*The Korean Economic Review*
Volume 35, Number 2, Summer 2019, 345-370

# Endogenous Timing with a Socially Responsible Firm*

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*This study considers a duopoly model in which a socially responsible (SR) firm competes with a private firm by incorporating environmental externality and clean technology. We analyze the endogenous market structure where both firms strategically, sequentially, or simultaneously decide quantities, which also affect abatement activities. We reveal that depending on the relative concerns on environment and consumer surplus, the SR firm can be less or more aggressive in production and abatement, and it may earn high profits. Thus, not only the significance of externality but also the instrumental conflict of social concerns are crucial factors in determining the equilibrium of endogenous timing game. Finally, we indicate that unless the concern for externality is high, the simultaneous and sequential move game with SR firm leadership are socially desirable.*

JEL Classification: L13, L31, Q5
Keywords: Endogenous Timing, Socially Responsible Firm, Environmental Externality

## I. Introduction

Conventional economic theory regards firms as entities whose sole objective is to maximize their profits. In the real world, however, many private firms have voluntarily and increasingly paid attention to corporate social responsibility (CSR).[1]

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*Received: Feb. 3, 2018. Revised: June 19, 2018. Accepted: Aug. 31, 2018.*

* We express our sincere gratitude to two anonymous referees for their helpful comments that helped improve the quality of the manuscript. This work is supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea (NRF-2017S1A5B8059731).

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[1] Many companies have participated in greenhouse gas reduction programs, issued various statements on CSR, and outlined activities in their annual reports. For example, see CSR trend report by PricewaterhouseCoopers (2010) and KPMG (2015).

Due to the expansion of CSR in most countries, many industries are characterized by the concurrent presence of for-profit (FP) and not-FP firms. Such presence indicates that the heterogeneity of objectives among firms emerges as an important research topic in the literature.[2]

The recent topic on CSR has received increasing attention from broad research on empirical and theoretical economics.[3] Many studies have also formulated theoretical approaches on CSR in the field of applied microeconomic theory.[4] Different competition models of oligopolies, where profit-maximizing firms compete with their rival firms that adopt CSR activities, are analyzed. Consumer surplus is also regarded as a proxy of firms' CSR concerns, in which CSR initiatives include profitability and consumer surplus. However, these approaches set aside the concern on environmental problem, which is becoming an essential part of CSR behavior; thus, it is a realistic representation of how CSR firms operate.[5]

In the formulation of the objective of CSR-initiated firms, we define a socially responsible (SR) firm that considers its profits and social concerns, which include not only consumer surplus but also environmental externality. These two social concerns have opposite effects on production and abatement. The concern on environment restrains the production of an SR firm and increases abatement, whereas the concern on consumer surplus expands production and decreases costly abatement. Thus, the commitment to social concerns may allow the SR firm to include different production strategies, which induce the SR firm to be more or less aggressive than its its rival firm. Furthermore, the competition with the SR firm may lead to different market structures and firm behaviors, depending on the two social concerns.

Interest is subtantial in the recent theoretical literature on endogenous timing game, which asks when firms are likely to play either a simultaneous or a sequential move game. Contrary to the fixed timing game with the given order of output decisions, such as Cournot game or Stackelberg game, endogenous timing game is derived from firms' decisions such a game. Committing firm movement is beneficial

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[2] For example, Martin and Osberg (2007), Katz and Page (2010), and Besley and Ghatak (2017) analyzed social enterprises as entities which balance making profits with a social mission. Chirco et al. (2013) revealed that behavioral heterogeneity may be the equilibrium outcome of the strategic interaction of ex-ante identical agents, whereas Matsumura and Ogawa (2014) and Cho and Lee (2017) investigated that heterogeneity may produce different market structures.

[3] For the intensive discussions on the empirical works on CSR, see Schreck (2011) and Crifo and Forget (2015). Lyon and Maxwell (2004) and Kitzmueller and Shimshack (2012) provided fruitful discussions on the practical and academic issues on CSR.

[4] Recent theoretical research on CSR activities includes Goering (2012, 2014), Kopel and Brand (2012), Brand and Grothe (2013, 2015), Chang et al. (2014), Kopel (2015), and Matsumura and Ogawa (2014, 2017).

[5] Recent analysis has emphasized that environmental concern is critical in the recent CSR codes of conduct. See the discussions in Lambertini and Tampieri (2015), Liu et al. (2015), Hirose et al. (2017), and Lee and Park (2019).

only if rivals do not commit because doing so may be costly otherwise. Thus, a fundamental trade-off exists between flexibility and commitment. Firms may fear the severe costs associated with the resulting "Stackelberg war," as Schelling (1960) emphasized; thus, coordination for waiting strategies is important in the endogenous timing game.[6]

Hamilton and Slutsky (1990) introduced a pre-play period in which two firms endogenously determine the order of their moves prior to the actual choice of production. Endogenous timing game has an extended game with an observable delay, that is, firms first select the timing of taking their actions in a pre-play game, which announces a production period (one of two periods), and then firms produce in the announced sequence.[7] Two possible periods are available for output choice, and each firm selects its output from only one of the two periods. In the first period, each firm simultaneously decides between being the leader or the follower. If both firms' choices are different, then the equilibrium of a sequential move game is yielded under the agreed timing. Otherwise, the equilibrium in a simultaneous move game emerges.

In this study, we consider a duopoly model in which an SR firm with social concerns competes against an FP private firm by incorporating environmental externality and clean technology. We then analyze the endogenous market structure in an observable delay game in which the role of a leader or a follower is determined endogenously. We examine whether the SR firm strategically, sequentially, or simultaneously decides quantities, which also affect abatement activities.

Our model formulation has a few interesting features compared with the previous literature. First, in the literature on mixed markets in which public and private firms coexist, most works analyze the case that public firms assign the same weights to consumer surplus and environmental problems.[8] By contrast, our model allows an SR firm to be more or less concerned with consumer surplus or environmental damage. This situation is realistic in the context of the private initiatives of CSR activities. Second, certain studies consider the portion of environmental problems in the objectives of public firms, but they mostly examine

[6] Certain important theoretical contributions to the literature on endogenous timing game include Robson (1990), Mailath (1993), Ellingsen (1995), Amir (1995), van Damme and Hurkens (1999, 2004), Matsumura (1999), and Normann (2002).

[7] The practical questions in the observable delay game are relevant to new markets where two or more firms can enter the market, which has been intensively used in many contexts of game theory, industrial organization, and public economics. They also extend to other economics fields, such as contest theory (Baik and Shogren, 1992; Hoffmann and Rota-Graziosi, 2012), spatial voting in politics (Osborne, 1993; Huck et al., 2006), experimental studies (Fonseca et al., 2006; Santos-Pinto, 2008; Nosenzo and Sefton, 2011), and cognitive behavioral studies (Carvalho and Santos-Pinto, 2014).

[8] Pal (1998) introduced the analysis in mixed markets, thus recent works have extensively examined the observable delay game. Recent research includes Amir and De Feo (2014), Matsumura and Ogawa (2014, 2017), Naya (2015), Din and Sun (2016), and Lee and Xu (2018).

the fixed timing game under government regulations, such as emission taxes and emission standards.[9] We extend their works to the endogenous timing game without government intervention from the viewpoint of voluntary adoption of CSR. Finally, most previous studies on environmental problems analyze the end-of-pipe technology where the choices of abatement activities and outputs are technically independent, whereas we consider a clean technology which integrates the interdependence between the two choices.[10]

In the presence of clean technology, depending on the relative concerns on environment and consumer surplus, the SR firm can be less or more aggressive in production and abatement. The SR firm may achieve a high profit even though it adopts abatement activities in a quantity-setting competition. Thus, being an SR firm can be a way of strategically committing to its profitable output.[11] Therefore, not only the significance of externality but also the instrumental conflict of social concerns are crucial factors in determining the equilibrium of endogenous timing game and welfare consequences.

The following are the main findings which contribute to previous literature. Regarding environmental concern, when the SR firm does not desire consumer surplus, a simultaneous move game is a unique equilibrium regardless of how significant the externality is. This result is consistent with Hamilton and Slutsky (1990), who first formulated an observable delay game in a private duopoly between two homogeneous FP firms without externality. We reveal that the analysis of a simultaneous move game remains useful in a duopoly with an SR firm having high environmental concerns. In addition, a simultaneous move game emerges in equilibrium if the *ex-ante* initial level of pollution emission is not significant, and the SR firm's concern for consumer surplus is not high but entirely accounts for the environmental externality it solely causes. This result also includes the analysis of Matsumura and Ogawa (2017), who considered environmental externality in a mixed duopoly and showed that a simultaneous move game emerges in equilibrium when the externality is significant. The SR firm reduces output and always obtains lower profits than FP firm in the equilibrium.

By constrast, when the SR firm is significantly concerned with consumer surplus, two sequential move games may be equilibria. If the SR firm cares for the

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[9] For recent analysis, see Pal and Saha (2014, 2015), Xu et al. (2016), and Xu and Lee (2015, 2018).

[10] Regarding green technology with the end-of-pipe abatement, which is additively separable with production process, see the recent analysis and discussion in Lee and Park (2011, 2018), Kim and Lee (2014, 2016), and Kim et al. (2018).

[11] This interpretation is related to the managerial delegation contract with sales targets, in which firms have incentives when placing a high weight on output to induce rivals, thus reducing their outputs in quantity-setting oligopolies. Recently, Lambertini and Tampieri (2015), Hirose et al. (2017), and Lee and Park (2019) discussed its relevance in environmental CSR. The difference of our analysis with the managerial delegation contract is that the motivation of CSR in our setting is not always profit-oriented strategy.

environment and accounts entirely for consumer surplus, then two sequential movements are the equilibria regardless of the externality. This result is consistent with Pal (1998), who analyzed an observable delay game in a mixed duopoly between an FP firm and a welfare-maximizing public firm. These equilibria emerge no matter how significant the externality is. We also reveal that the SR firm increases output if it does not care for externality significantly; thus, the firm can obtain higher profits than its profit-seeking competitor in both leadership equilibria.[12] This result includes the analysis of Lambertini and Tampieri (2015), who considered a duopoly where an SR firm competes with FP firms in a Cournot game. Thus, their analysis holds in an endogenous choice game when the SR firm does not significantly account for consumer surplus regardless of the externality.

Finally, when the social concern on consumer surplus is not that high, the simultaneous move game is socially desirable only if the environmental concern is also relatively small. When the concern for consumer is high, SR leadership is payoff dominant (and risk dominant) and is socially desirable unless the environmental concern is high. However, when the concern for consumer is intermediate, FP leadership is payoff dominant (and risk dominant) but not socially desirable regardless of the environmental concern.

The remainder of this paper is organized in the following manner. In Section II, we formulate a Cournot duopoly model with an SR firm and an FP firm. In Sections III and IV, we analyze a fixed timing game and an endogenous timing game, respectively. In Section V, we examine illustrative cases and discuss equilibrium and welfare consequences. In Section VI, we conclude the discussion.

## II. Model

We consider two firms in a quantity-setting game. One of the firms is an SR firm, (hereafter referred to as firm 0). This firm not only maximizes its profits but also makes an effort to decrease the pollution generated by its production and cares for consumer surplus. The other is an FP firm (hereafter referred to as firm 1) that maximizes only its profits.

Both polluting firms produce homogeneous goods and compete in quantities. Firms 0 and 1 sell their output $q_0 \geq 0$ and $q_1 \geq 0$, respectively, at the market clearing price $p(Q) = 1 - Q$, where $Q = q_0 + q_1$. We assume that both firms have identical technologies, and the production cost function takes a quadratic form,[13]

[12] Recent literature on CSR also supports that firms can increase their profits under the strategic CSR. For example, see Kopel and Brand (2012), Brand and Grothe (2013, 2015), Liu et al. (2015), Lambertini and Tampieri (2015), and Lee and Park (2019).

[13] As Matsumura and Okamura (2015) indicated, the quadratic cost model in the literature on mixed oligopoly is popular, but the model may yield specific results compared with the constant

$c(q_i) = \frac{1}{2}q_i^2$, ($i = 0,1$).

Suppose that each unit of production generates $e$ pollution emissions. However, the SR firm can make an abatement effort $a$ per unit production to reduce pollution emissions. $e$ indicates how significant pollution externality is in an industry. For example, the emissions of the financial sector are not as substantial as those of the mining, textile, or clothing industries. We also assume that both products in the same industry emit the same type of *ex-ante* pollutants. Unlike other studies which consider end-of-pipe abatement technologies, we consider a cleaner production technology in our analysis.[14] The pollution generated by firm 0 after abatement effort is $E_0 = (e - a) \cdot q_0$. The expenditure function of this abatement effort is $\frac{a^2}{2}$. In practice, most pollutants cannot be decreased completely; therefore, we assume $e > a \geq 0$. Pollution abatement is costly, so profit-maximizing firm 1 makes no effort in the absence of environmental regulation; thus, its pollution is $E_1 = e \cdot q_1$.

The profit of SR firm is given by $\pi_0 = p \cdot q_0 - \frac{1}{2}q_0^2 - \frac{a^2}{2}$. We assume that the SR firm maximizes profits plus a fraction of consumer surplus $(CS)$, and as SR firm cares for the environment, it places a weight on the pollution that its production generates. Thus, the payoff that SR firm maximizes is as follows:

$$V_0 = \pi_0 - \gamma E_0 + \theta CS, \tag{1}$$

where $CS = \frac{Q^2}{2}$. Parameters $\theta \in [0,1]$ and $\gamma \in [0,1]$ measure the SR firm's degree of consumer and environmental concern, respectively. Both concerns are exogenously given.

FP firm only seeks profit maximization.

$$\pi_1 = p \cdot q_1 - \frac{1}{2}q_1^2 \tag{2}$$

The analysis of the observable two-stage delay game by Hamilton and Slutsky (1990) is considered. In the first stage, each firm simultaneously decides whether to move early or late. In the second stage, the game played is simultaneous if both firms select the same period and sequential otherwise.

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marginal cost model. In our analysis, the quadratic cost model assures interior solutions without loss of general economic insights.

[14] End-of-pipe technology refers to an equipment setup by a firm that can reduce gross pollution but can leave the firm's output unchanged. For example, see Wang and Wang (2009), Pal and Saha (2015), Xu et al. (2016), and Lee and Xu (2018). Clean technology involves a change in a firm's production process whereby it generates low pollution per output. Therefore, the output and abatement decisions of clean technology are intertwined. See Chiou and Hu (2001), Ulph and Ulph (2007), Jinji (2013), and Tsai et al. (2016).

# III. Fixed Timing Game

We first consider a fixed-timing game in which SR and FP firms compete in quantities in a simultaneous and sequential move game, respectively. In the following analysis, we assume that the three fixed timing games have interior solutions.

**Assumption**: For any $\gamma \in [0,1]$ and $\theta \in [0,1]$,

$$e \in (\underline{e}, \overline{e}) \text{, where } \underline{e} \equiv \max\left\{\frac{2\gamma(3+\theta)}{21-4\theta}, \frac{\gamma(2+\theta)}{2(4-\theta)}\right\} \text{ and}$$

$$\overline{e} \equiv \min\left\{\frac{5+2\theta-\theta^2-\gamma^2(2+\theta)}{\gamma(8-3\gamma^2-2\theta)}, \frac{2(3+\theta)}{9\gamma}\right\}.$$

## 3.1. Simultaneous Move Game

The SR firm independently selects its abatement effort level ($a$) and output ($q_0$), whereas FP firm chooses its production ($q_1$). To solve the first-order conditions for maximizing the payoffs of SR and FP firms in (1) and (2), respectively, we obtain the following equilibrium quantities and abatement level.

$$q_0^c = \frac{2-3e\gamma+\theta}{8-3\gamma^2-2\theta}, \quad a^c = \gamma \cdot q_0^c, \quad q_1^c = \frac{2+e\gamma-\gamma^2-\theta}{8-3\gamma^2-2\theta}, \tag{3}$$

where superscript "$c$" denotes the Cournot game. The equilibrium profits, payoffs, and welfare are as follows:

$$\begin{aligned}
\pi_0^c &= \frac{-\sigma_1+(2+\theta)(6-5\theta-\gamma^2(6+\theta))}{2(8-3\gamma^2-2\theta)^2} \\
V_0^c &= \frac{\sigma_2+12\theta+\gamma^4\theta-5\theta^2-\gamma^2(4+8\theta-\theta^2)+12}{2(8-3\gamma^2-2\theta)^2} \\
\pi_1^c &= \frac{3(2+e\gamma-\gamma^2-\theta)^2}{2(8-3\gamma^2-2\theta)^2} \\
W^c &= \frac{\sigma_3+2\gamma(2+\theta)^2-\gamma^2(32+2\theta+\theta^2)+2(20-8\theta-\theta^2)+4\gamma^4}{2(8-3\gamma^2-2\theta)^2},
\end{aligned} \tag{4}$$

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are as presented in Appendix A.[15]

[15] For expositional convenience, we provide $\sigma_i$ ($i=1,\ldots,8$) in Appendix A.

### 3.2. SR Firm as a Stackelberg Leader

Firm 0 selects its output and abatement levels, and then firm 1 sequentially chooses its output level. The following are equilibrium quantities and abatement level.

$$q_0^{sl} = \frac{6-9e\gamma+2\theta}{21-9\gamma^2-4\theta}, \quad a^{sl} = \gamma \cdot q_0^{sl}, \quad q_1^{sl} = \frac{5+3e\gamma-3\gamma^2-2\theta}{21-9\gamma^2-4\theta}, \tag{5}$$

where superscript " $sl$ " denotes the equilibrium outcome in the Stackelberg game with SR firm leadership. The resulting profits, payoffs, and welfare are as follows:

$$\begin{aligned}
\pi_0^{sl} &= \frac{\sigma_4 + 4(21-8\theta-5\theta^2-\gamma^2(27+12\theta+\theta^2))}{2(21-9\gamma^2-4\theta)^2} \\
V_0^{sl} &= \frac{9e^2\gamma^2-4e\gamma(3+\theta)+(5-\gamma^2)\theta+4}{42-18\gamma^2-8\theta} \\
\pi_1^{sl} &= \frac{3(5+3e\gamma-3\gamma^2-2\theta)^2}{2(21-9\gamma^2-4\theta)^2} \\
W^{sl} &= \frac{\sigma_5 + 4(70+9\gamma^4-23\theta-2\theta^2+2\gamma(3+\theta)^2-\gamma^2(66+3\theta+\theta^2))}{2(21-9\gamma^2-4\theta)^2}
\end{aligned} \tag{6}$$

### 3.3. FP Firm as a Stackelberg Leader

Firm 1 chooses its output level, and then firm 0 selects its output and abatement levels. The following are the equilibrium quantities and abatement level.

$$\begin{aligned}
q_0^{fl} &= \frac{5+2\theta-\theta^2-\gamma^2(2+\theta)-e\gamma(8-3\gamma^2-2\theta)}{(3-\gamma^2-\theta)(7-3\gamma^2-\theta)}, \\
a^{fl} &= \gamma \cdot q_0^{fl}, \quad q_1^{fl} = \frac{2+e\gamma-\gamma^2-\theta}{7-3\gamma^2-\theta},
\end{aligned} \tag{7}$$

where superscript " $fl$ " denotes the equilibrium outcome in the Stackelberg game with FP firm leadership. The resulting profits, payoffs, and welfare are as follows:

$$\begin{aligned}
\pi_0^{fl} &= \frac{-\sigma_6+\eta_1}{2(3-\gamma^2-\theta)^2(7-3\gamma^2-\theta)^2} \\
V_0^{fl} &= \frac{\sigma_7+\eta_2}{2(3-\gamma^2-\theta)(7-3\gamma^2-\theta)^2}
\end{aligned}$$

$$\pi_1^{fl} = \frac{(2 + e\gamma - \gamma^2 - \theta)^2}{2(3 - \gamma^2 - \theta)(7 - 3\gamma^2 - \theta)}$$

$$W^{fl} = \frac{\sigma_8 + \eta_3}{2(3 - \gamma^2 - \theta)^2 (7 - 3\gamma^2 - \theta)^2}, \tag{8}$$

where $\eta_1$, $\eta_2$, and $\eta_3$ are as presented in Appendix B.[16]

# IV. Endogenous Timing Game

We discuss the first-stage choice in an endogenous timing game. Each firm $i (i = 0,1)$ simultaneously decides whether to move early $(t_i = 1)$ or late $(t_i = 2)$. If both firms select the same period, the equilibrium is a simultaneous move game. Otherwise, the equilibrium is a sequential move game. Table 1 provides the payoff matrix of the observable delay game.

**[Table 1]** Payoff Matrix of the Observable Delay Game

| Firm $0/1$ | $t_1 = 1$ | $t_1 = 2$ |
|---|---|---|
| $t_0 = 1$ | $(V_0^c, \pi_1^c)$ | $(V_0^{sl}, \pi_1^{sl})$ |
| $t_0 = 2$ | $(V_0^{fl}, \pi_1^{fl})$ | $(V_0^c, \pi_1^c)$ |

Given that the payoff of a firm when it is the leader is never smaller than its payoff in the simultaneous move game, $V_0^{sl} \geq V_0^c$ and $\pi_1^{fl} \geq \pi_1^c$, and $(t_0, t_1) = (2,2)$ never constitutes an equilibrium unless $V_0^{sl} = V_0^c$ and $\pi_1^{fl} = \pi_1^c$. However, $V_0^{sl} = V_0^c$ and $\pi_1^{fl} = \pi_1^c$ never hold simultaneously. Under these conditions, the equilibrium outcomes are as follows:

(a) $(t_0, t_1) = (2,1)$ emerges as an equilibrium if $V_0^c \leq V_0^{fl}$.

(b) $(t_0, t_1) = (1,2)$ emerges as an equilibrium if $\pi_1^c \leq \pi_1^{sl}$.

(c) $(t_0, t_1) = (1,1)$ emerges as an equilibrium if $V_0^c \geq V_0^{fl}$ and $\pi_1^c \geq \pi_1^{sl}$.

Let $\overline{\theta}_0(\gamma) \equiv \frac{\eta_4}{2(7-3\gamma^2)}$, defined on the interval $[0,1]$, and $e_0(\theta,\gamma) \equiv \frac{\eta_5}{\gamma\eta_6}$, where $e_0 : [0,1] \times (0,1] \to \mathbb{R}$ monotonically decreases on $\theta$, such as $e_0(\theta,\gamma) > 0$ if $0 \leq \theta < \overline{\theta}_0$, $e_0(\overline{\theta}_0,\gamma) = 0$ and $e_0(\theta,\gamma) < 0$ if $\overline{\theta}_0 < \theta \leq 1$. We then obtain the following lemma.

**Lemma 1** $V_0^c \geq V_0^{fl}$ *for any* $\gamma \in (0,1]$ *if* $0 \leq \theta < \overline{\theta}_0(\gamma)$ *and* $0 < e \leq e_0(\theta,\gamma)$.

[16] For expositional convenience, we provide $\eta_i (i = 1,\ldots,6)$ in Appendix B.

*When* $\gamma = 0$*, it holds for any* $e \in (0,\infty)$ *if* $0 \le \theta < \overline{\theta}_0(0)$.
*Proof:* see Appendix C.

Let $\overline{\theta}_1(\gamma) \equiv \frac{2}{3-\gamma^2}$, defined on the interval $[0,1]$, and $e_1(\theta,\gamma) \equiv \frac{2-(3-\gamma^2)\theta}{\gamma(3-2\theta)}$, where $e_1 : [0,1]\times(0,1] \to \mathbb{R}$ monotonically decreases on $\theta$, such as $e_1(\theta,\gamma) > 0$ if $0 \le \theta < \overline{\theta}_1(\gamma)$, $e_1(\overline{\theta}_1,\gamma) = 0$, and $e_1(\theta,\gamma) < 0$ if $\overline{\theta}_1(\gamma) < \theta \le 1$. We obtain the following lemmas.

**Lemma 2** $\pi_1^c \ge \pi_1^{sl}$ *for any* $\gamma \in (0,1]$*, if* $0 \le \theta < \overline{\theta}_1(\gamma)$ *and* $0 < e \le e_1(\theta,\gamma)$. *When* $\gamma = 0$*, it holds for any* $e \in (0,\infty)$ *if* $0 \le \theta < \overline{\theta}_1(0)$.
*Proof:* see Appendix C.

**Lemma** *3 a)* $e_1(\theta,\gamma) > e_0(\theta,\gamma)$ *for any* $\theta \in [0,1)$ *and* $\gamma \in (0,1]$
*b)* $e_1(1,\gamma) = e_0(1,\gamma) = \gamma - \frac{1}{\gamma} \le 0$ *and* $\gamma \in (0,1]$
*c)* $\overline{\theta}_1(\gamma) > \overline{\theta}_0(\gamma)$ *for any* $\gamma \in (0,1]$
*Proof:* see Appendix C.

From Lemmas 1–3, we obtain the following main results.

**Proposition 1** *In a mixed duopoly with an SR firm, we have the following.*
*(i)* For any $\gamma \in (0,1]$ and $e \in (\underline{e},\overline{e})$:
  (a) If $0 \le \theta < \overline{\theta}_0(\gamma)$ and $e < e_0(\theta,\gamma)$, then the only equilibrium of the game is the simultaneous movement, that is, $(t_0,t_1) = (1,1)$.
  (b) If $0 \le \theta < \overline{\theta}_0(\gamma)$ and $e = e_0(\theta,\gamma)$, then either the simultaneous movement $(t_0,t_1) = (1,1)$ or the sequential move outcome, in which FP firm is the leader, $(t_0,t_1) = (2,1)$, is the equilibrium outcome.
  (c) If $\overline{\theta}_1(\gamma) \le \theta \le 1$ or $e \ge e_1(\theta,\gamma)$, then either the SR or FP firm can be the Stackelberg leader of the game. That is, $(t_0,t_1) = (1,2)$ and $(t_0,t_1) = (2,1)$ are the equilibrium outcomes.
  (d) Otherwise, one sequential move outcome in which the FP firm is the leader, $(t_0,t_1) = (2,1)$, is the unique equilibrium outcome.
*(ii)* When $\gamma = 0$, the equilibrium outcomes for any $e \in (0,\infty)$ are as follows:
  (a) If $0 \le \theta < \overline{\theta}_0(0)$, then $(t_0,t_1) = (1,1)$.
  (b) If $\theta = \overline{\theta}_0(0)$, then $(t_0,t_1) = (1,1)$ and $(t_0,t_1) = (2,1)$.
  (c) If $\overline{\theta}_1(0) \le \theta \le 1$, then $(t_0,t_1) = (1,2)$ and $(t_0,t_1) = (2,1)$.
  (d) Otherwise, $(t_0,t_1) = (2,1)$.
*Proof:* see Appendix D.

For easy explanations of Proposition 1, we examine four special but interesting cases for illustration without losing further economic insights. First, consider a case with $\theta = 0$, where the SR firm does not care for consumer surplus but does for the environment. Proposition 1(i) states that regardless of how significant the externality is, the only equilibrium of the game is the simultaneous movement (see Appendix E). This result is consistent with the observable delay game in a private duopoly without environmental externality, as formulated by Hamilton and Slutsky (1990). In a duopoly with an SR firm that has high concern for the environment, the analysis of a simultaneous move game is useful. In our setting, the SR firm reduces output to reduce its pollution. Thus, $\pi_1^c > \pi_0^c$ for $\gamma \in (0,1]$ because the SR firm is less aggressive in production and obtains lower profits than the FP firm.

Second, consider a case with $\gamma = 1$, where the SR firm cares for consumer surplus but entirely accounts for the environmental externality it solely causes. Proposition 1(i) states that depending on how much the SR firm cares for consumer surplus, three different equilibria emerge in the endogenous timing game (see Appendix E). This result includes the analysis of Matsumura and Ogawa (2017), who considered environmental externality in a mixed duopoly where a welfare-maximizing public firm competes with an FP firm and showed that a simultaneous move (sequential move) game emerges in equilibrium when the externality is significant (insignificant). The conditions for having the simultaneous move outcome are $0 \le \theta < \theta_0(1)$ and $\underline{e} < e \le e_0(\theta,1)$. That is, if the concern on consumer surplus is not high and the *ex-ante* initial level of pollution emission is not significant but the SR firm significantly cares for the environment, both firms opt to move early. Thus, a simultaneous movement emerges in equilibrium. Otherwise, sequential movements are equilibrium outcomes. In these three cases, we confirm that the SR firm reduces output and $\pi_1 > \pi_0$ for $\theta \in (0,1]$ and $e \in (\underline{e}, \overline{e})$ because the SR firm is more aggressive in production and obtains lower profits than the FP firm.

Third, consider a case with $\theta = 1$, where the SR firm cares for the environment but entirely accounts for consumer surplus. Proposition 1(i) states that regardless of how significant the externality is, two sequential movements are the equilibrium outcomes of the game (see Appendix E). This result is consistent with Pal (1998), who considered a mixed duopoly without environmental externality. We also suggest that the SR firm increases output if it does not care for externality significantly, thus $\pi_0 > \pi_1$ only if $\gamma < 0.577$ for both leadership equilibria. A necessary condition for the SR firm to obtain higher profits than its profit-seeking competitor in both leadership equilibria is that it must not significantly care for externality. This result supports the analysis of Lambertini and Tampieri (2015), who considered a mixed Cournot oligopoly where an SR firm competes with FP firms. They revealed that SR firms can produce higher output and obtain higher profits than FP firms if the market size is sufficiently large and SR firms completely account for consumer surplus.

Finally, consider a case with $\gamma = 0$, where the SR firm is never concerned with the environment but cares for consumer surplus. Proposition 1(ii) states that depending on how much the SR firm is concerned with consumer surplus, three different equilibria emerge in the endogenous timing game (see Appendix E). Furthermore, the condition for having the simultaneous move outcome is $\theta < \overline{\theta}_0(0)$. Regardless of the externality, if the SR firm does not significantly account for consumer surplus, then both firms opt to move early, and a simultaneous movement emerges in equilibrium. Otherwise, sequential movements are the equilibrium outcomes. The condition for having both sequential move outcomes is $\overline{\theta}_1(0) < \theta \le 1$. Regardless of the externality, if the SR firm significantly accounts for consumer surplus, then one firm opts to move early and the other firm later in the equilibrium. In these three different equilibria, the SR firm increases output, so $\pi_0 > \pi_1$ for $\theta \in (0,1]$. Thus, the SR firm always obtains higher profits than the FP firm.

Overall, depending on the relative concerns on environment and consumer surplus, the SR firm can be less or more aggressive in production and abatement. Hence, not only the significance of externality but also the instrumental conflict of social concerns are crucial factors in determining the equilibrium of endogenous timing game. Furthermore, the SR firm may achieve a high profit even though it adopts abatement activities in a quantity-setting competition. Being an SR firm can be one way of strategically committing to its profitable output. Thus, CSR activities can be either altruistic motivation or profit-initiated instrumental motivation, depending on the weights on social concern.s Hence, our analysis can also shed light on the economic motivations of the behaviors of SR firms.

# V. Discussion: Equilibrium and Welfare

We discuss the welfare consequences of the equilibrium choices in the endogenous timing game. Without loss of the generality, we simplify the analysis by setting $e = \frac{1}{2} \in (\underline{e}, \overline{e})$, which satisfies the conditions for the interior solutions for any $\gamma \in (0,1)$ and $\theta \in (0,1)$.

**Proposition 2** *For any* $0 < \gamma < 1$,

(a) If $0 \le \theta \le \overline{\theta}_2(\gamma)$, where $\gamma\eta_6(\overline{\theta}_2(\gamma)) - 2\eta_5(\overline{\theta}_2(\gamma)) = 0$, then the simultaneous move outcome, $(t_0, t_1) = (1,1)$, is an equilibrium.

(b) If $\theta = \overline{\theta}_2(\gamma)$, then either the simultaneous move outcome, $(t_0, t_1) = (1,1)$, or the sequential move outcome in which the FP firm is the leader, $(t_0, t_1) = (2,1)$, is an equilibrium.

(c) If $\overline{\theta}_2(\gamma) < \theta < \frac{4-3\gamma}{2(3-\gamma(1+\gamma))}$, then one sequential move outcome in which the FP firm is the leader, $(t_0, t_1) = (2,1)$, is the unique equilibrium.

(d) If $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))} \le \theta < 1$, then either the SR or FP firm can be the Stackelberg leader, $(t_0, t_1) = (1,2)$ or $(t_0, t_1) = (2,1)$, in the equilibrium.

*Proof:* see Appendix F.

**[Figure 1]** Equilibrium of the Endogenous Timing Game with $e = \frac{1}{2}$

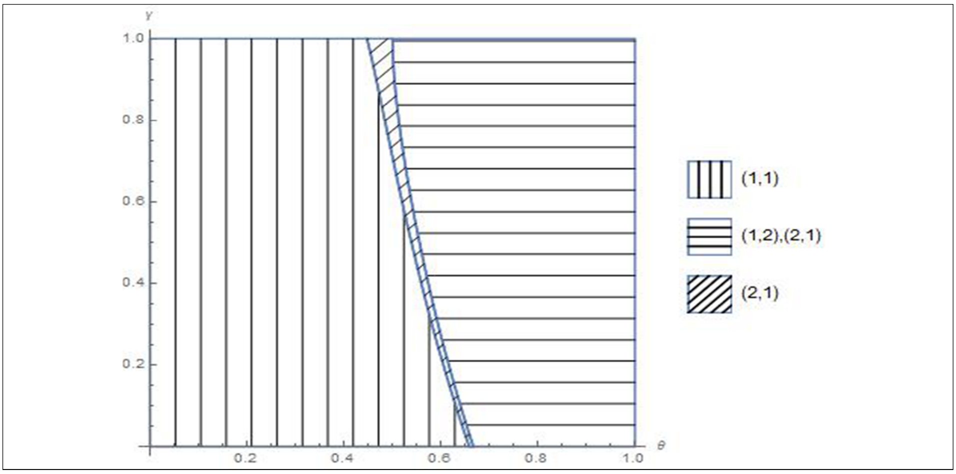



Figure 1 illustrates the equilibrium of the endogenous timing game. The region with vertical lines represents the zone where (1,1) is the equilibrium; the region with horizontal lines represents the zones where (1,2) and (2,1) are equilibria. Otherwise, (2,1) is the equilibrium. The equilibrium depends on the concern for consumer surplus. The simultaneous move game emerges when it is small, whereas the sequential move game with FP firm leadership emerges when it is intermediate. However, when the concern for consumer surplus is large, that is, $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))} \le \theta < 1$, then two equilibria exist: SR firm leadership, $(t_0, t_1) = (1,2)$ and FP firm leadership, $(t_0, t_1) = (2,1)$.

**Proposition 3** *For any* $0 < \gamma < 1$,

(a) The payoff dominance is FP firm leadership if $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))} < \theta < \theta_{p1}$, where $\pi_1^{sl}(\theta_{p1}) = \pi_1^{fl}(\theta_{p1})$.

(b) The payoff dominance is SR firm leadership if $\theta_{p0} < \theta < 1$, where $V_0^{sl}(\theta_{p0}) = V_0^{fl}(\theta_{p0})$.

(c) The payoff dominance is risk dominance.

*Proof:* see Appendix G.

The sequential move game with FP firm leadership is in payoff and risk dominant equilibrium if the concern for consumer surplus is relatively small, whereas the sequential move game with SR firm leadership is in payoff and risk dominant equilibrium if the concern for consumer surplus is relatively large. Proposition 3(c) supports the findings of Matsumura and Ogawa (2009), who showed that payoff dominant equilibrium is risk dominant equilibrium in the general observable delay game.

**Proposition 4** *For any* $0<\gamma<1$,

(a) $W^{c} > W^{fl}$ for all $\theta \in (0,1)$.

(b) $W^{sl} \le W^{c}$ if $\min\{\theta_{W_{sc}}(\gamma), \frac{4-3\gamma}{2(3-\gamma(1+\gamma))}\} \le \theta \le \max\{\theta_{W_{sc}}(\gamma), \frac{4-3\gamma}{2(3-\gamma(1+\gamma))}\}$, where $\theta_{W_{sc}}$ is $W^{c}(\theta_{W_{sc}}) = W^{sl}(\theta_{W_{sc}})$ and $W^{sl} > W^{c}$ elsewhere.

(c) $W^{fl} \ge W^{sl}$ if $\theta_{-} \le \theta \le \theta_{+}$, where $\theta_{-}$ and $\theta_{+}$ are $W^{fl}(\theta_{-}) = W^{sl}(\theta_{-})$ and $W^{fl}(\theta_{+}) = W^{sl}(\theta_{+})$, respectively.

The following corollary is a result of $\theta_{+} < \frac{4-3\gamma}{2(3-\gamma(1+\gamma))}$.

**Corollary 1** If $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))} \le \theta < 1$, then $W^{sl} > W^{fl}$.

Figure 2 shows the regions where the welfare obtained in a simultaneous competition, $W^{c}$, is the largest. The regions where the sequential game with the SR firm leadership yields the largest welfare are also displayed. FP firm leadership is never the best in terms of welfare.

**[Figure 2]** Welfare Comparison with $e=\frac{1}{2}$

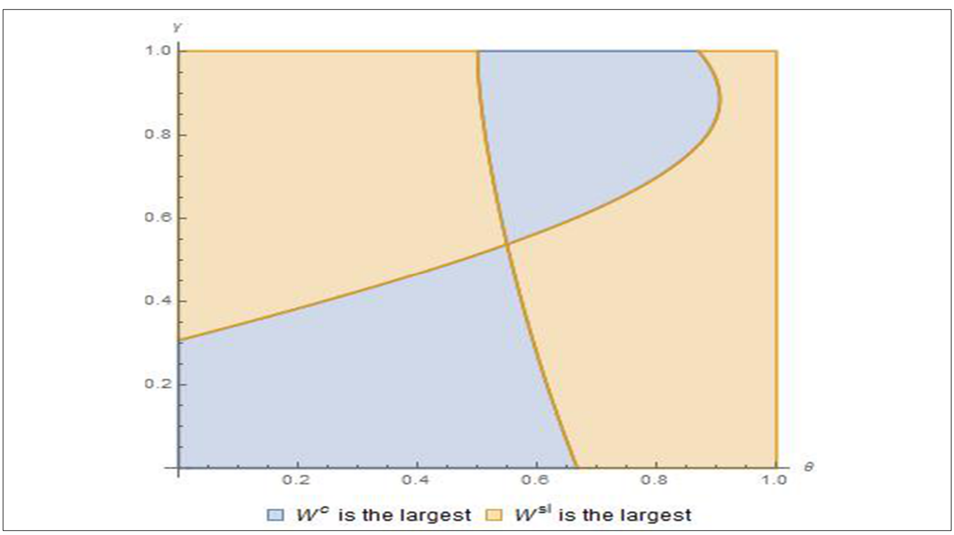
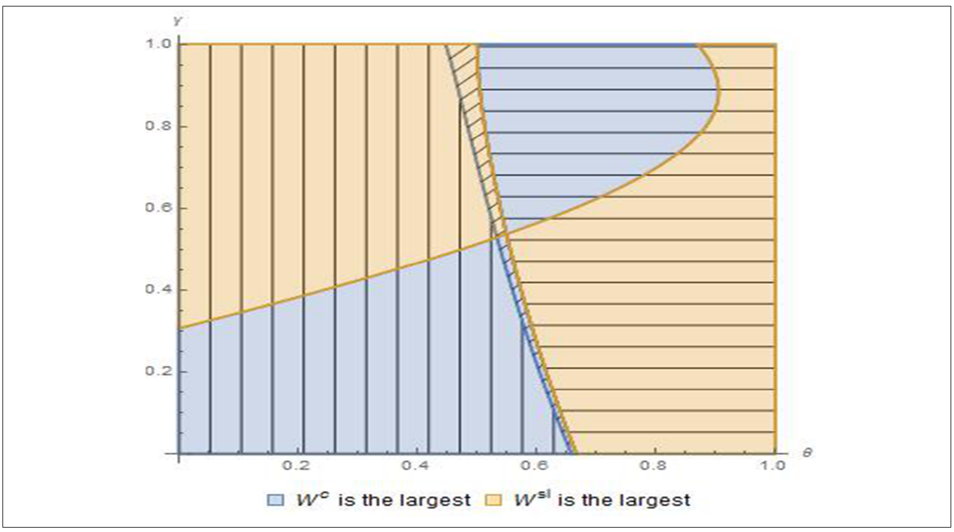


Figure 3 provides the comparison between equilibrium and welfare ranks. The region with vertical lines represents the zone where (1,1) is the equilibrium; the region with horizontal lines represents the zones where (1,2) and (2,1) are equilibria. Otherwise, (2,1) is the equilibrium. When the social concern on consumer surplus is not high, the simultaneous move game is the equilibrium, which is socially desirable only if the environmental concern is also relatively small. By contrast, when the concern for consumer is not that small, two sequential move games are the equilibria. When the concern for consumer is high, SR leadership is payoff dominant (and risk dominant) and socially desirable unless the environmental concern is high. However, when the concern for consumer is intermediate, FP leadership is payoff dominant (and risk dominany) but is not socially desirable regardless of the environmental concern.

**[Figure 3]** Comparison between Equilibrium and Welfare




# VI. Concluding Remarks

This study considered the heterogeneity of objectives among firms in which an SR firm is concerned with not only consumer surplus but also environmental externality in the presence of clean technology. We examined how behavioral heterogeneity and the significance of abatement technology induce the equilibrium outcome of the endogenous choice on different market structures. We found that two social concerns on consumer surplus and environment have opposite effects on production and abatement. Thus, the commitment to social concerns may lead to different market structures in the equilibrium of endogenous timing game. The firm's CSR initiatives can play significant roles in the market competition and have detrimental effects on social welfare.

Our main findings are summarized as follows: (i) When the SR firm is concerned with externality, depending on the significance of the *ex-ante* initial level of pollution emission, a simultaneous movement emerges in equilibrium. In addition, the SR firm reduces output and always obtains lower profits than FP firm. (ii) When the SR firm is significantly concerned with consumer surplus, two sequential move games may also be equilibria, and the SR firm increases output and obtains higher profits than FP firm. (iii) The desirability of the equilibrium depends on the social concern on consumer surplus when the environmental concern is relatively small. (iv) FP leadership is not socially desirable regardless of the environmental concern.

Our analysis also has a limitation because of the simple structure of our modeling with linear demand and quadratic cost functions. The model should be further examined in general settings in the context of different strategies, such as price and quality within the framework of differentiated products.[17] Future research must investigate the real-world situation and analyze the effects of government intervention on CSR behaviors.

[17] In the literature on endogenous timing game in duopolies, the results are mostly reversed depending on whether firms compete in price or quantity and whether firms compete in a private or mixed duopoly. For example, in a private duopoly with symmetric payoffs, firms decide simultaneously under quantity competition and sequentially under price competition, whereas in a mixed duopoly with asymmetric payoffs, firms decide sequentially under quantity competition and simultaneously under price competition. Recent research on the endogenous competition structure between Cournot–Bertrand comparison in mixed oligopolies has become rich and diverse. See Matsumura and Ogawa (2014, 2017), Ghosh and Mitra (2014), Liu et al. (2015), Haraguchi and Matsumura (2016), Xu et al. (2016), and Lee and Xu (2018).

## Appendix A. Values of $\sigma_i$

$$\sigma_1 \equiv 3e^2\gamma^2(7+3\gamma^2)+2e\gamma(2-11\theta-3\gamma^2(4+\theta));$$
$$\sigma_2 \equiv e^2\gamma^2(27-9\gamma^2-8\theta)-2e\gamma(18+\theta-2\theta^2-2\gamma^2(3+\theta));$$
$$\sigma_3 \equiv e^2\gamma(32-14\gamma+6\gamma^2-9\gamma^3-8\theta)-2e(3\gamma^4+\gamma(4-8\theta)+8(4-\theta)$$
$$-8\gamma^2(1-\theta)-\gamma^3(11+3\theta));$$
$$\sigma_4 \equiv -27e^2\gamma^2(7+3\gamma^2)+e\gamma(132\theta+36\gamma^2(6+\theta));$$
$$\sigma_5 \equiv 3e^2\gamma(84-42\gamma+18\gamma^2-27\gamma^3-16\theta)-e(462+54\gamma^4-88\theta-18\gamma^3(11+2\theta)$$
$$-12\gamma^2(9-8\theta)+6\gamma(7-16\theta));$$
$$\sigma_6 \equiv e^2\gamma^2(144+9\gamma^6-68\theta+8\theta^2-3\gamma^4(9-4\theta)-\gamma^2(46+6\theta-4\theta^2))$$
$$+2e\gamma(15--87\theta+41\theta^2-5\theta^3-3\gamma^6(4+\theta)+\gamma^4(64-8\theta-5\theta^2)$$
$$-\gamma^2(91-74\theta+5\theta^2+2\theta^3));$$
$$\sigma_7 \equiv e^2\gamma^2((8-3\gamma^2)^2-29\theta+11\gamma^2\theta+3\theta^2)-2e\gamma(40-7\theta^2+\theta^3+2\gamma^4(3+\theta)$$
$$-\gamma^2(31+5\theta-3\theta^2));$$
$$\sigma_8 \equiv e^2\gamma(6\gamma^6-9\gamma^7+\gamma^5(34-12\theta)-2\gamma^4(1-\theta)-\gamma(98-48\theta+6\theta^2)$$
$$+\gamma^3(10+14\theta-4\theta^2)-4\gamma^2(29-12\theta+\theta^2)+2(105-71\theta+15\theta^2-\theta^3))$$
$$-2e(231+3\gamma^8-173\theta+41\theta^2-3\theta^3-\gamma^7(11+3\theta)-\gamma^6(25-13\theta)$$
$$+\gamma^5(61-4\theta-5\theta^2)+\gamma^4(104-83\theta+15\theta^2)+4\gamma(7-18\theta+8\theta^2-\theta^3)$$
$$-\gamma^3(95-58\theta+\theta^2+2\theta^3)-\gamma^2(243-195\theta+53\theta^2-5\theta^3)).$$

## Appendix B. Values of $\eta_i$

$$\eta_1 \equiv 75-50\theta-32\theta^2+22\theta^3-3\theta^4-\gamma^6(12+8\theta+\theta^2)+\gamma^4(72+28\theta-11\theta^2-2\theta^3)$$
$$-\gamma^2(135-46\theta^2+6\theta^3+\theta^4);$$
$$\eta_2 \equiv 25+32\theta-\gamma^6\theta-22\theta^2+3\theta^3+\gamma^4(4+11\theta-2\theta^2)-\gamma^2(20+34\theta-14\theta^2+\theta^3);$$
$$\eta_3 \equiv 280+4\gamma^8-240\theta+42\theta^2+8\theta^3-2\theta^4+2\gamma^5(2+\theta)^2-\gamma^6(54-4\theta+\theta^2)$$
$$+2\gamma(-5-2\theta+\theta^2)^2-4\gamma^3(10+9\theta-\theta^3)+2\gamma^4(120-31\theta+\theta^2-\theta^3)$$
$$-\gamma^2(437-226\theta+16\theta^2+\theta^4).$$

Let $\alpha = 19755-67932\gamma^2+100440\gamma^4-83146\gamma^6+42020\gamma^8-13236\gamma^{10}+2531\gamma^{12}-268\gamma^{14}+12\gamma^{16}$. Then,

$$\eta_4 \equiv 44-32\gamma^2+6\gamma^4-\frac{2\sqrt[3]{2}(99-175\gamma^2+113\gamma^4-31\gamma^6+3\gamma^8)}{\sqrt[3]{96-399\gamma^2+531\gamma^4-316\gamma^6+87\gamma^8-9\gamma^{10}+\sqrt{-(14-13\gamma^2+3\gamma^4)^2\alpha}}}$$

$$-2^{2/3}\sqrt[3]{96-399\gamma^2+531\gamma^4-316\gamma^6+87\gamma^8-9\gamma^{10}+\sqrt{-(14-13\gamma^2+3\gamma^4)^2\alpha}}\ ;$$

$$\eta_5 \equiv 82-63\gamma^2+12\gamma^4-3\theta(55-52\gamma^2+17\gamma^4-2\gamma^6)+(66-48\gamma^2+9\gamma^4)\theta^2-(7-3\gamma^2)\theta^3\ ;$$

$$\eta_6 \equiv 6(4(1-\gamma^2)+\gamma^4)+(1-\theta)(12\gamma^4+4\theta^2+(1-\theta)(41-15\gamma^2)+62-57\gamma^2)>0\ .$$

## Appendix C. Proofs of Lemmas

**Lemma 1**

$$V_0^c - V_0^{fl} = \frac{(1-\theta)(e\gamma+1-\gamma^2+1-\theta)}{2(3-\gamma^2-\theta)(7-3\gamma^2-\theta)^2(8-3\gamma^2-2\theta)^2}(\eta_5 - e\gamma\eta_6)\,.$$

a) If $\gamma\in(0,1]$, then the sign of the difference $V_0^c-V_0^{fl}$ is the sign of $\eta_5-e\gamma\eta_6$, which is positive if and only if $0\le\theta<\overline{\theta}_0(\gamma)\equiv\frac{\eta_4}{2(7-3\gamma^2)}$ and $0<e<e_0\equiv\frac{\eta_5}{\gamma\eta_6}$.
b) If $\gamma=0$, then the sign of the difference $V_0^c-V_0^{fl}$ is the sign of $\eta_5(\gamma=0)=82-165\theta+66\theta^2-7\theta^3$, which does not depend on $e$ and is positive if and only if $0\le\theta<\overline{\theta}_0(0)\approx 0.658$.

**Lemma 2**

$$\pi_1^c-\pi_1^{sl}=\frac{3(2-(3-\gamma^2)\theta-e\gamma(3-2\theta))(e\gamma(45-18\gamma^2-10\theta)+(1-\theta)(47-25\gamma^2-8\theta)+(35-18\gamma^2)(1-\gamma^2))}{2(8-3\gamma^2-2\theta)^2(21-9\gamma^2-4\theta)^2}\,.$$

a) If $\gamma\in(0,1]$, then the sign of the difference $\pi_1^c-\pi_1^{sl}$ is the sign of $2-(3-\gamma^2)\theta-e\gamma(3-2\theta)$, which is positive if and only if $0\le\theta<\overline{\theta}_1(\gamma)\equiv\frac{2}{3-\gamma^2}$ and $0<e<e_1\equiv\frac{2-3\theta+\gamma^2\theta}{\gamma(3-2\theta)}$.
b) If $\gamma=0$, then the sign of the difference $\pi_1^c-\pi_1^{sl}$ is the sign of $2-(3-\gamma^2)\theta$, which does not depend on $e$ and is positive if and only if $0\le\theta<\overline{\theta}_1(0)=\frac{2}{3}$.

**Lemma 3**
a) For any $\gamma\in(0,1]$,

$$e_1-e_0=\frac{(1-\theta)^2(8-3\gamma^2-2\theta)(1+(1-\gamma^2)\theta)}{\gamma(3-2\theta)\eta_6}>0 \quad \text{for any} \quad 0\le\theta<1\,.$$

b) By substitution of $\theta=1$, $e_1(1,\gamma)=e_0(1,\gamma)=\gamma-\frac{1}{\gamma}$.
c) From Lemma 3 a) and b), $e_1(\overline{\theta}_0,\gamma)>e_0(\overline{\theta}_0,\gamma)=0=e_1(\overline{\theta}_1,\gamma)$; $e_1$ monotonically decreases on $\theta$, thus $\overline{\theta}_1(\gamma)>\overline{\theta}_0(\gamma)$.

## Appendix D. Proof of Proposition

Using Lemmas 1–3, we obtain the following table.

**[Table D]** Payoff Comparison

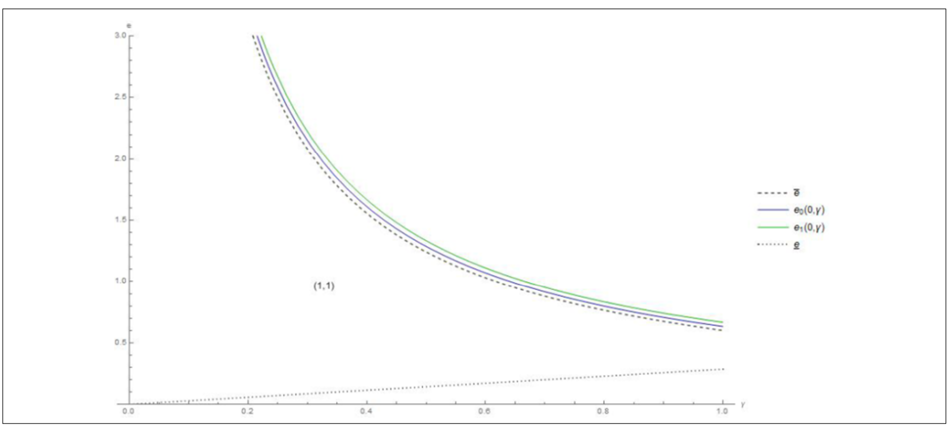
| $\theta / e$ | $e \le e_0$ | $e_0 < e \le e_1$ | $e > e_1$ |
|---|---|---|---|
| $0 < \theta < \overline{\theta}_0$ | $V_0^c \ge V_0^{fl}$ & $\pi_1^c \ge \pi_1^{sl}$ | $V_0^c < V_0^{fl}$ & $\pi_1^c \ge \pi_1^{sl}$ | $V_0^c < V_0^{fl}$ & $\pi_1^c < \pi_1^{sl}$ |
| $\overline{\theta}_0 < \theta < \overline{\theta}_1$ | $V_0^c < V_0^{fl}$ & $\pi_1^c \ge \pi_1^{sl}$ | $V_0^c < V_0^{fl}$ & $\pi_1^c \ge \pi_1^{sl}$ | $V_0^c < V_0^{fl}$ & $\pi_1^c < \pi_1^{sl}$ |
| $\theta > \overline{\theta}_1$ | $V_0^c < V_0^{fl}$ & $\pi_1^c < \pi_1^{sl}$ | $V_0^c < V_0^{fl}$ & $\pi_1^c < \pi_1^{sl}$ | $V_0^c < V_0^{fl}$ & $\pi_1^c < \pi_1^{sl}$ |

## Appendix E. Cases

### E1. Case with $\theta = 0$

A graphical representation is shown in Figure E.1.

**[Figure E.1]** Endogenous Timing Game Equilibrium with $\theta = 0$



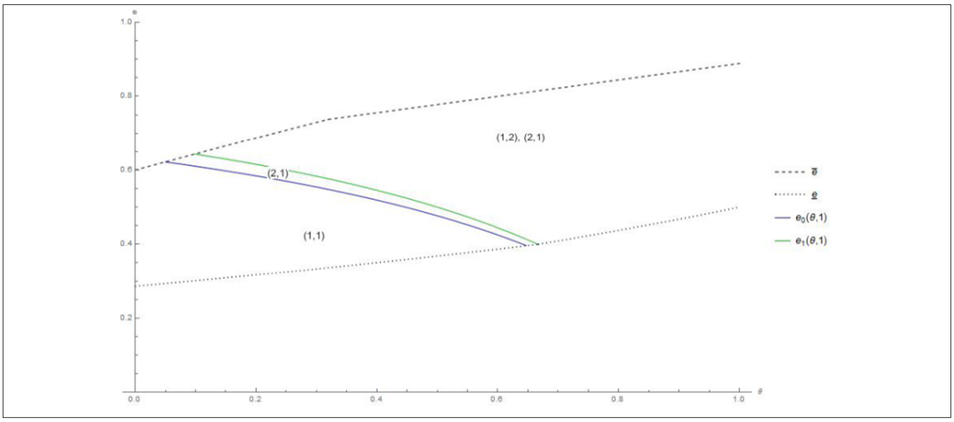

**Remark 1** *When* $\theta = 0$*, the only equilibrium of the game is the simultaneous movement, that is,* $(t_0, t_1) = (1,1)$.
**Remark 2** *When* $\theta = 0$*,* $\pi_1^c > \pi_0^c$ *for* $\gamma \in (0,1]$.

### E2. Case with $\gamma = 1$

**Remark 3** *When* $\gamma = 1$*, the equilibrium outcomes of the endogenous timing game are as follows:*
a) If $0 \le \theta < \overline{\theta}_0(1)$ and $e \le e_0(\theta,1)$, then $(t_0, t_1) = (1,1)$.
b) If $\overline{\theta}_1(1) < \theta \le 1$ or $e > e_1(\theta,1)$, then $(t_0, t_1) = (1,2)$ and $(t_0, t_1) = (2,1)$.
c) Otherwise, $(t_0, t_1) = (2,1)$.

A graphical representation is displayed in Figure E.2.

**[Figure E.2]** Endogenous Timing Game Equilibrium with $\gamma = 1$




**Remark 4** *When* $\gamma = 1$*, we obtain the following.*

a) $\pi_1^c > \pi_0^c$ .

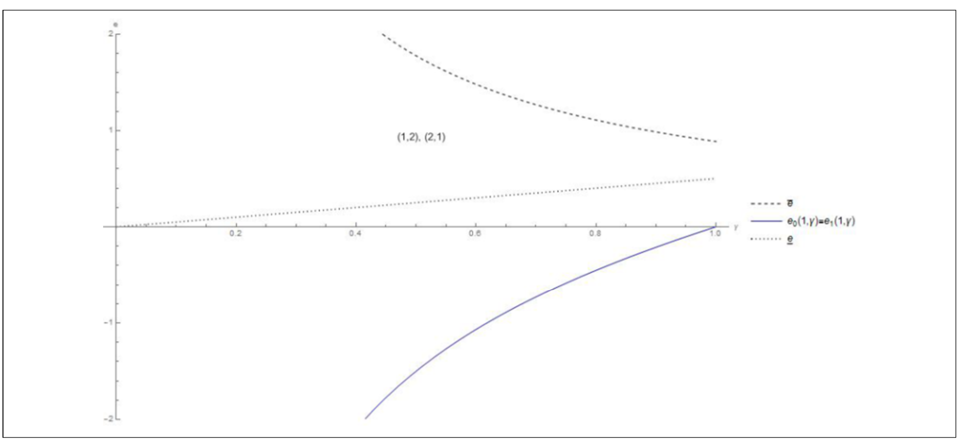
b) $\pi_1^{fl} > \pi_0^{fl}$ .

c) $\pi_1^{sl} > \pi_0^{sl}$ .

## E3. Case with $\theta = 1$

A graphical representation is provided in Figure E.3.

**[Figure E.3]** Endogenous Timing Game Equilibrium with $\theta = 1$




**Remark 5** *When* $\theta = 1$*, the equilibrium outcomes of the game are either the SR or FP firm can be the Stackelberg leader of the game, that is,* $(t_0, t_1) = (1, 2)$ *or*

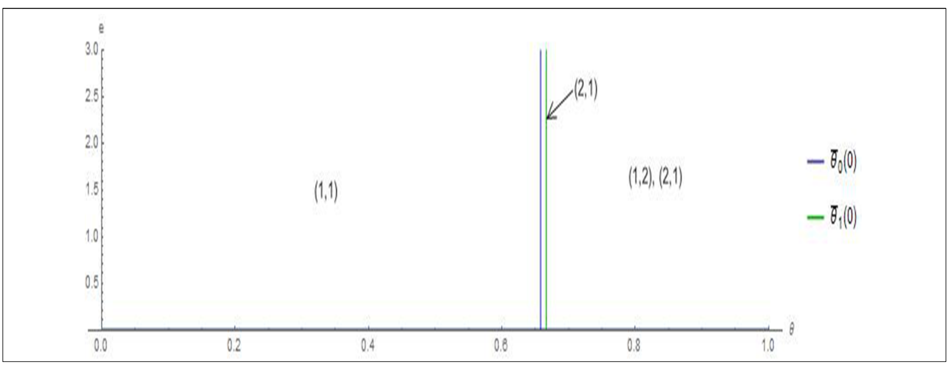
$(t_0, t_1) = (2,1)$, *respectively.*
**Remark 6** *When* $\theta = 1$, we have:
a) $\pi_0^{sl} > \pi_1^{sl}$ for $\gamma \in (0,.577)$ and $e \in (\max\{\underline{e}, \xi_1\}, \xi_2)$, and $\pi_1^{sl} > \pi_0^{sl}$ otherwise.
b) $\pi_0^{fl} > \pi_1^{fl}$ for $\gamma \in (0,.577)$ and $e \in (\xi_3, \xi_4)$, and $\pi_1^{fl} > \pi_0^{fl}$ otherwise.

*Proof.* When $\theta = 1$, we obtain:
a) Let $\xi_1 \equiv \frac{13+51\gamma^2-\sqrt{289-1173\gamma^2+999\gamma^4-243\gamma^6}}{9\gamma(8+3\gamma^2)}$ and $\xi_2 \equiv \frac{13+51\gamma^2+\sqrt{289-1173\gamma^2+999\gamma^4-243\gamma^6}}{9\gamma(8+3\gamma^2)}$. Thus,
$\pi_0^{sl} - \pi_1^{sl} = \frac{5-106\gamma^2-27\gamma^4-27e^2\gamma^2(8+3\gamma^2)+6e\gamma(13+51\gamma^2)}{2(17-9\gamma^2)^2} > 0$ for $\gamma \in (0,.577)$ and $max\{\underline{e}, \xi_1\} < e < \xi_2$.
b) Let $\xi_3 \equiv \frac{2(1+3\gamma^2)-\sqrt{4-16\gamma^2+13\gamma^4-3\gamma^6}}{\gamma(8+3\gamma^2)}$ and $\xi_4 \equiv \frac{2(1+3\gamma^2)+\sqrt{4-16\gamma^2+13\gamma^4-3\gamma^6}}{\gamma(8+3\gamma^2)}$. Thus,
$\pi_0^{fl} - \pi_1^{fl} = \frac{\gamma(-\gamma(5+\gamma^2)+4e(1+3\gamma^2)-e^2\gamma(8+3\gamma^2))}{6(2-\gamma^2)^2} > 0$ for $\gamma \in (0,.577)$ and $\xi_3 < e < \xi_4$.

### E4. Case with $\gamma = 0$

A graphical representation is shown in Figure E.4.

**[Figure E.4]** Endogenous Timing Game Equilibrium with $\gamma = 0$




**Remark 7** *Let* $\theta \in (0,1]$. *When* $\gamma = 0$, *we have* $\pi_0 > \pi_1$ *at every equilibrium of the endogenous timing game.*
*Proof.* When the SR firm sets $\gamma = 0$, then
a) $\pi_0^c - \pi_1^c = \frac{(3-\theta)\theta}{(4-\theta)^2} > 0$ for any $0 < \theta < \overline{\theta}_0(0)$.
b) $\pi_0^{sl} - \pi_1^{sl} = \frac{9+(149-32\theta)\theta}{2(21-4\theta)^2} > 0$ for any $\overline{\theta}_1(0) < \theta \le 1$.
c) $\pi_0^{fl} - \pi_1^{fl} = \frac{-1+(21-4\theta)\theta}{2(7-\theta)^2} > 0$ for any $\overline{\theta}_0(0) \le \theta \le 1$.

## Appendix F. Proof of Proposition 2

(a) Let $\overline{\theta}_2(\gamma) \in (0,1)$, such that if $\theta \lesseqgtr \overline{\theta}_2(\gamma)$, then $\frac{1}{2} \lesseqgtr \frac{\eta_5(\theta)}{\gamma\eta_6(\theta)}$. Thus, $\overline{\theta}_2(\gamma) < \overline{\theta}_0(\gamma)$.
If $0 < \theta < \overline{\theta}_2(\gamma) < \overline{\theta}_0(\gamma)$, then $e = \frac{1}{2} < e_0(\theta, \gamma) = \frac{\eta_5}{\gamma\eta_6}$. The conditions of

Proposition 1(i)(a) are satisfied.

(b) If $0<\theta=\overline{\theta}_2(\gamma)<\overline{\theta}_0(\gamma)$, then $e=\frac{1}{2}=e_0(\theta,\gamma)=\frac{\eta_5}{\gamma\eta_6}$. The conditions of Proposition 1(i)(b) are satisfied.

(c) If $\theta \lesseqgtr \frac{4-3\gamma}{2(3-\gamma(1+\gamma))}$, then $e=\frac{1}{2}\lesseqgtr e_1(\theta,\gamma)$. If $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))}\le\theta<1$, then $e=\frac{1}{2}\ge e_1(\theta,\gamma)$. The conditions of Proposition 1(i)(c) are satisfied.

(d) $\overline{\theta}_1\ge\frac{4-3\gamma}{2(3-\gamma(1+\gamma))}$ for any $\gamma\in(0,1)$. If $\overline{\theta}_2<\theta<\frac{4-3\gamma}{2(3-\gamma(1+\gamma))}$, then $\overline{\theta}_2<\theta<\overline{\theta}_1$ and $e_0(\theta,\gamma)<e=\frac{1}{2}<e_1(\theta,\gamma)$. The conditions of Proposition 1(i)(d) are satisfied.

## Appendix G. Proof of Proposition 3

(a) The equilibrium $(t_0,t_1)=(2,1)$ is payoff dominant if $V_0^{sl}<V_0^{fl}$ and $\pi_1^{sl}<\pi_1^{fl}$.

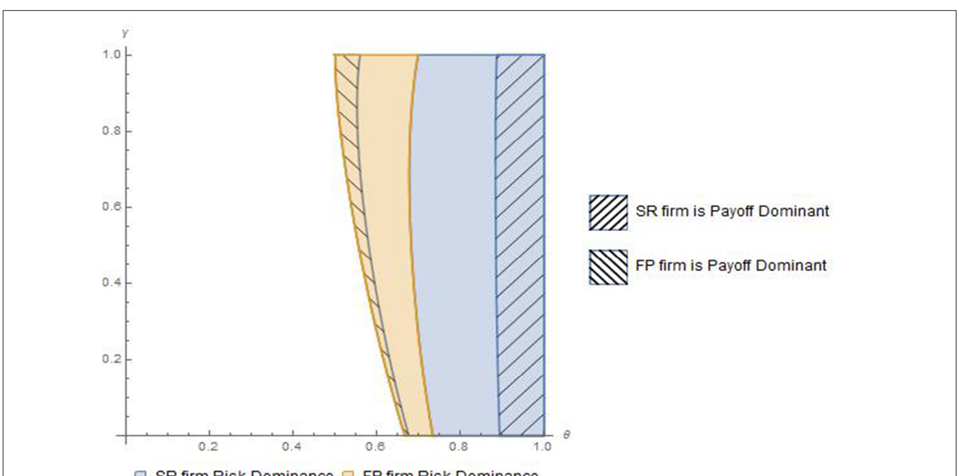
(b) The equilibrium $(t_0,t_1)=(1,2)$ is payoff dominant if $V_0^{sl}>V_0^{fl}$ and $\pi_1^{sl}>\pi_1^{fl}$.

(c) The equilibrium $(t_0,t_1)=(2,1)$ is risk dominant if

$$(V_0^{sl}(\theta_{r0})-V_0^{c}(\theta_{r0}))(\pi_1^{sl}(\theta_{r0})-\pi_1^{c}(\theta_{r0}))>(V_0^{fl}(\theta_{r0})-V_0^{c}(\theta_{r0}))(\pi_1^{fl}(\theta_{r0})-\pi_1^{c}(\theta_{r0})).$$

FP firm leadership is risk dominant if $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))}<\theta<\theta_{r0}$, whereasSR firm leadership is risk dominant if $\theta_{r0}<\theta<1$, where $(V_0^{sl}(\theta_{r0})-V_0^{c}(\theta_{r0}))(\pi_1^{sl}(\theta_{r0})-\pi_1^{c}(\theta_{r0}))=(V_0^{fl}(\theta_{r0})-V_0^{c}(\theta_{r0}))(\pi_1^{fl}(\theta_{r0})-\pi_1^{c}(\theta_{r0}))$. Figure G.1 illustrates that $\frac{4-3\gamma}{2(3-\gamma(1+\gamma))}<\theta_{p1}<\theta_{r0}<\theta_{p0}<1$ is satisfied for any $\gamma\in(0,1)$.

**[Figure G.1]** Payoff Dominance vs. Risk Dominance



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