{6}\overline{\theta}+\frac{1}{2}E_{\theta}(m) \tag{28}$$

$$\sigma_j^{\ddagger} = \frac{1}{3}(a-c)+\frac{1}{3}\overline{\theta}, \tag{29}$$

where $E_{\theta}(m) = \mathbb{E}[\theta \mid \mu^{\ddagger}(\theta)=m]$.

For the investor's optimal choice in Stage 1, suppose that firm i plays $\sigma_i^{\ddagger}$ in (28) and firm j plays $\sigma_j^{\ddagger}$ in (29). Given that the investor sends a message $m \in \overline{M}(\mu^{\ddagger})$, firm i and firm j's (ex-post) profits can be calculated as follows:

$$\begin{aligned}\Pi_i^{\ddagger}(m) &= (a-c+\theta-\sigma_i^{\ddagger}(m)-\sigma_j^{\ddagger})\sigma_i^{\ddagger}(m) \\ &= \frac{1}{36}(2(a-c)+6\theta-\overline{\theta}-3E_{\theta}(m))(2(a-c)-\overline{\theta}+3E_{\theta}(m)) \qquad (30)\\ \Pi_j^{\ddagger}(m) &= (a-c+\theta-\sigma_i^{\ddagger}(m)-\sigma_j^{\ddagger})\sigma_j^{\ddagger} \\ &= \frac{1}{36}(2(a-c)+6\theta-\overline{\theta}-3E_{\theta}(m))(2(a-c)+2\overline{\theta}). \qquad (31)\end{aligned}$$

Thus, firm i and firm j's (ex-post) payoffs are as follows:

$$\begin{aligned} U_i^{\ddagger}(m) &= (1-\beta_i)\Pi_i^{\ddagger}(m) \\ &= \frac{1}{36}(1-\beta_i)(2(a-c)+6\theta-\overline{\theta}-3E_\theta(m))(2(a-c)-\overline{\theta}+3E_\theta(m)) \end{aligned} \tag{32}$$

$$\begin{aligned} U_j^{\ddagger}(m) &= (1-\beta_j)\Pi_j^{\ddagger}(m) \\ &= \frac{1}{36}(1-\beta_j)(2(a-c)+6\theta-\overline{\theta}-3E_\theta(m))(2(a-c)+2\overline{\theta}). \end{aligned} \tag{33}$$

In addition, the investor observing θ receives the (ex-post) payoff of

$$\begin{aligned} V^{\ddagger}(m) &= \beta_i\Pi_i^{\ddagger}(m)+\beta_j\Pi_j^{\ddagger}(m) \\ &= \frac{1}{36}(2(a-c)+6\theta-\overline{\theta}-3E_\theta(m)) \\ &\quad \times(\beta_i(2(a-c)-\overline{\theta}+3E_\theta(m))+\beta_j(2(a-c)+2\overline{\theta})) \end{aligned} \tag{34}$$

by sending message $m\in\overline{M}(\mu^{\ddagger})$. If the investor's strategy $\mu^{\ddagger}$ constitutes an equilibrium, then for each $\theta\in[0,\hat{\theta}]$, any $m\in\text{supp}(\mu^{\ddagger}(\theta))$ and $m'\in\overline{M}(\mu^{\ddagger})$ satisfy $V^{\ddagger}(m)\geq V^{\ddagger}(m')$. That is, the investor observing θ maximizes its payoff by sending a message $m\in\text{supp}(\mu^{\ddagger}(\theta))$.

For the investor's equilibrium strategy $\mu^{\ddagger}$, let $\mathcal{E}_\theta(\mu^{\ddagger})$ be the set of expectations on θ that can be formed from the observable messages under $\mu^{\ddagger}$. That is,

$$\mathcal{E}_\theta(\mu^{\ddagger})=\{\mathbb{E}[\theta\,|\,\mu^{\ddagger}(\theta)=m]:m\in\overline{M}(\mu^{\ddagger})\}. \tag{35}$$

Lemma 2 *For the investor's equilibrium strategy* $\mu^{\ddagger}$, $\mathcal{E}_\theta(\mu^{\ddagger})$ *in (35) is finite.*

Proof. Let $\mu^{\ddagger}$ be the investor's equilibrium strategy. For convenience, for message $m\in\overline{M}(\mu^{\ddagger})$, let $E_\theta(m)=\mathbb{E}[\theta\,|\,\mu^{\ddagger}(\theta)=m]$ be the expectation on θ that can be formed under $\mu^{\ddagger}$ given that m is observed. Consider $m\in\overline{M}(\mu^{\ddagger})$ and $m'\in\overline{M}(\mu^{\ddagger})$ with $E_\theta(m')\neq E_\theta(m)$. This means that $E_\theta(m')$ and $E_\theta(m)$ are two different elements in $\mathcal{E}_\theta(\mu^{\ddagger})$. Without loss of generality, let $E_\theta(m')<E_\theta(m)$. Note that $m\in\overline{M}(\mu^{\ddagger})$ implies the nonemptyness of the set $\{\theta\,|\,m\in\text{supp}(\mu^{\ddagger}(\theta))\}$. Given that $E_\theta(m)=\mathbb{E}[\theta\,|\,\mu^{\ddagger}(\theta)=m]$ takes an expectation of θ on the set $\{\theta\,|\,m\in\ \text{supp}(\mu^{\ddagger}(\theta))\}$, $m\in\overline{M}(\mu^{\ddagger})$ implies the existence of $\theta^{o}\in[0,\hat{\theta}]$ satisfying $m\in\ \text{supp}(\mu^{\ddagger}(\theta^{o}))$ and $\theta^{o}\leq E_\theta(m)$. Given that $(\mu^{\ddagger},\sigma_1^{\ddagger},\sigma_2^{\ddagger})$ is an equilibrium, we can see from (34) that

$$V^{\ddagger}(m)-V^{\ddagger}(m')$$

$$= \frac{\beta_i}{2}(E_\theta(m') - E_\theta(m))\left(\frac{E_\theta(m') + E_\theta(m)}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right) - \theta^o\right) \geq 0 . \tag{36}$$

Given that $E_\theta(m') < E_\theta(m)$ and $\theta^o \leq E_\theta(m)$, (36) implies

$$\frac{2\beta_j(a-c+\overline{\theta})}{3\beta_i} \leq E_\theta(m) - E_\theta(m') . \tag{37}$$

Since $m \in \overline{M}(\mu^\ddagger)$ and $m' \in \overline{M}(\mu^\ddagger)$ are arbitrary and satisfy $0 \leq E_\theta(m') < E_\theta(m) \leq \hat{\theta}$, (37) implies the result. ■

Lemma 2 implies that for any equilibrium $(\mu^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$, the collection of expectations on θ under the belief consistent with $\mu^\ddagger$ can be represented as for some $K \in \mathbb{N}$, $\mathcal{E}_\theta(\mu^\ddagger) = \{\overline{\theta}_1, \ldots, \overline{\theta}_K\}$ with $\overline{\theta}_1 < \cdots < \overline{\theta}_K$. In particular, $\overline{\theta}_k \in \mathcal{E}_\theta(\mu^\ddagger)$ means that for some $\theta \in [0, \hat{\theta}]$, $\overline{\theta}_k = E_\theta(m)$ for some $m \in \text{supp}(\mu^\ddagger(\theta))$.

Moreover, there is no equilibrium in which the investor plays a truth-telling strategy μ^t and in which the investor constantly completely reveals its private information on θ (that is, $\mu^t(\theta) = \theta$ for any θ). That is, the investor does not completely reveal its private information in any equilibrium. Note that this situation does not rule out the possibility of equilibrium, in which the investor partially reveals its information to firm i. Proposition 2 shows the existence of such equilibria and characterizes them.

Proposition 2 *Let* $(\mu^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$ *be an equilibrium with* $\mathcal{E}_\theta(\mu^\ddagger) = \{\overline{\theta}_1, \ldots, \overline{\theta}_K\}$. *For each* $k = 1, \ldots, K-1$, *let*

$$\theta_k = \frac{\overline{\theta}_k + \overline{\theta}_{k+1}}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right). \tag{38}$$

Then, the following holds:

(a) $0 \equiv \theta_0 < \theta_1 < \cdots < \theta_{K-1} \leq \theta_K \equiv \hat{\theta}$, *and for each* $k = 1, \ldots, K$,

$$\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k]. \tag{39}$$

(b) *There are disjoint subsets* $M_1, \ldots, M_K$ *of* $[0, \hat{\theta}]$, *such that for each* k, $\theta \in (\theta_{k-1}, \theta_k)$ *implies* $\text{supp}(\mu^\ddagger(\theta)) \subset M_k$.

(c) *For message* $m \in M_k$,

$$\sigma_i^{\ddagger}(m) = \frac{1}{3}\left(a-c\right) - \frac{1}{6}\overline{\theta} + \frac{1}{2}\overline{\theta}_k \text{ and } \sigma_j^{\ddagger} = \frac{1}{3}(a-c) + \frac{1}{3}\overline{\theta}. \tag{40}$$

In addition, if there exists $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K-1}) \in \mathbb{R}_+^{K-1}$ *satisfying (38) and (39), then an equilibrium satisfying (b) and (c) exists.*[10]

Proof. The proof of the first assertion is found in the Appendix. For the proof of the second assertion, we here provide an equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$. Suppose that $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K-1})$ satisfies (38) and (39). To construct an equilibrium satisfying (b) and (c), let $\{\Theta_1, \ldots, \Theta_K\}$ be a partition of $[0, \hat{\theta}]$ such that $\Theta_1 = [\theta_0, \theta_1]$ and $\Theta_k = (\theta_{k-1}, \theta_k]$ for each $k = 2, \ldots, K$. Then, there are disjoint closed intervals $M_1, \ldots, M_K$ such that for each k, $M_k \subset (\theta_{k-1}, \theta_k)$. Consider the investor's strategy $\mu^{\ddagger}$ such that for any $\theta \in \Theta_k$, $\mu^{\ddagger}(\theta)$ is uniformly distributed on M_k. Under the belief consistent with $\mu^{\ddagger}$, firm i receiving message $m \in M_k$ believes that θ is distributed with a conditional density $f(\theta \mid \theta \in \Theta_k)$, and so it has an expectation on θ as $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \le \theta \le \theta_k]$. When firm i receives a message $m \notin \bigcup_{k=1}^{K} M_k$, it forms an expectation of θ as $\overline{\theta}_1$.[11] Then, the arguments used to obtain (29) and (28) show that $(\sigma_i^{\ddagger}, \sigma_j^{\ddagger})$ in (40) maximizes each firm i's (*resp.* firm j's) expected payoff given that the other firm j plays $\sigma_j^{\ddagger}$ (*resp.* the other firm i plays $\sigma_i^{\ddagger}$). In addition, for messages $m \in M_k$ and $m' \in M_{k'}$ with $k' \neq k$, it is easy to see from (34) that

$$V^{\ddagger}(m) - V^{\ddagger}(m') = \frac{\beta_i}{2}(\overline{\theta}_{k'} - \overline{\theta}_k)\left(\frac{\overline{\theta}_{k'} + \overline{\theta}_k}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right) - \theta\right). \tag{41}$$

Consider $\theta \in \Theta_k$. Note that $\theta_{k-1} \le \theta \le \theta_k$ holds. If $\overline{\theta}_{k'} < \overline{\theta}_k$, then (38) implies

$$\begin{aligned} &\frac{\overline{\theta}_{k'} + \overline{\theta}_k}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right) - \theta \\ &\le \frac{\overline{\theta}_{k-1} + \overline{\theta}_k}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right) - \theta = \theta_{k-1} - \theta \le 0. \end{aligned} \tag{42}$$

If $\overline{\theta}_{k'} > \overline{\theta}_k$, then (38) implies

[10] If $K = 1$, then the conditions (38) and (39) are redundant for the existence of equilibrium.

[11] The perfect Bayesian equilibrium does not require consistency of belief off the equilibrium path. Thus, firm i can have any belief when it receives message $m \notin \bigcup_{k=1}^{K} M_k$.

$$\frac{\bar{\theta}_{k'}+\bar{\theta}_k}{2}+\frac{\beta_j}{\beta_i}\left(\frac{a-c+\bar{\theta}}{3}\right)-\theta$$
$$\geq \frac{\bar{\theta}_{k+1}+\bar{\theta}_k}{2}+\frac{\beta_j}{\beta_i}\left(\frac{a-c+\bar{\theta}}{3}\right)-\theta=\theta_k-\theta\geq 0\,. \tag{43}$$

Thus, $V^{\ddagger}(m)-V^{\ddagger}(m')\geq 0$ holds. This means that the investor observing $\theta\in\Theta_k$ does not have an incentive to deviate from $\mu^{\ddagger}$. ■

In the equilibrium described in Proposition 2, the investor creates a partition of $[0,\hat{\theta}]$ consisting of intervals with threshold $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_{K-1})$ and sends messages that inform the interval to which a realization of θ belongs. In Proposition 2, (a) provides the properties of threshold for the partition of $[0,\hat{\theta}]$ in the equilibrium. (b) describes the investor's strategy. When the investor observes $\theta\in(\theta_{k-1},\theta_k)$, it sends a (random) message whose outcomes belong to M_k.[12] For example, the investor observing $\theta\in(\theta_{k-1},\theta_k)$ sends a random message that is uniformly distributed on a closed interval $M_k\subset(\theta_{k-1},\theta_k)$. Since M_k's are disjoint, firm i receiving a message $m\in M_k$ can determine the interval containing θ. Once firm i is informed that θ belongs to (θ_{k-1},θ_k), its expectation on θ is formed as in (39) under the belief consistent with the investor's strategy. Lastly, the equilibrium strategies for the firms are described in (c).

Proposition 2 also implies that the equilibria with the same $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_{K-1})$ are outcome equivalent in that they yield the same outcome. To prove this result, note that the distribution of outcomes in an equilibrium depends on the distribution of $\bar{\theta}_k$. From (c) in Proposition 2, given the distribution of θ, the distribution of $\bar{\theta}_k$ depends only on and so does the distribution of the outcomes.

An equilibrium $(\mu^{\ddagger},\sigma_1^{\ddagger},\sigma_2^{\ddagger})$ in Proposition 2 is referred to as a *partition equilibrium with size* K.[13] K represents the number of elements in the partition of $[0,\hat{\theta}]$ that is generated by $\mu^{\ddagger}$. With a slight abuse of notation, a partition of $[0,\hat{\theta}]$ is denoted by $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_{K-1})\in\mathbb{R}_+^{K-1}$, where an element of partition $\boldsymbol{\theta}$ is a set Θ_k satisfying $(\theta_{k-1},\theta_k)\subset\Theta_k\subset[\theta_{k-1},\theta_k]$.

There may be multiple partition equilibria with the same size but having different information partitions. However, if the distribution of θ satisfies a relevant condition, then every partition equilibrium with size K yields the unique information partition and so the unique outcome in terms of payoff.[14]

[12] It is possible that, in an equilibrium, the investor observing $\theta=\theta_k$ sends (random) messages $m\in M_k\cup M_{k+1}$ because it is indifferent on the outcomes with $\bar{\theta}_k$ and $\bar{\theta}_{k+1}$.

[13] A partition equilibrium is initially suggested by Crawford and Sobel (1982).

[14] The condition for the uniqueness of information partition in equilibrium is that, for any $x,x',y,y'\in[0,\bar{\theta}]$ with $x\neq x'$ and $y\neq y'$, $|\mathbb{E}[\theta\mid x\leq\theta\leq y']-\mathbb{E}[\theta\mid x\leq\theta\leq y]|<|y'-y|$ and $|\mathbb{E}[\theta\mid x'\leq\theta\leq y]-\mathbb{E}[\theta\mid x\leq\theta\leq y]|<|x'-x|$ hold. Uniform distribution is an example for the

A partition equilibrium with size 1 is an *uninformative equilibrium* in the sense that the investor's message does not change the firm's prior belief on θ. In addition, we refer to a partition equilibrium with size $K \geq 2$ as an *informative equilibrium* because the investor's message transmits information that θ belongs to a proper subset of $[0,\hat{\theta}]$. Proposition 3 provides the conditions for the existence of an informative equilibrium.

Proposition 3 *For each* (β_i,β_j)*, there exists an integer* $K(\beta_i,\beta_j)\in\mathbb{N}$*, such that for any* $K \leq K(\beta_i,\beta_j)$*, there exists a partition equilibrium with size* K*. In addition, if* $(3/2)(\hat{\theta}/(a-c+\overline{\theta}))<(\beta_j/\beta_i)$*, then* $K(\beta_i,\beta_j)=1$*; if* $(\beta_j/\beta_i)<(3/2)(\hat{\theta}/(a-c+\overline{\theta}))$*, then* $K(\beta_i,\beta_j)\geq 2$*.*

Proof. See Appendix.[15] ■

In Proposition 3, a condition for the existence of an informative partition equilibrium is that the investor's share of firm i is sufficiently large compared with its share of firm j. If the investors's share of firm i is sufficiently small compared with that of firm j, then an informative equilibrium does not exist. The relative ratio of β_i and β_j matters for the existence of informative equilibrium. A decrease in β_j/β_i can be interpreted as firm i's profit becomes important to the investor's portfolio, thereby resulting in the investor's interest becoming substantially similar to that of firm i. Thus, as β_j/β_i decreases, the investor may become willing to share its information with firm i.

If there is a partition equilibrium with size $K \geq 2$, a partition equilibrium with size $K' < K$ can also be constructed. That is, if there is an equilibrium in which the investor partially reveals its information, there is also an equilibrium in which the investor reveals less information. Proposition 3 also implies the existence of uninformative equilibrium with $K=1$ regardless of (β_i,β_j).

The condition for the existence of an equilibrium informative to firm i means that the investor's share of firm j is sufficiently small compared with its share of firm i. This implies the next proposition.

Proposition 4 *Given* (β_i,β_j)*, if there exists an informative equilibrium in which the investor sends a message only to firm* i*, there does not exist an informative*

distributions satisfying this condition.

[15] Some proofs, including the present one, have similar features to those in Cho (2013), in which a fixed point argument is used. However, the validity of the proofs depends on the specific form of the sender's payoff function. Cho (2013) extends the model suggested by Crawford and Sobel (1982) to a situation with two senders (local governments) and one receiver (central government). Since the payoff functions in our model are different from those in Cho (2013), the results in this paper cannot be directly obtained through the proofs in Cho (2013).

equilibrium in which the investor sends a message only to firm j.

Proof. Suppose that there exists a partition equilibrium with size $K \geq 2$ when the investor sends messages only to firm i. Proposition 3 and (2) imply

$$\frac{\beta_j}{\beta_i} \leq \frac{3}{2}\left(\frac{\hat{\theta}}{a-c+E_\theta}\right) < \frac{1}{2} < \frac{\beta_i}{\beta_j}. \tag{44}$$

Thus, we obtain the result.[16] ■

Consider a partition equilibrium $(\mu^{\ddagger},\sigma_1^{\ddagger},\sigma_2^{\ddagger})$ with partition $\boldsymbol{\theta}=(\theta_1,\ldots,\theta_{K-1}) \in \mathbb{R}_+^{K-1}$. From (32), (33), and (34), when the investor observes $\theta \in (\theta_{k-1},\theta_k)$, firm i, firm j, and the investor's (ex-post) payoffs are as follows, respectively:

$$U_i^{\ddagger} = \frac{1}{36}(1-\beta_i)(2(a-c)+6\theta-\overline{\theta}-3\overline{\theta}_k)(2(a-c)-\overline{\theta}+3\overline{\theta}_k) \tag{45}$$

$$U_j^{\ddagger} = \frac{1}{36}(1-\beta_j)(2(a-c)+6\theta-\overline{\theta}-3\overline{\theta}_k)(2(a-c)+2\overline{\theta}) \tag{46}$$

$$\begin{aligned} V^{\ddagger} &= \frac{1}{36}(2(a-c)+6\theta-\overline{\theta}-3\overline{\theta}_k) \\ &\quad \times(\beta_i(2(a-c)-\overline{\theta}+3\overline{\theta}_k)+\beta_j(2(a-c)+2\overline{\theta})), \end{aligned} \tag{47}$$

where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k]$. Taking expectations on (45), (46), and (47), we derive firm i, firm j, and the investor's ex-ante payoffs are as follows, respectively:[17]

$$\mathbb{E}[U_i^{\ddagger}] = \frac{1}{9}(1-\beta_i)(a-c+\overline{\theta})^2 + \frac{1}{4}(1-\beta_i)\mathbb{E}[(\overline{\theta}_k-\overline{\theta})^2] \tag{48}$$

$$\mathbb{E}[U_j^{\ddagger}] = \frac{1}{9}(1-\beta_j)(a-c+\overline{\theta})^2 \tag{49}$$

$$\mathbb{E}[V^{\ddagger}] = \frac{1}{9}(\beta_i+\beta_j)(a-c+\overline{\theta})^2 + \frac{1}{4}\beta_i\mathbb{E}[(\overline{\theta}_k-\overline{\theta})^2], \tag{50}$$

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[16] The second inequality in (44) relies on the condition in (2). As explained in Section 3, it may be easier to obtain an informative equilibrium if the condition in (2) is not satisfied. That is, violating the condition in (2), the upper bound of β_j / β_i for the existence of informative equilibrium increases and may be greater than 1/2. If this happens, there can be an informative equilibrium in both situations, in which the investor sends a message only to firm i and only to firm j.

[17] In deriving (48), (49), and (50), notice that $\mathbb{E}[\overline{\theta}_k\theta]=\mathbb{E}[\overline{\theta}_k^2]$, where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k]$.

where $\mathbb{E}[(\bar{\theta}_k - \bar{\theta})^2]$ is the variance of the conditional expectation $\bar{\theta}_k$. We can compare the ex-ante payoffs between an uninformative equilibrium and an informative equilibrium, in which the investor partially reveals its information.

Proposition 5 *The investor and firm i get higher ex-ante payoffs at an informative equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ compared with an uninformative equilibrium $(\mu'^{\ddagger}, \sigma_i'^{\ddagger}, \sigma_j'^{\ddagger})$. Firm j receives the same ex-ante payoff at the informative equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ and the uninformative equilibrium $(\mu'^{\ddagger}, \sigma_i'^{\ddagger}, \sigma_j'^{\ddagger})$.*

Proof. Since $\mathbb{E}[(\bar{\theta}_k - \bar{\theta})^2] = \mathbb{E}[(\bar{\theta} - \bar{\theta})^2] = 0$ for an uninformative equilibrium $(\mu'^{\ddagger}, \sigma_i'^{\ddagger}, \sigma_j'^{\ddagger})$ and $\mathbb{E}[(\bar{\theta}_k - \bar{\theta})^2] > 0$ for a partition equilibrium with size $K \geq 2$, the results follow from (48), (49), and (50). ■

In Section 3, it is shown that when the investor sends messages to both firms using a public channel of information transmission, every equilibrium $(\mu^{\dagger}, \sigma_1^{\dagger}, \sigma_2^{\dagger})$ is uninformative. Thus, the outcome of the equilibrium $(\mu^{\dagger}, \sigma_1^{\dagger}, \sigma_2^{\dagger})$ is the same as the outcome of the uninformative equilibrium $(\mu'^{\ddagger}, \sigma_i'^{\ddagger}, \sigma_j'^{\ddagger})$ in Proposition 5. This result directly leads to Corollary 1.

Corollary 1 *The investor and firm i receive a higher ex-ante payoffs at an informative equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ in the situation in which the investor sends messages only to firm i than at an equilibrium $(\mu^{\dagger}, \sigma_1^{\dagger}, \sigma_2^{\dagger})$ in the situation that the investor publicly sends messages to the firms. Firm j receives the same ex-ante payoff at an informative equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ and at an equilibrium $(\mu^{\dagger}, \sigma_1^{\dagger}, \sigma_2^{\dagger})$.*

Corollary 1 implies that the investor will not be worse off using the private channel compared with the public channel of information transmission.[18] Thus, the investor (weakly) prefers the private channel to the public channel of information transmission and will choose to use a private channel if afforded the option. Firm i also prefers the private channel to the public channel of information transmission. Thus, the investor can easily obtain firm i's agreement on sharing private information between themselves. Since firm j is indifferent between the private and public channels of information transmission, utilizing a private channel will improve the Pareto efficiency (among the investor and the firms) in view of the ex-ante payoffs.[19]

We next discuss the effects of β_i and β_j on the investor's payoff. In particular,

[18] Eliaz and Forges (2015) also find that the private communication rather than public communication is optimal to the planner who cares about the joint profit of the firms. However, their result relies on the planner's commitment on the truthfulness of messages.

[19] In Section 5, we also discuss that the use of private channel to transmit information improves the consumer surplus.

we are interested in how the investor's payoff is affected by the change in relative share of the firms. To answer the question, we restrict (β_i, β_j) to satisfy $\beta_i + \beta_j = \overline{\beta}$ for some $\overline{\beta} > 0$. Because the firms are ex-ante identical, this restriction can be interpreted as a fixed total amount of investment in the firms. Under this assumption, increasing β_i can be interpreted as the investor making a larger investment in firm i than in firm j. Because the investor's optimal decisions are invariant with $\overline{\beta}$, we assume without loss of generality that

$$\beta_i + \beta_j = 1. \tag{51}$$

Consider a partition equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ with partition $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K-1}) \in \mathbb{R}_+^{K-1}$ for $K \geq 2$. Under the condition in (51), the investor's ex-ante payoff at $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ becomes as follows:

$$\mathbb{E}[V^{\ddagger}] = \frac{1}{9}(a - c + \overline{\theta})^2 + \frac{1}{4}\beta_i \mathbb{E}[(\overline{\theta}_k - \overline{\theta})^2]. \tag{52}$$

Proposition 6 shows how $\mathbb{E}[V^{\ddagger}]$ changes as β_i increases.

Proposition 6 *Suppose that for* $\overline{\beta}_i \in (0,1)$*, there exists a partition equilibrium with size* $K \geq 2$*. Then, for any* $\beta_i \in [\overline{\beta}_i, 1)$*, there exists a partition equilibrium with the same size* K*. In addition, the investor's ex-ante payoff increases as* β_i *increases from* $\overline{\beta}_i$.

Proof. See the Appendix. ■

Proposition 6 states that the existence of a partition equilibrium with size K ensures the existence of a partition equilibrium with the same size K when the investor increases its share of firm i. Thus, $K(\beta_i, 1-\beta_i)$, the maximum size of equilibrium partitions, is non-decreasing in β_i. Because the larger size of the equilibrium partition can be interpreted as the investor revealing more of its private information, this observation coincides with the following intuition: when the investor has a large share of firm i, the optimal outputs between the investor and firm i become similar, and the investor's incentive to conceal its private information is reduced.

Proposition 6 also states that given a size of equilibrium partition, the investor's ex-ante payoff increases as it invests more in firm i.[20] The investor's payoff

[20] If the distribution of θ satisfies the condition in footnote 12, every partition equilibrium with size K is unique in terms of payoffs. If this is the case, Proposition 6 also implies that the investor is better off in every partition equilibrium with the same size.

depends on the variance of the conditional expectation on the market conditions multiplied by the investor's share of firm i. An increase in the investor's share of firm i reduces the difference in the preferences between the investor and firm i. Thus, the investor with a greater share of firm i is willing to reveal its private information more accurately to firm i. This reduces the variance of the conditional expectation on the market conditions and increases the ex-ante payoff to firm i and the investor.[21]

V. Discussion

In the previous sections, we focus on the investor's incentive to share its private information with firms and the effect of the investor's information sharing on the payoffs. One may be interested in how the consumer surplus and social welfare are affected by the investor's information sharing with firms. Given a market output Q in an equilibrium, the price p is determined by the market demand in (1). Thus, the consumer surplus can be defined as follows:

$$CS = \int_0^Q (a+\theta-Q-p)\,dQ = \frac{1}{2}Q^2. \tag{53}$$

From Propositions 1 and 2, given a realization of $\theta \in (\theta_{k-1}, \theta_k)$, the market output $Q^\dagger$ at an uninformative equilibrium $(\mu^\dagger, \sigma_1^\dagger, \sigma_2^\dagger)$ under a public channel and the market output $Q^\ddagger$ at an informative equilibrium $(\mu^\ddagger, \sigma_1^\ddagger, \sigma_2^\ddagger)$ under a private channel are as follows, respectively:

$$Q^\dagger = \frac{2}{3}(a-c) + \frac{2}{3}\overline{\theta} \tag{54}$$

$$Q^\ddagger = \frac{2}{3}(a-c) + \frac{1}{6}\overline{\theta} + \frac{1}{2}\overline{\theta}_k, \tag{55}$$

where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \le \theta \le \theta_k]$. The ex-ante consumer surplus at each equilibrium is determined as follows, respectively:

$$\mathbb{E}[CS^\dagger] = \mathbb{E}\left[\frac{1}{2}(Q^\dagger)^2\right] = \frac{2}{9}(a-c-\overline{\theta})^2 \tag{56}$$

[21] One may be interested in the effect of β_i on firm i's and firm j's ex-ante payoffs. From (48) and (49), we can easily see that firm j's ex-ante payoff increases as β_i increases, but it is not clear whether an increase in β_i improves firm i's ex-ante payoff.

$$\mathbb{E}[CS^{\ddagger}] = \mathbb{E}\left[\frac{1}{2}(Q^{\ddagger})^2\right] = \frac{2}{9}(a-c-\overline{\theta})^2 + \frac{1}{8}\mathbb{E}[(\overline{\theta}_k - \overline{\theta})^2]. \tag{57}$$

Since $\mathbb{E}[CS^{\dagger}] < \mathbb{E}[CS^{\ddagger}]$, the informative equilibrium is beneficial to the demand side (consumers) and supply side (the firms and the investor). In addition, the investor's sharing information with a firm improves the social welfare that consists of surplus on the demand and the supply sides.

This study considers two methods for the investor to transmit information. The investor using a public channel sends the same message to both firms or the investor using a private channel sends a message only to one firm (firm i). However, the investor may also send a message $m_1 \in [0, \hat{\theta}]$ to firm 1 and another message $m_2 \in [0, \hat{\theta}]$ to firm 2. In this case, the investor's strategy can be represented as a function μ that assigns a pair of (random) messages $(\mu_1(\theta), \mu_2(\theta))$ to each $\theta \in [0, \hat{\theta}]$, where $\mu_i(\theta)$ is a (random) message that the investor observing θ sends to firm i. After the investor sends messages to the firms, each firm simultaneously decides its output. Evidently, there is also an equilibrium in this situation, but we have some difficulties in characterizing the equilibrium.

For example, let $(\mu^o, \sigma_1^o, \sigma_2^o)$ be an equilibrium and consider the situation that each firm receives a message from the investor and has to choose the output. Suppose that the investor's strategy $\mu^o = (\mu_1^o, \mu_2^o)$ generates different partitions on $[0, \hat{\theta}]$ for each firm. Let $\overline{\Theta}_i$ be the information partition on $[0, \hat{\theta}]$ generated by μ_i^o. For each θ, let $\Theta_i(\theta)$ be an element of $\overline{\Theta}_i$ to which θ belongs. If the meet of $\overline{\Theta}_1$ and $\overline{\Theta}_2$ is the partition containing only $[0, \hat{\theta}]$, the firms fail to form a common belief on θ except that θ belongs to $[0, \hat{\theta}]$.[22] For message $m_i \in \text{supp}(\mu_i(\theta))$, firm i forms a belief that θ belongs to some $\Theta_{ik} \in \overline{\Theta}_i$. With a slight abuse of notation, for message $m_i \in \text{supp}(\mu_i(\theta))$, let $\Theta_i(m_i)$ be an element of $\overline{\Theta}_i$ to which θ belongs under the belief of firm i receiving m_i from the investor. We can find an equilibrium strategy (σ_1^o, σ_2^o) for the firms as follows. Given a message $m_i \in \text{supp}(\mu_i(\theta))$, for each t, let $K_i^t(m_i)$ be firm i's t-order conditional expectation on θ, which is defined sequentially as follows:

$$\begin{aligned}
&K_i^1(m_i) = \mathbb{E}[\theta \mid \Theta_i(m_i)], \\
&K_i^2(m_i) = \mathbb{E}[\mathbb{E}[\theta \mid \overline{\Theta}_j] \mid \Theta_i(m_i)], \\
&K_i^3(m_i) = \mathbb{E}[\mathbb{E}[\mathbb{E}[\theta \mid \overline{\Theta}_i] \mid \overline{\Theta}_j] \mid \Theta_i(m_i)], \\
&\qquad\qquad\vdots
\end{aligned} \tag{58}$$

Firm i's equilibrium strategy σ_i^o has to satisfy that for each $m \in \text{supp}(\mu_i^o(\theta))$:

[22] For partitions $\overline{\Theta}_1$ and $\overline{\Theta}_2$ of $[0, \hat{\theta}]$, the *meet* of $\overline{\Theta}_1$ and $\overline{\Theta}_2$ is the finest common coarsening of $\overline{\Theta}_1$ and $\overline{\Theta}_2$.

$$\sigma_i^o(m_i) = \frac{1}{3}(a-c) + \frac{1}{2}\sum_{t=1}^{\infty}\left(-\frac{1}{2}\right)^{t-1} K_i^t(m_i). \tag{59}$$

Note that if the meet of $\overline{\Theta}_1$ and $\overline{\Theta}_2$ is the partition containing only $[0,\hat{\theta}]$, it is possible that $K_i^t(m_i)$, $t = 1,2,\ldots$, have different values. In addition, the investor's payoff by sending a message $m_i \in \text{supp}(\mu_i^o(\theta))$ also depends on $K_i^t(m_i)$, $t = 1,2,\ldots$. To find the condition for the investor not to deviate from μ_i^o, we may have to explicitly determine how $K_i^t(m_i)$ depends on messages m_i. However, this appears to be a difficult task.

Although we cannot fully characterize the equilibria, we can find some properties of the equilibria for the situation that the investor sends separate messages to each firm. First, there does not exist an equilibrium $(\mu^o, \sigma_1^o, \sigma_2^o)$, in which the investor's strategy is informative and generates the same information partition for the firms. If σ_i^o and σ_j^o generate the same information partition on $[0,\hat{\theta}]$, the firms with a belief consistent with μ^o have a common belief on the distribution of θ. The arguments similar to those proving Lemma 2 can be applied to verify the assertion. Second, there is an equilibrium $(\mu^o, \sigma_1^o, \sigma_2^o)$ in which μ_i^o is informative and μ_j^o is uninformative. For μ_i^o to be informative, the condition $(\beta_j/\beta_i) < (3/2)((\hat{\theta} - \overline{\theta})/(a-c+\overline{\theta}))$ in Proposition 3 should hold. Then, one can show that the investor's strategy $\mu^o = (\mu_i^o, \mu_j^o)$ such that μ_i^o is the same as $\mu^{\ddagger}$ in Proposition 2 and, for each θ, $\mu_j^o(\theta)$ is uniformly distributed on $[0,\hat{\theta}]$ constitutes an equilibrium. In this case, firm i's strategy σ_i^o is the same as $\sigma_i^{\ddagger}$ in (40) and firm j's strategy σ_j^o is $\sigma_j^o(m_j) = (1/3)(a-c) + (1/3)\overline{\theta}$ for any m_j. Arguments similar to those in Section 4 can be applied to prove this assertion. We also note that there is an uninformative equilibrium $(\mu^o, \sigma_1^o, \sigma_2^o)$ in which the investor constantly sends random messages $\mu_i^o(\theta)$ and $\mu_j^o(\theta)$ that are independently and uniformly distributed on $[0,\hat{\theta}]$.

The results of this paper can be extended to the situation with more than two firms. Suppose that there are n firms in the market and the market demand is given by $p = a + \theta - \sum_{k=1}^{n} q_k$, where θ is private information to the investor and $a-c$ is sufficiently large compared with the uncertainty in the market condition. Each firm i's payoff and the investor's payoff are defined similarly to (4) and (5). The investor sends a common message to a subset of the firms, which can be the set containing all firms or a set containing only one firm. The firms are assumed to know who receives the message from the investor. In this environment, we can also characterize the equilibrium. Similar arguments in Section 3 show that when the investor sends a message to more than one firm, there is an equilibrium in which the investor sends an uninformative message, and the equilibrium is unique in terms of the outcome of the game. In addition, when the investor sends a message only to firm i and the investor's share of firm i is sufficiently large compared with its shares of the other firms, there exist equilibria in which the investor sends an informative message. In these equilibria, the investor's strategy generates a partition on $[0,\hat{\theta}]$ that has a finite number of elements. The properties of these equilibria are similar to those of the equilibria in Section 4.

VI. Conclusion

This paper studies the incentives of an investor to disclose its private information on market conditions to the firms in an oligopoly. Because there is a discrepancy in the optimum level of outputs between the investor and the firms, the investor has an incentive to use its private information strategically. In particular, we consider two cases of investor's information transmission to the firms. In the first case, the investor publicly provides its information to the firms. In this case, the investor does not reveal any of its information to the firms in any equilibrium. This result occurs because the discrepancy in the optimum level of outputs between the investor and the firms is so large that the investor cannot increase its payoff by manipulating the transmission of its private information. In the second case, the investor privately transmits its information to a specific firm. In this case, there can be an equilibrium in which the investor partially reveals its information to the firm. Such an equilibrium exists when the investor invests relatively more in a firm than another firm. In addition, the payoffs of the investor and the firm receiving information from the investor are improved in an equilibrium with the investor's partial revelation of information compared with an equilibrium, in which the investor does not reveal any of its information.

In this paper, we consider the model in which the investor's shares of the firms are exogenously given. One may be interested in the situation in which the investor and the firms decide the investor's share of the firms through a bargaining procedure. As discussed in Section 4, when the investor transmits its information privately to firm i, an increase in the investor's share of firm i improves the investor's (ex-ante) payoff given that the total amount of investment is fixed. However, an increase in the investor's share of firm i can decrease firm i's payoff, although firm i can take advantage of the investor's private information. Thus, some bargaining procedure can play a role in determining the investor's share of firm i. Our model also assumes that the investor transmits its information either publicly or privately. However, it sounds more realistic that the investor provides a different quality of information to the firms using both public and private channels. For example, the investor makes a public report on market conditions and provides private consultancy services to its invested firms. Studies on the behavior of the investor and firms in such environments are left for future research.

In addition, the firms in this paper compete with each other in a Cournot oligopoly and produce perfect substitutes. This means that the firm's actions are strategic substitutes. However, we can consider the situation, in which the firm's actions are strategic complements because they produce complementary goods. As see in the previous studies (e.g., Angeletos and Pavan (2007) and Eliaz and Serrano (2014)), the effects of public and private information on social welfare, which is defined as the sum of the individual payoffs, may be different depending on whether the individual's actions are strategic substitutes or strategic complements. The approach in this paper can be used to study how the investor's incentive to transmit its information is affected by whether the firms produces substitutes or complements when the information is not verifiable. However, various limitations have prevented us from analyzing it in thisstudy. Thus, we leave it for future research.

Appendix

Proof of Proposition 2. For each $k=1,\ldots,K$, let $M_k=\{m\in\bar{M}(\mu^{\ddagger}):\mathbb{E}[\theta\,|\,\mu^{\ddagger}(\theta)=m]=\bar{\theta}_k\}$ be a set of messages that induce expectation $\bar{\theta}_k$ for θ under the investor's strategy $\mu^{\ddagger}$ and let $\Theta_k=\{\theta:\text{supp}(\mu^{\ddagger}(\theta))$ has an element $m\in M_k\}$ be a set of θ that can induce expectation $\bar{\theta}_k$ under $\mu^{\ddagger}$. Note that $\{M_1,\ldots,M_K\}$ is a partition of $\bar{M}(\mu^{\ddagger})$ and $\bigcup_{k=1}^{K}\Theta_k=[0,\hat{\theta}]$.

For messages $m\in M_k$ and $m'\in M_{k'}$ with $k'\neq k$, $V^{\ddagger}(m)-V^{\ddagger}(m')$ can be represented as (41). If the investor observes $\theta\in\Theta_k\cap\Theta_{k'}$, then it constantly sends messages that yield the same expected payoff under $\mu^{\ddagger}$. This means that $V^{\ddagger}(m)-V^{\ddagger}(m')=0$. Since $\bar{\theta}_k\neq\bar{\theta}_{k'}$, (41) implies that $\theta\in\Theta_k\cap\Theta_{k'}$ satisfies

$$\theta=\frac{\bar{\theta}_{k'}+\bar{\theta}_k}{2}+\frac{\beta_j}{\beta_i}\left(\frac{a-c+\bar{\theta}}{3}\right). \tag{60}$$

This means that there is at most one element in $\Theta_k\cap\Theta_{k'}$.

For the investor not to deviate from $\mu^{\ddagger}$, for any $\theta\in\Theta_k$, $V^{\ddagger}(m)-V^{\ddagger}(m')\geq 0$ should be satisfied for any $m\in M_k$ and $m'\in M_{k'}$ with $k\neq k'$. Since $\bar{\theta}_1<\cdots<\bar{\theta}_K$ holds, $V^{\ddagger}(m)-V^{\ddagger}(m')\geq 0$ implies that for each $k=1,\ldots K-1$,

$$\sup\Theta_k\leq\frac{\bar{\theta}_k+\bar{\theta}_{k+1}}{2}+\frac{\beta_j}{\beta_i}\left(\frac{a-c+\bar{\theta}}{3}\right)\leq\inf\Theta_{k+1}. \tag{61}$$

Given that $\bigcup_{k=1}^{K}\Theta_k=[0,\hat{\theta}]$ holds and $\Theta_k\cap\Theta_{k+1}$ has at most one element, (61) implies that for each $k=1,\ldots,K-1$,

$$\sup\Theta_k=\inf\Theta_{k+1}=\frac{\bar{\theta}_k+\bar{\theta}_{k+1}}{2}+\frac{\beta_j}{\beta_i}\left(\frac{a-c+\bar{\theta}}{3}\right)=\theta_k. \tag{62}$$

This implies $(\theta_{k-1},\theta_k)\subset\Theta_k\subset[\theta_{k-1},\theta_k]$. Since

$$\begin{aligned}\bar{\theta}_k&=\mathbb{E}[\bar{\theta}_k|m\in M_k]=\mathbb{E}[\mathbb{E}[\theta\,|\,\mu^{\ddagger}(\theta)=m]\,|\,m\in M_k]\\&=\mathbb{E}[\theta\,|\,m\in M_k]=\mathbb{E}[\theta\,|\,\theta_{k-1}\leq\theta\leq\theta_k]\end{aligned} \tag{63}$$

holds, (a) is proved.

Suppose that for some $\theta\in(\theta_{k-1},\theta_k)$, there is $m\in\text{supp}(\mu^{\ddagger}(\theta))$ that does not belong to M_k. Since the collection of M_k forms a partition of $\bar{M}(\mu^{\ddagger})$, m belongs to $M_{k'}$ with $k'\neq k$. This implies $\theta\in\Theta_{k'}$, which is a contradiction.

Thus, $m \in M_k$ holds. This proves (b). For (c), apply the argument used in obtaining (29) and (28) . ■

Proof of Proposition 3. For $K \geq 2$, let

$$Y_{K-1} = \{(\theta_1, \ldots, \theta_{K-1}) \in \mathbb{R}^{K-1} : 0 \equiv \theta_0 \leq \theta_1 \leq \cdots \leq \theta_{K-1} \leq \theta_K \equiv \hat{\theta}\}. \tag{64}$$

Let $G_{K-1} : Y_{K-1} \to \mathbb{R}^{K-1}$ be a function defined by, for $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K-1}) \in Y_{K-1}$,

$$G_{K-1,k}(\boldsymbol{\theta}) = \frac{\overline{\theta}_k + \overline{\theta}_{k+1}}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a - c + \overline{\theta}}{3}\right) \tag{65}$$

for each $k = 1, \ldots, K-1$, where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k]$. Y_{K-1} is compact and convex. If G_{K-1} has a fixed point in Y_{K-1} for some $K \geq 2$, let $K(\beta_i, \beta_j)$ be the maximum of such K's. Otherwise, let $K(\beta_i, \beta_j) = 1$. A fixed point $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K-1})$ of G_{K-1} satisfies that for each k,

$$\theta_{k-1} + \frac{\beta_j}{\beta_i}\left(\frac{a - c + \overline{\theta}}{3}\right) \leq \frac{\overline{\theta}_k + \overline{\theta}_{k+1}}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a - c + \overline{\theta}}{3}\right) = \theta_k. \tag{66}$$

This implies

$$K \leq \frac{\beta_i}{\beta_j}\left(\frac{3}{a - c + \overline{\theta}}\right)\hat{\theta}. \tag{67}$$

Thus, $K(\beta_i, \beta_j)$ exists. The existence of the partition equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ with size K follows from Proposition 2.

Suppose that G_{K-1} has a fixed point $\boldsymbol{\theta}^* = (\theta_1^*, \ldots, \theta_{K-1}^*)$ for $K \geq 3$. Let $K' = K - 1$ and let

$$\overline{Y}_{K'-1} = \left\{(\theta_1, \ldots, \theta_{K'-1}) \in \mathbb{R}^{K'-1} : \begin{array}{l} 0 \equiv \theta_0 \leq \theta_1 \leq \cdots \leq \theta_{K'} \equiv \hat{\theta}, \quad \text{and} \\ \theta_k^* \leq \theta_k \leq \theta_{k+1}^* \quad \text{for } k = 1, \ldots, K'-1 \end{array}\right\}. \tag{68}$$

$\overline{Y}_{K'-1}$ is compact and convex. In addition, $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K'-1}) \in \overline{Y}_{K'-1}$ implies that for $k = 1, \ldots, K'-1$,

$$\theta_k^* = \frac{\mathbb{E}[\theta \mid \theta_{k-1}^* \leq \theta \leq \theta_k^*] + \mathbb{E}[\theta \mid \theta_k^* \leq \theta \leq \theta_{k+1}^*]}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a - c + \overline{\theta}}{3}\right)$$

$$\leq \frac{\mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k] + \mathbb{E}[\theta \mid \theta_k \leq \theta \leq \theta_{k+1}]}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right)$$

$$\leq \frac{\mathbb{E}[\theta \mid \theta_k^* \leq \theta \leq \theta_{k+1}^*] + \mathbb{E}[\theta \mid \theta_{k+1}^* \leq \theta \leq \theta_{k+2}^*]}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right) = \theta_{k+1}^*. \quad (69)$$

This means that $G_{K'-1}(\boldsymbol{\theta}) \in \overline{Y}_{K'-1}$ for each $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K'-1}) \in \overline{Y}_{K'-1}$. Then, the continuity of $G_{K'-1}$ on $\overline{Y}_{K'-1}$ implies the existence of fixed point $\boldsymbol{\theta}^{**} \in \overline{Y}_{K'-1}$ for $G_{K'-1}$. Thus, Proposition 2 establishes the existence of partition equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ with size $K < K(\beta_i, \beta_j)$.

We can see from (67) that if $(3/2)(\hat{\theta}/(a-c+\overline{\theta})) < (\beta_j/\beta_i)$, $K < 2$ holds. This implies $K(\beta_i, \beta_j) = 1$. To find a condition for $K(\beta_i, \beta_j) \geq 2$, let $\Gamma : [0, \hat{\theta}] \to \mathbb{R}_+$ be a function defined by

$$\Gamma(\theta_1) = \frac{\mathbb{E}[\theta \mid 0 \leq \theta \leq \theta_1] + \mathbb{E}[\theta \mid \theta_1 \leq \theta \leq \hat{\theta}]}{2} + \frac{\beta_j}{\beta_i}\left(\frac{a-c+\overline{\theta}}{3}\right) - \theta_1. \quad (70)$$

Note that $(\beta_j/\beta_i) < (3/2)(\hat{\theta}/(a-c+\overline{\theta}))$ implies $\Gamma(\hat{\theta}) < 0$ and $\Gamma(0) > 0$. Thus, the continuity of Γ ensures the existence of $\theta_1 \in [0, \hat{\theta}]$ satisfying $\Gamma(\theta_1) = 0$. Then, Proposition 2 implies the existence of partition equilibrium $(\mu^{\ddagger}, \sigma_1^{\ddagger}, \sigma_2^{\ddagger})$ with size 2. ■

Lemma 3 is useful in proving Proposition 6,.

Lemma 3 *If, for* $\beta_i' \in (0,1)$ *there is an equilibrium* $(\mu'^{\ddagger}, \sigma_1'^{\ddagger}, \sigma_2'^{\ddagger})$ *with a partition* $\boldsymbol{\theta}' = (\theta_1', \ldots, \theta_{K-1}')$, *then for any* $\beta_i'' \in (\beta_i', 1)$, *there exists an equilibrium* $(\mu''^{\ddagger}, \sigma_1''^{\ddagger}, \sigma_2''^{\ddagger})$ *with a partition* $\boldsymbol{\theta}'' = (\theta_1'', \ldots, \theta_{K-1}'')$ *satisfying* $\theta_k'' \leq \theta_k'$ *for each* k.

Proof. For $\beta_i' \in (0,1)$, let $(\mu'^{\ddagger}, \sigma_1'^{\ddagger}, \sigma_2'^{\ddagger})$ be an equilibrium in which $\mu^{\ddagger}$ generates an information partition $\boldsymbol{\theta}' = (\theta_1', \ldots, \theta_{K-1}')$ on $[0, \hat{\theta}]$. We define a set $\tilde{Y}_{K-1}$ as follows:

$$\tilde{Y}_{K-1} = \left\{ (\theta_1, \ldots, \theta_{K-1}) \in \mathbb{R}^{K-1} : \begin{array}{l} 0 \equiv \theta_0 \leq \theta_1 \leq \cdots \leq \theta_K \equiv \hat{\theta} \quad \text{and} \\ \theta_k \leq \theta_k' \quad \text{for each} \quad k = 1, \ldots, K-1 \end{array} \right\}. \quad (71)$$

$\tilde{Y}_{K-1}$ is compact and convex. For $\beta_i'' \in (\beta_i', 1)$, let $G_{K-1}'' : \tilde{Y}_{K-1} \to \mathbb{R}^{K-1}$ be a function defined by for $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{K-1}) \in \tilde{Y}_{K-1}$,

$$G''_{K-1,k}(\boldsymbol{\theta}) = \frac{\overline{\theta}_k + \overline{\theta}_{k+1}}{2} + \frac{1-\beta''_i}{\beta''_i}\left(\frac{a-c+\overline{\theta}}{3}\right) \tag{72}$$

for each k, where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k]$. Then, for each $\boldsymbol{\theta} \in \tilde{Y}_{K-1}$, it is satisfied that for each k,

$$\begin{aligned} G''_{K-1,k}(\boldsymbol{\theta}) &= \frac{\overline{\theta}_k + \overline{\theta}_{k+1}}{2} + \frac{1-\beta''_i}{\beta''_i}\left(\frac{a-c+\overline{\theta}}{3}\right) \\ &\leq \frac{\overline{\theta}'_k + \overline{\theta}'_{k+1}}{2} + \frac{1-\beta'_i}{\beta'_i}\left(\frac{a-c+\overline{\theta}}{3}\right) = \theta'_k, \end{aligned} \tag{73}$$

where $\overline{\theta}_k = \mathbb{E}[\theta \mid \theta_{k-1} \leq \theta \leq \theta_k]$ and $\overline{\theta}'_k = \mathbb{E}[\theta \mid \theta'_{k-1} \leq \theta \leq \theta'_k]$. This implies that $G''_{K-1}(\boldsymbol{\theta}) \in \tilde{Y}_{K-1}$ for each $\boldsymbol{\theta} \in \tilde{Y}_{K-1}$. Since G'_{K-1} is continuous on $\tilde{Y}_{K-1}$, G''_{K-1} has a fixed point $\boldsymbol{\theta}''$ in $\tilde{Y}_{K-1}$. Then, Proposition 2 implies the result. ■

Proof of Proposition 6. The existence of partition equilibrium with size K follows from Lemma 3. For each $\beta_i \in [\overline{\beta}_i, 1)$, let $\boldsymbol{\theta}(\beta_i) = (\theta_1(\beta_i), \ldots, \theta_{K-1}(\beta_i)) \in \mathbb{R}^{K-1}$ be a partition of $[0, \hat{\theta}]$ generated by an equilibrium. For each $k = 1, \ldots, K$, let $\overline{\theta}_k(\beta_i)$ $= \mathbb{E}[\theta \mid \theta_{k-1}(\beta_i) \leq \theta \leq \theta_k(\beta_i)]$, where $\theta_0(\beta_i) = 0$ and $\theta_K(\beta_i) = \hat{\theta}$. Note that

$$\begin{aligned} &\frac{d\mathbb{E}[\overline{\theta}_k(\beta_i)^2]}{d\beta_i} \\ &= \frac{d}{d\beta_i}\left(\sum_{k=1}^{K}\int_{\theta_{k-1}(\beta_i)}^{\theta_k(\beta_i)} \overline{\theta}_k(\beta_i)^2 f(\theta) d\theta\right) \\ &= \sum_{k=1}^{K}(\theta_k(\beta_i)^2 f(\theta_{k-1}(\beta_i)) - 2\overline{\theta}_k(\beta_i)\theta_{k-1}(\beta_i) f(\theta_{k-1}(\beta_i)))\frac{d\theta_{k-1}(\beta_i)}{d\beta_i} \\ &\quad -\sum_{k=1}^{K}(\overline{\theta}_k(\beta_i)^2 f(\theta_k(\beta_i)) - 2\overline{\theta}_k(\beta_i)\theta_k(\beta_i) f(\theta_k(\beta_i)))\frac{d\theta_k(\beta_i)}{d\beta_i} \\ &= \sum_{k=1}^{K-1}(\overline{\theta}_{k+1}(\beta_i)^2 f(\theta_k(\beta_i)) - 2\overline{\theta}_{k+1}(\beta_i)\theta_k(\beta_i) f(\theta_k(\beta_i)))\frac{d\theta_k(\beta_i)}{d\beta_i} \\ &\quad -\sum_{k=1}^{K-1}(\overline{\theta}_k(\beta_i)^2 f(\theta_k(\beta_i)) - 2\overline{\theta}_k(\beta_i)\theta_k(\beta_i) f(\theta_k(\beta_i)))\frac{d\theta_k(\beta_i)}{d\beta_i} \\ &= \sum_{k=1}^{K-1}\left((\overline{\theta}_{k+1}(\beta_i) - \overline{\theta}_k(\beta_i))((\overline{\theta}_{k+1}(\beta_i) + \overline{\theta}_k(\beta_i)) - 2\theta_k(\beta_i)) f(\theta_k(\beta_i))\frac{d\theta_k(\beta_i)}{d\beta_i}\right). \end{aligned} \tag{74}$$

The third equality in (74) comes from $(d\theta_0 / d\beta_i)(\beta_i) = 0$ and $(d\theta_K / d\beta_i)(\beta_i) = 0$. Lemma 3 implies that for each k, $(d\theta_k / d\beta_i)(\beta_i) < 0$ holds for each $\beta_i \in (\overline{\beta}_i, 1)$. In addition, for each k $\overline{\theta}_{k+1}(\beta_i) - \overline{\theta}_k(\beta_i) > 0$ and $(\overline{\theta}_{k+1}(\beta_i) + \overline{\theta}_k(\beta_i)) - 2\theta_k(\beta_i) < 0$ hold. Hence, we have $d\mathbb{E}[\overline{\theta}_k(\beta_i)^2] / d\beta_i > 0$.

Let $V^{\ddagger}(\beta_i)$ be the investor's (ex-post) payoff at the partition equilibrium for β_i. Note that

$$\mathbb{E}[V^{\ddagger}(\beta_i)] = \frac{1}{9}(a - c + \overline{\theta})^2 + \frac{1}{4}\beta_i(\mathbb{E}[\overline{\theta}_k(\beta_i)^2] - \overline{\theta}^2). \tag{75}$$

Since $\mathbb{E}[\overline{\theta}_k(\beta_i)^2] - \overline{\theta}^2 = \mathbb{E}[\overline{\theta}_k(\beta_i) - (\overline{\theta})^2] \geq 0$ holds, we obtain

$$\frac{d\mathbb{E}[V^{\ddagger}(\beta_i)]}{d\beta_i} = \frac{1}{4}\left((\mathbb{E}[\overline{\theta}_k(\beta_i)^2] - \overline{\theta}^2) + \beta_i \frac{d(\mathbb{E}[\overline{\theta}_k(\beta_i)^2]}{d\beta_i} \right) > 0. \tag{76}$$

This completes the proof. ■

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