The Korean Economic Review
Volume 35, Number 2, Summer 2019, 307-344.

Endogenous Growth and Equilibrium Cycles under Altruistic and Envious Preferences

Hyun Park*

This study examines how habit persistence in preferences affects short- and long-run economic growth in a dynamic competitive economy and addresses the possibility of equilibrium cycles in the presence of habit persistence in preferences. The study shows a continuum of transitional competitive equilibrium paths, each of which converges to a unique balanced growth path when external habits influence a household's preferences and when a firm's productivities increase in public capital services financed by consumption taxes. The self-fulfilling sunspot equilibrium path exhibits equilibrium cycles under altruistic or envious preferences in a growing competitive economy with exogenous fiscal policies. The study also demonstrates that a distortionary fiscal policy, including consumption tax, acts as a stabilizer for sunspot-driven fluctuations among rational expectation agents, selects a unique transitional path around the unique balance growth path, and eradicates equilibrium cycles in the social optimum.

JEL Classification: D91, E60, H30, O38
Keywords: Equilibrium Cycles, Consumption Tax, Habits, Indeterminacy, Public Capital, Stabilizer

I. Introduction

Habits have long occupied economists' attention in how they affect consumption–saving decisions, economic development and growth, asset pricing and risk premium, and macroeconomic stabilities and business cycles.[1] Numerous

Received: March 21, 2017. Revised: March 27, 2018. Accepted: June 25, 2018.
* Professor, Department of Economics, Kyung Hee University, 001 Hoeki-Dong, Dongdaemoon-Gu, Seoul, 130–701, South Korea; Phone: +82-2-961-9375; Fax: +82-2-966-7426; E-mail: econhyun@khu.ac.kr. I thank Costas Azariadis, Yukyung Jung, Hyeon Ok Lee, Junsoo Lee, Kyu Yub Lee, Jeremy Greenwood, Tetsu Haruyama, Heraklis Polemarchakis, and Mitsuyashi Yangihara for comments and suggestions. I also thank seminar participants in University of Alabama, University of Pennsylvania, Kobe University, Korea University, and Nagoya University. I also acknowledge the 2018 Sabbatical Grant from Kyung Hee University.

[1] Selective works include Abel (1990), Campbell and Cochrane (1999), Gali (1994), Boldrin, Christiano, and Fisher (2001), Dupor and Liu (2003), and Binsbergen (2016), among several others.

studies provide convincing support for consumption externalities, although empirical evidence on the importance of habits is sparse.[2] Recently, the introduction of unstable equilibrium paths has motivated an increased interest in habit persistence in preferences. Instability coexists with multiplicity, which can explain equilibrium cycles driven by an agent's self-fulfilling rational expectations in a dynamic competitive economy with the same economic fundamentals, including preferences and technology with the same initial capital stock.[3]

In the spirit of the underlying mechanism of sunspot-driven endogenous equilibrium cycles, this study explores the dynamic aspects of an endogenously growing economy in the presence of habits in preferences. First, I examine how habit persistence in altruistic and envious preferences affects short- and long-run economic growth in a dynamic competitive economy. Second, I investigate the possibility of equilibrium cycles of a balanced growth path in external habit persistence in preferences. I also posit the internalization of those external habits and derive an optimal tax and spending policy to restore a decentralized competitive equilibrium to the social optimum allocation. Finally, I search for the condition in which the social optimum allocation associated with a distortionary fiscal policy is unique and stable and thus displays no equilibrium cycles in a perpetually growing economy.

This analysis is based on a simple dynamic general equilibrium model with habit persistence in preferences and fiscal policies including distortionary tax and productive spending. The main features of the model are as follows. The production function is the constant returns to scale to its private inputs of labor and capital at given productive public capital services. This technology permits a perpetually growing economy. The felicity function is multiplicative and represents either altruism/admiration or envy/jealousy in Joneses habit preferences. External habit formation is specified as a combination of contemporaneous economy-wide average consumption and the stream of past habit stocks in the competitive economy. External habits act as consumption externalities in a decentralized competitive economy, and they justify the introduction of a social planning economy in which a government intends to design a dynamic optimal fiscal policy to improve social welfare and stabilize economic fluctuations.

Concerning an optimal fiscal policy in the Joneses habit economies, I deviate from the usual tax scheme of income, capital, and/or labor taxes in earlier studies

---

All these works are based on a unique, stable, and thus determinate equilibrium path.

[2] For examples, see Stadt, Kapteyn, and Geer (1985), Osborn (1988), and Fuhrer (2000). These studies examine how individual utility depends on current consumption, which is related to past consumption or others' current and past consumption.

[3] For example, Benhabib and Famer (1994) and Schmitt-Grohe and Uribe (1997) offer a new mechanism for propagating and magnifying existing shocks, including fiscal policies, preferences, and technology.

(e.g., Christiano and Harrison, 1999; Ljungqvist and Uhlig, 2000; Alonso-Carrera, Cabellé, and Raurich, 2005). In the presence of habit persistence, I consider consumption tax as a policy instrument for regulating market failure in a decentralized competitive economy. Consumption tax is introduced for the purpose of this study because an external habit stock generates consumption externalities, thereby causing market failure in a Joneses habit economy. I examine whether consumption tax is sufficient to restore efficiency in the competitive Joneses habit equilibrium. More importantly, I investigate the role of consumption tax as a stabilizer for competitive equilibrium cycles in the corresponding social planning economy.[4]

The contributions of this study are as follows. I revisit earlier studies to characterize the effect of habit persistence on short- and long-run economic growth. Along with a generalized utility function with external habit stocks, I identify each Joneses preference as the representation of altruism/admiration or envy/jealousy and the complementarity or substitutability between the contemporaneous average consumption and evolution of habit stocks in preferences. Together with the specification of external habit formation, the multiplicative felicity function also captures catching-up-with-the-Joneses, keeping-up-with-the-Joneses, or running-away-from-the-Joneses preferences.[5] I then derive the generalized growth condition in terms of the effective elasticity of intertemporal substitution—a combination of the usual elasticity of intertemporal substitution and the different types of habit persistence in preferences—and the input factor productivity including productive public capital services in technology.

This study shows that together with the nature of habit stocks in a multiplicative utility function, the effective elasticity of intertemporal substitution determines short- and long-run economic growth and dictates the dynamic properties of competitive equilibrium allocations. For instance, when habit stocks exhibit catching-up-with-the-Joneses or keeping-up-with-the-Joneses preferences, an increase in habit stocks increases short- and long-run economic growth, whereas the opposite argument is true when habit stocks exhibit running-away-from-the-

[4] Prior literature analyzes the role of fiscal policy on habit externalities. For examples, see Ljungqvist and Uhlig (2000), Turnovsky and Monteiro (2007), and Koehne and Kuhn (2015). Their analyses are either explicitly or implicitly based on the stability of dynamics in the Joneses habit economies.

[5] This property is from the fact that habit stocks in multiplicative preferences affect the marginal utility of the current consumption. However, habit stocks in subtractive preferences can be interpreted as a critical level of consumption so that the current consumption yields no additional felicity/utility unless the current consumption is larger than habit stocks. Hence, habit stocks in subtractive preferences capture a level effect on the current consumption but does not change the marginal effect of the current consumption. As a result, external habit stocks can be viewed as a comparison felicity of consumption, whereas internal habit stocks can be regarded as a minimum sustainable level of consumption.

Joneses preferences. This generalized result clarifies the mixed effect of habit stocks on economic growth in the Joneses habit literature (e.g., Alonso-Carrera, Cabellé, and Raurich, 2005; Alvarez-Cuadrado, Monteiro, and Turnovsky, 2004).

This study also derives the sufficient condition for indeterminate competitive equilibria in the presence of habit persistence in preferences and productive public capital services.[6] Local indeterminacy arises when the effective elasticity of intertemporal substitution is larger than the inverse of a share of private capital with respect to the realized rate of returns to public capital services. That is, the high effective elasticity of intertemporal substitution indicates that habit persistence is essential for the emergence of indeterminacy. Noticeably, indeterminacy arises in the presence of both positive and negative consumption externalities—herein, altruism/admiration and envy/jealousy in preferences, respectively—in a growing economy with productive government spending. As the dynamic competitive economy with Joneses preferences is indeterminate, the sunspot-driven self-fulfilling equilibrium paths emerges.

Furthermore, the nature of equilibrium cycles in this study differs from earlier findings of habit persistence in the real business-cycle literature. Habit persistence has been introduced as an instrument to explain, among other things, a propagation mechanism, the equity premium puzzle, and stock market behaviors (see Constantinides, 1990; Abel, 1990; Campbell and Cochran, 1999; Binsbergen, 2016). In contrast with previous studies, habit persistence in this study destabilizes an endogenously growing economy even in the absence of any stochastic stocks in the economic fundamentals. As in the usual sunspot equilibrium models (e.g., Benhabib and Farmer, 1994; Schmitt-Grohe and Uribe, 1997), I argue that habit persistence can be an additional source of equilibrium cycles in Joneses habit economies.[7]

The study thereafter postulates that optimal fiscal policies select a unique transitional and stationary path among multiple competitive equilibrium paths and thus prevent agents' ex ante expectations from self-fulfilling sunspot equilibrium allocations. These optimal fiscal policies in the presence of Joneses habits in preferences can suppress aggregate instability in a decentralized competitive economy. This result comes from the fact that the optimal fiscal policies take into account the full effect of external habits in consumption by imposing the distortive consumption tax for financing productive public capital services to support long-run

[6] See Benhabib and Gali (1995) for a general survey on multiplicity, indeterminacy, aggregate stability, equilibrium cycles, and macroeconomic properties. The present study follows a similar argument on the relation between stability and indeterminacy as in Schmitt-Grohe and Uribe (1997).

[7] Thomas (2004) surveys the implications of real business cycle and sunspot models on the volatility and frequency of real variables and the correlation between these variables and the corresponding prices. See also Jermann (1998), Boldrin, Christiano, and Fisher (2001), and Ascari, Phaneuf, and Sims (2015).

economic growth. The important implication is that optimal fiscal policy not only improves economic efficiency by choosing the efficient allocation among many decentralized competitive equilibria, but also suppresses equilibrium cycles by eradicating the continuum of ex ante animal spirit allocations in a growing Joneses habit economy.[8]

The remainder of the paper proceeds as follows. Section 2 reviews the related literature on indeterminacy and fiscal policies for stabilization. Section 3 lays out the basic model, describing how habit persistence in preferences affects the dynamics of a decentralized competitive economy and characterizes the long-run growth effects of habit formation. Section 4 examines indeterminacy and equilibrium cycles of a balanced growth path in a decentralized competitive economy. Section 5 internalizes external habits and derives distortive tax and spending policies that enable the decentralized competitive economy to replicate the social optimum. Section 6 establishes the unique and stable social optimum allocation with distortionary fiscal policy. Section 7 provides the concluding remarks.

II. Literature Review

This study follows the basic structure of the prior literature on endogenous growth and aggregate instability in models of habit formation. However, the underlying features and main results of this study differ substantially in several key aspects of internal and external habit formation, subtractive and multiplicative preferences, exogenous and endogenous labor market, various tax and spending policies, and local and global indeterminacy. On the basis of these aspects, this section surveys earlier studies of Joneses habit preferences and fiscal policies.

Habit persistence is introduced either as an internal or external form. The former is in Constantinides (1990), Carroll, Overland, and Weil (2000), and Chen (2007), whereas the latter is in Gail (1994) and Turnovsky and Monteiro (2007). Abel (1990) and Alvarez-Cuadrado, Monteiro, and Turnovsky (2004) consider both external and internal habit formation. In a model of external habits, Chen, Hsu, and Mino (2010) require endogenous labor supply to show aggregate instability. The critical element of their results is the interaction between production and/or consumption externalities and labor–leisure choices. In particular, the role of labor–leisure choices on indeterminacy is well understood in economic growth models, including

[8] Christiano and Harrison (1999) and Koehne and Kuhn (2015) discuss in detail social efficiency and welfare gain from stabilization fiscal and monetary policies. In general, welfare gain is larger in an endogenous fluctuation model than that in a real business cycle or a New Keynesian price rigidity business cycle model.

those of Schmitt-Grohe and Uribe (1997) and Benhabib and Farmer (1994). The literature clearly suggests that no labor–leisure choice strengthens the indeterminacy property. Noticeably, indeterminacy in this study does not rely on the endogenous labor supply in a competitive economy with Joneses habit preferences and distortionary fiscal policies.

Following Abel (1990), Gali (1994), and Ljungqvist and Uhlig (2000), a multiplicative form of felicity is adopted to compare stability property with those in the literature (e.g., Boldrin, Christiano, and Fisher, 2001; Chen, 2007). To the contrary, Constantinides (1990) and Campbell and Cochrane (1999) introduce a subtractive form, which has no change in marginal utility of a current consumption in terms of habit stocks. Thus, to the best of my knowledge, no indeterminacy arises with subtractive felicity unless one introduces an additional feature in a model of habit persistence. For example, Park (2013) shows endogenous equilibrium cycles in a growing economy with subtractive habit preferences. Unlike consumption tax in the present study, Park (2013) employs distortive income taxation. Furthermore, Park's (2013) income tax does not play the role of stabilizer where a social optimum allocation is indeterminate and generates endogenous equilibrium cycles.

The role of fiscal policies is important for indeterminacy in a social optimum allocation. In the earlier literature on endogenous business cycle literature, Schmitt-Grohe and Uribe (1997) and Farmer and Guo (1996) show that distortionary (endogenous or exogenous) fiscal policies can be as a source of aggregate instability in an imperfectly competitive economy with consumption and/or production externalities. However, Christiano and Harrison (1999) and Guo and Lansing (1998) design progressive income taxation and lump-sum transfers as a stabilization scheme to obtain a unique allocation in an imperfectly competitive economy with production externalities and increasing returns. Social optimum consumption taxes in this study resemble their progressive scheme of time-varying income taxation for stabilization.

Following Schmitt-Grohe and Uribe (1997), earlier works, including Giannitsarou (2007), Nourry, Seegmuller, and Venditti (2013), and Kamiguchi and Tamai (2011), examine which tax is a stabilizer under the proper conditions on an elasticity of intertemporal substitution in a dynamic competitive economy without habit formation. In particular, Giannitsarou (2007) finds that consumption tax is a stabilizer. However, Nourry, Seegmuller, and Venditti (2013) point out that Giannitsarou's claim is true only when the income effect is small and the elasticity of intertemporal substation is large. Kamiguchi and Tamai (2001) also show that indeterminacy arises less likely under consumption tax in a growth model with the productive public capital because the consumption tax, as opposed to other income taxes, does not facilitate the increasing returns to scale via the endogeneity of labor supply. In contrast with the present study on Joneses habit preferences, the consumption tax in Kamiguchi and Tamai (2001) does not act as a stabilizer for equilibrium cycles in the sense that their business cycles are not eradicated by their tax policies in the corresponding social optimum equilibrium.[9]

Lee and Park (2015) also examine whether the socially optimal allocation is indeterminate when the fiscal policies consist of the income (not consumption) tax and productive public capital in subtractive (not multiplicative) Joneses habit preference (see footnote 5 for the property of multiplicative and subtractive preferences). As a consequence, although the present study shows that the fiscal policies with consumption tax stabilize the corresponding competitive equilibrium, the socially optimal fiscal policies in Lee and Park (2015) continuously destabilize the aggregate economy with the income tax and the presence of Joneses habit preferences.

Cazzavillan (1996) and Park and Philippopoulos (2004) extend this competitive economy to a decentralized economy in which a government provides consumptive public spending and productive public services with distortive capital income taxes. Their consumptive public spending acts as an additional policy distortion, which causes another market failure, thus generating aggregate instability in a decentralized competitive market with productive fiscal policies. The above studies are complementary to the present study in a sense that habit formation plays a role in consumption externalities by generating indeterminacy in the decentralized competitive economy and then recovering determinacy in the social optimum allocation. In the same spirit, Palivos, Yip, and Zhang (2003) also study how government policies can select an optimal allocation from two balanced growth paths of a competitive economy. Unlike the present study with the distortionary fiscal policy, their scheme requires—in addition to endogenous labor supply—a lump-sum transfer mechanism in an optimal fiscal policy as in Christiano and Harrison (1999) and Guo and Lansing (1998).

III. Decentralized Competitive Economy

I consider a closed dynamic competitive economy with a private sector and a government sector. The private sector consists of a representative household and a representative firm. At any point in time t, a representative household derives its felicity $u(c(t),h(t))$ not only from its current consumption $c(t)$ but also from the current level of a reference habit stock $h(t)$ formed by an economy-wide average consumption. Each household inelastically supplies a unit of labor $l(t)=1$, saves in

[9] Chen and Hsu (2009) implement a consumption tax policy when consumption externalities affect an endogenous time discount function (rather than a habit persistent felicity function). Furthermore, they focus only on a saddle stable transitional path even though it is possible that indeterminacy emerges in their model.

the form of capital accumulation, and rents out its capital stocks $k(t)$ to private firms. A representative firm produces output $y(t)$ by employing privately provided labor $l(t)=1$ and private capital $k(t)$ and taking advantage of the government's capital services $g(t)$. The government imposes the time-invariant consumption tax $\tau(t)=\tau$, but no capital and labor tax to finance productive government services $g(t)$.[10] All agents are endowed with perfect foresight over time and act competitively in a decentralized competitive market. The government has a time-consistent precommitment policy instrument. The population does not grow, and capital does not depreciate. No uncertainty exists, and time is continuous on an infinite horizon.

The representative household's felicity function $u(c(t),h(t))$ is specified an iso-elastic multiplicative felicity function:

$$u(c(t),h(t))=\frac{1}{1-\sigma}\left[\frac{c(t)}{h(t)^{\gamma}}\right]^{1-\sigma}, \tag{1}$$

where $\sigma>0$ is the inverse of an elasticity of intertemporal substitution, and $\gamma<1$ is a weight of habit stocks to the current consumption. The parameter $\sigma>0$ ensures a concave felicity function for external habit stocks and allows both the elastic and inelastic intertemporal substitution. The assumption on $\gamma<1$ implies the nonsatiation on felicity of a sustaining increase in consumption.[11] Notice that the value of γ can be negative. The positive value of γ represents envy or jealousy whereby agents derive disutility from habit stocks, and the negative value of γ characterizes altruism or admiration of preferences. In general, the marginal rate of substitution between $c(t)$ and $h(t)$ is negative (*resp.* positive) when habit persistence in preferences represents altruism/admiration (*resp.* envy/jealousy).

The dynamic of habit stocks evolves according to $h(t)=\varphi\int_{-\infty}^{t}e^{-\delta(t-s)}\overline{c}(s)ds$, where $\overline{c}(s)$ denotes a level of average aggregate consumption in the decentralized competitive economy.[12] I assume that the initial habit stock $h(0)=h_0$ is given to each household. Differentiating the habit formation equation with respect to time

---

[10] In general, consumption tax is less distortionary than either capital or labor input tax, thereby satisfying the efficiency principle of optimum taxation.

[11] Parameter γ with a value not larger than 1 allows the current consumption to uniformly increase lifetime utility. See also Alonso-Carrera, Cabellé, and Raurich (2005) for a detailed discussion on a range of this parameter γ for the convexity condition on the utility function.

[12] Rather than external habits in the current model, Constantinides (1990) and Carroll, Overland, and Weil (2000) consider internal habits. In addition, Alvarez-Cuadrado, Monteiro, and Turnovsky (2004) introduce habit stocks as a combination of internal habits (own consumption) and external habits (others' consumption). Moreover, habit stocks can be formed not by consumption but by labor/leisure as in Azariadis, Chen, Lu, and Wang (2013).

yields the dynamic evolution function of habit formation:

$$\dot{h}(t) = \varphi\bar{c}(t) - \delta h(t)\,, \tag{2}$$

where a dot over a variable denotes the time derivative. The parameter $\varphi \geq 0$ is a coefficient that forms the average consumption into the habit stocks, and the parameter $\delta \geq 0$ indicates a rate of depreciation of habit stocks. The increasing value of φ indicates the increasing weight of the aggregate consumption in the recent past habit stocks, and the depreciation rate δ captures the rate of habit persistence in preferences.

Along with the felicity function defined in Eq. (1), the dynamic habit equation includes keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences. When δ goes to ∞ with some positive value of φ, the stock of habits is equivalent to the contemporaneous consumption; thus, the habit dynamics with the felicity function capture a continuous-time version of keeping-up-with-the-Joneses preferences along with the conditions on the felicity function. When φ is close but not equal to zero with some finite positive value of δ, the stock of habits consists of the accumulation of past aggregate consumption. Thus, the habit dynamics with the felicity function capture a continuous-time version of catching-up-with-the-Joneses preferences. The felicity function with respect to individual consumption and external habit stocks is a combination of the current value of individual consumption, the contemporaneous average consumption, and the accumulated level of the past average aggregate consumption.

Now I set up the household's optimization problem. The household saves in the form of assets $k(t)$, thus receiving interest income $r(t)k(t)$, where $r(t)$ is a market asset return in a competitive capital market. The household also inelastically supplies its labor services $l(t) = 1$ so that wage income is $w(t)l(t) = w(t)$, where $w(t)$ is a market wage rate in the competitive labor market.[13] It pays the proportional consumption tax $\tau > 0$ over time. The household receives a net dividend $d(t)$ from the ownership of firms. There is no tax on dividends and no asset depreciation. Thus, the representative household has intertemporal budget constraints, given the initial capital stock $k(0) = k_0$, as

$$\dot{k}(t) = r(t)k(t) + w(t) + d(t) - (1+\tau)c(t)\,. \tag{3}$$

Formally, given a time-invariant consumption tax rate τ, a capital rental rate $r(t)$, a wage rate $w(t)$, and a profit share $d(t)$ from each firm in the decentralized

[13] As surveyed in Section 2, the inelastic labor supply assumption is not so innocuous in a sense that distortive effects of consumption externalities on economic growth depend on the elasticity of the labor supply (Turnovsky and Monteiro, 2007).

competitive economy, the representative household optimizes its discounted lifetime utility with the rate of time preference $\rho > 0$:

$$\int_0^\infty u(c(t), h(t)) e^{-\rho t} dt \,, \tag{4}$$

subject to intertemporal budget constraints in Eq. (3), taking the existing habit stocks $h(t)$ as a predetermined level of a reference stock by economy-wide average consumption $\overline{c}(t)$ and given the initial level of habit stock $h(0) = h_0$ and capital stock $k(0) = k_0$.

The current valued Hamiltonian equation $H(c, k, \lambda; h, \tau, r, w, d)$ for the representative household problem is defined as follows:

$$H(c, k, \lambda; h, \tau, r, w, d) \equiv \frac{1}{1-\sigma}\left[\frac{c}{h^\gamma}\right]^{1-\sigma} + \lambda[rk + w + d - (1+\tau)c] \,,$$

where λ is the multiplier for the household's budget constraints. (Hereafter, I omit a time index unless it causes confusion.) Hence, the Euler's necessary conditions for the household problem are

$$c^{-\sigma} h^{-\gamma(1-\sigma)} = (1+\tau)\lambda \,, \tag{5a}$$

$$\dot{\lambda} = \rho\lambda - r\lambda \,, \tag{5b}$$

$$\dot{k} = rk + w + d - (1+\tau)c \,, \tag{5c}$$

and the dynamic equation of habit stocks in Eq. (2), taken exogenously by households. As previously indicated, $h(t)$ is an externality to the household, the Hamiltonian equation is convex in $\{c, k\}$, given the parameter values $\sigma > 0$ and $\gamma < 1$ of the felicity function. Therefore, the conditions of Eqs. (5a)–(5c), along with the transversality condition below, are also the sufficient condition for the household's optimization.

First, taking logarithms and time derivatives with respect to time for Eq. (5a) yields $-\sigma \dot{c} / c - \gamma(1-\sigma)\dot{h} / h = \dot{\lambda} / \lambda$. Then, by using Eqs. (2), (5a), and (5b), I obtain

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[r - \rho - \gamma(1-\sigma)\left[\varphi\frac{\overline{c}}{h} - \delta\right]\right] > 0 \,. \tag{6}$$

This equation summarizes the consumption dynamics for the representative household's problem. The consumption growth rate is affected by the weight $-\gamma(1-\sigma)$ of the evolution of habit stocks: $\frac{\dot{h}}{h} = [\varphi\overline{c} / h - \delta]$ in Eq. (2). Hence, the set of joint conditions on a type of habit stocks and an elasticity of intertemporal substitution influence the consumption and investment decisions, thereby contributing to the rich dynamics of the competitive equilibrium. Moreover, the consumption dynamic equation in Eq. (6) induces that the transversality condition for the household problem becomes that $0 < \frac{1}{\sigma}[r-\rho-\gamma(1-\sigma)[\varphi\frac{\bar{c}}{h}-\delta]] < \rho$. That is, the rate of consumption/capital growth is smaller than the rate of time preferences, so the lifetime utility is bounded above in a persistently growing economy.

The representative firm has a production technology with constant returns to scale to its private inputs, capital $k(t)$ and labor $l(t)$, with both factors having a positive but diminishing marginal product. For simplicity, I assume that the demand for labor is exogenous over time, say $l(t)=1$. In addition, the final output also depends on public capital services $g(t)$, which are assumed to be nonexcludable and nonrival. I assume no adverse congestion effect of government spending. That is, each firm use of public capital services does not diminish the quality or quantity available to other firms in the decentralized competitive economy. This technology is well known in public finance growth models, and the public capital services are productive, thus playing a role of an engine of long-run growth (Barro, 1990). I further specify the production technology as

$$y = Ag^{1-\alpha}k^{\alpha}, \tag{7}$$

where $A>0$ is the total factor productivity, $0<\alpha\le 1$ is the capital share of output, and $0 \le 1-\alpha < 1$ is the productivity of public capital services. Along with the iso-elastic felicity functional form in Eq. (1), this specification of technology permits the existence of a balanced growth path in a growing economy.

Given public capital services $g(t)$, the representative firm maximizes the following static profit function $\pi(t)$ at the time $t \ge 0$:

$$\pi = y - rk - w, \tag{8}$$

subject to the production technology of Eq. (7) at a given market rate of interest $r(t)$ and a wage rate $w(t)$ in the decentralized competitive market economy. The first-order conditions are satisfied with respect to the real rate of return to capital $r(t)$ and the wage rate $w(t)$, respectively, as

$$r = \alpha A g^{1-\alpha}k^{\alpha-1}, \tag{9a}$$

$$w = y - rk = (1-\alpha)Ag^{1-\alpha}k^{\alpha}. \tag{9b}$$

I assume that the consumption tax revenue is used for productive public services.

That is, government expenditure is not a first-best policy. I also assume that the government balances its budget at each point in time. Thus, the government's budget constraints satisfy

$$g(t) = \tau c(t) . \tag{10}$$

The balanced budget rules allow no ambiguity in coordination between the government's tax and spending policies. Along with the different consumption level, the economy's corresponding tax revenue changes over time; therefore, government expenditure is not predetermined but endogenously determined under balanced budget constraints.[14] Given that the production function satisfies the Inada condition, the consumption tax revenue and public capital services cannot be zero. The fiscal tax and spending policy is nontrivial in the competitively growing economy.

Formally, under the balanced budget conditions in Eq. (10), the set of optimization conditions in Eqs. (7), (9a), and (9b) for the producer's problem can be respectively expressed as

$$y = A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha} , \tag{11a}$$

$$r = \alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1} , \tag{11b}$$

$$w = (1-\alpha)A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha} . \tag{11c}$$

The market solution exhibits rich dynamics because consumption influences equilibrium output and wage and interest rates in the decentralized competitive economy. This solution deviates from the equilibrium property in a standard public finance AK–growth model with capital or output taxation. Moreover, no wedge exists between the market rate of interest and the social rate of interest because the net social rate of returns $r^*(t)$ after consumption tax, $r^*(t) \equiv \frac{\partial y}{\partial k} = \alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1}$, from Eq. (11a) is identical to the competitive interest rate in Eq. (11b). This property of consumption taxation under balanced growth constraints implies that no distortion arises between the realized market returns to the capital investment and the private asset returns perceived by the agent's saving/investment decisions in the decentralized competitive economy. Therefore, unlike capital and labor income taxes, consumption taxes are not a direct cause of the existence of multiple equilibria in a public finance growth model.[15] Nevertheless, the interest

[14] Schmitt-Grohe and Uribe (1997) and Nourry, Seegmuller, and Venditti (2013) assume that government expenditure is predetermined so that a tax rate is endogenously determined under the balanced budget constraints. Their alternative condition is also capable to show indeterminacy and aggregate instability.

[15] By contrast, in a usual public finance growth model with capital income tax and public services as in Barro (1990), such a wedge can cause aggregate instability under Ramsey fiscal policies (Park and

and wage rates are dependent on private consumption, and the consumption tax rates are not asymptotically linear to private capital stocks. Later sections discuss implications of this nonlinearity in capital concerning the uniqueness and stability property of a balanced growth path in decentralized competitive and social planning economies.

I now characterize the equilibrium allocation $\{c(t), h(t), k(t)\}$ in a decentralized competitive economy with any feasible exogenous tax and spending policy $\{\tau, g(t)\}$. Without loss of generality, all agents are assumed to be identical and the number of agents is assumed to be one. First, by substituting Eqs. (2) and (11b) into Eq. (6), and $\overline{c}(t) = c(t)$ at the symmetric equilibrium, the consumption policy function at any point in time t is

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[\alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1} - \rho - \gamma(1-\sigma)\left[\varphi\frac{c}{h} - \delta\right]\right]. \tag{12a}$$

Second, by combining Eqs. (10), (11b), and (11c) and $d(t) = \pi(t)$ at the equilibrium,[16] the representative household's rate of capital accumulation in Eq. (3) is

$$\frac{\dot{k}}{k} = A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1} - (1+\tau)\frac{c}{k}. \tag{12b}$$

Third, again, under the symmetric equilibrium condition $\overline{c}(t) = c(t)$ for all $t \geq 0$, Eq. (2) is

$$\frac{\dot{h}}{h} = \varphi\frac{c}{h} - \delta. \tag{12c}$$

Finally, I impose the transversality condition in the growing equilibrium such that

$$\frac{1}{\sigma}\left[\alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1} - \rho - \gamma(1-\sigma)\left[\varphi\frac{c}{h} - \delta\right]\right] < \rho. \tag{12d}$$

Therefore, the decentralized competitive equilibrium for any feasible government policies satisfies the dynamic system of Eqs. (12a)–(12d), dictating the agent's maximization conditions including government budget constraints and market-

Philippopoulos, 2004).

[16] The net profit (and thus the net dividend) becomes zero because $\pi(t) = rk + w = 0$ from Eqs. (11b) and (11c).

clearing conditions.[17] The following proposition depicts the competitive equilibrium under habit persistence in preferences and exogenous fiscal policies with consumption taxes and productive government spending.

Proposition 1: *Under the assumptions on the felicity, production, and habit evolution function, the decentralized competitive equilibrium* $\{c(t), h(t), k(t)\}$ *exists under any feasible fiscal policies with the time-invariant consumption tax* τ *and government expenditure* $g(t)$ *if and only if the sequence of* $\{c(t), h(t), k(t)\}$ *satisfies the system of the dynamic Eqs. (12a)–(12d) from the initial capital stock* k_0 *and the initial habit stock* h_0.

Now, I examine the growth property of the competitive equilibrium. The consumption Eq. (12a) pertains to the aggregate economic growth condition, which is determined by combining the difference between the realized interest rate $r(t) = \alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1}$ and the rate of time preferences ρ with the evolution $\dot{h}(t)$ of reference habit stocks with the weight of $-\gamma(1-\sigma)$. When no habit in preferences exists (i.e., $\gamma = 0$), the consumption dynamics are dictated exclusively by the difference between the realized rate of returns $r(t)$ and the rate of time preferences ρ (i.e., $r(t) - \rho = \alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1} - \rho \geq 0$). As previously mentioned, the realized rate of returns $r(t)$ is a function of consumption, capital stocks, and tax rates (see Eq. (11b)), all of which come from the productive public services financed by the consumption tax under the balanced budget constraints.

Earlier studies show that when the sign of $-\gamma(1-\sigma)$ is positive as in Eq. (12a), habit persistence represents catching-up-with-the-Joneses (Abel, 1990) or keeping-up-with-the-Joneses preferences (Gali, 1994). Under this condition, there are two possible cases: (i) Habit preferences exhibiting envy/jealousy (i.e., $0 < \gamma < 1$) under the elasticity σ^{-1} of intertemporal substitution is less than 1 and (ii) habit preferences exhibiting altruism/admiration (i.e., $\gamma < 0$) under the elasticity σ^{-1} of intertemporal substitution is larger than 1. Symmetrically, when the sign of $-\gamma(1-\sigma)$ is negative, habit persistence represents running-away-from-the-Joneses preferences (Dupor and Liu, 2003). Under this condition, the two possible cases also arise: (i) Habit preferences exhibiting envy/jealousy (i.e., $0 < \gamma < 1$) under the elasticity σ^{-1} of intertemporal substitution is larger than 1, and (ii) habit preferences exhibiting altruism/admiration (i.e., $\gamma < 0$) under the elasticity σ^{-1} of intertemporal substitution is less than 1. Therefore, from Eq. (12a), envy/jealousy preferences under catching/keeping-up-with-the-Joneses imply that a high elasticity of intertemporal substitutes is accompanied by a low rate of economic growth,

[17] This closed-form solution characterizes the properties for Joneses habit economies including economic growth and stabilities of habit formation. Recently, Chilarescu (2016) has extended the closed solution in this study to that in a two-sector growth model with habit formation.

whereas altruism/admiration preferences under running-away-from-the-Joneses imply that a high elasticity of intertemporal substitution leads to a high rate of economic growth in the decentralized competitive economy.[18]

The following lemma summarizes the previous arguments and demonstrates the set of all possible growth rates in the decentralized competitive economy with altruism/admiration and envy/jealousy habits in preferences.

Lemma 1: *When Joneses habit preferences are altruistic (resp. envious), the high elasticity of intertemporal substitution* σ^{-1} *positively (resp. negatively) affects short- and long-run economic growth in the decentralized competitive economy with any feasible fiscal policies.*

Now, I focus on the long-run properties of the competitive equilibrium in a growing competitive economy. First, the balanced growth path is defined as follows: (i) Consumption $c(t)$ and capital $k(t)$ grow at a constant rate, and (ii) the stock of habits $h(t)$ grows at a constant rate. By applying the usual method to the long-run analysis, let $z(t) \equiv c/k$ and $x(t) \equiv h/c$. I then transform a three-dimensional dynamic system of $\{c,h,k\}$ into a two-dimensional dynamic system of $\{z,x\}$. This transformation is to analyze a perpetually growing economy, where the growth rate of two new variables $z(t)$ and $x(t)$ converges to zero at a balanced growth path. Under the condition that consumption is a one-to-one trade-off from capital accumulation, the consumption–capital ratio remains constant in a balanced growth path. In addition, the habit–consumption ratio is constant in the balanced growth path because habit formation is a linear function of consumption. Hence, the reduced dynamic system of $\{z,x\}$ satisfies $\dot{z}/z=0$ and $\dot{x}/x=0$ in the balanced growth path $\{c,h,k\}$.

Suppose that the realized rate of interest is defined as $r(z;\tau) \equiv \alpha A \tau^{1-\alpha} z^{1-\alpha}$ in terms of z and τ. A simple manipulation for dynamic Eqs. (12a)–(12c) in the competitive equilibrium leads to

$$\frac{\dot{z}}{z} = (1+\tau)z - \frac{1}{\sigma}\left[\left[\frac{\sigma-\alpha}{\alpha}\right] r(z;\tau) + \rho - (\sigma-\xi)\left[\varphi\frac{1}{x} - \delta\right]\right], \tag{13a}$$

$$\frac{\dot{x}}{x} = \frac{1}{\sigma}\left[-r(z;\tau) + \rho + \xi\left[\varphi\frac{1}{x} - \delta\right]\right], \tag{13b}$$

where $\xi \equiv \gamma + \sigma(1-\gamma)$.[19]

I redefine a steady state $\{\tilde{z},\tilde{x}\}$ corresponding to a balanced growth path

[18] This conclusion is from the ceteris paribus effect on the interest rate and the habit evolution because I cannot explicitly solve the implicit functions Eqs. (12a)–(12c).

[19] Notice that Eq. (13a) is from Eqs. (12a) and (12b); similarly, Eqs. (12a) and (12c) yield Eq. (13b).

$\{\tilde{c},\tilde{h},\tilde{k}\}$. (Hereafter, a tilde over a variable denotes that the variable is in the balanced growth path.) If $\{\tilde{z},\tilde{x}\}$ is in the interior of any feasible allocations given any feasible exogenous tax rate τ,[20] then the steady state equations of $\{\tilde{z},\tilde{x}\}$ satisfy

$$(1+\tau)\tilde{z}=\left[\frac{\xi-\alpha}{\xi\alpha}\right]r(\tilde{z};\tau)+\frac{\rho}{\xi}, \tag{14a}$$

$$\tilde{x}=\frac{\varphi\xi}{r(\tilde{z};\tau)-\rho+\delta\xi}. \tag{14b}$$

Eqs. (14a)–(14b) show that a unique solution exists for $\tilde{z}$ in Eq. (14a). Then, I also pin down $\tilde{x}$ from Eq. (14b), thus obtaining the unique balanced growth path $\{\tilde{z},\tilde{x}\}$. First, when $\xi-\alpha>0$, the left-hand side of Eq. (14a) is a linearly increasing function of $\tilde{z}$, whereas $r(\tilde{z};\tau)$ in the right-hand side of Eq. (14a) is a monotonically increasing concave function in $\tilde{z}$. Thus, a unique interior solution for $\tilde{z}$ exists in Eq. (14a), in which substituting $\tilde{z}$ into Eq. (14b) determines the unique solution $\tilde{x}$ in Eq. (14b). Second, when $\xi-\alpha<0$, the right-hand side of Eq. (14a) is monotonically decreasing and a convex function of $\tilde{z}$. I also find the unique interior solution for $\tilde{z}$ and $\tilde{x}$, given the linear function of $\tilde{z}$ in the left-hand side of Eq. (14a). Hence, the unique balanced growth path exists regardless of the elasticity of intertemporal substitution (i.e., $0<\sigma<1$ or $\sigma>1$) and of altruistic or envious preferences (i.e., $\gamma<0$ or $0<\gamma<1$).

Moreover, by using Eqs. (12a) and (14b), I obtain the long-run growth rate $\tilde{\Gamma}_{CE}$ as

$$\tilde{\Gamma}_{CE}=\frac{\dot{c}}{c}=\frac{\dot{k}}{k}=\frac{\dot{h}}{h}=\frac{1}{\xi}[r(\tilde{z};\tau)-\rho]. \tag{14c}$$[21]

By observing the expression of Eq. (14c), $\xi^{-1}>0$, $\xi=\gamma+\sigma(1-\gamma)$, can be thought of as the "effective" elasticity of intertemporal substitution in the presence of habit persistence. The effective elasticity of intertemporal substitution ξ^{-1} encompasses altruistic (i.e., $\gamma<0$) and envious (i.e., $0<\gamma<1$) preferences. In later sections, I find that the effective elasticity of intertemporal substitution plays a critical role not only for endogenous growth but also for aggregate instability in the competitive equilibrium with habit persistence in preferences.

The following proposition summarizes the long-run property in the

[20] A later discussion explicitly specifies the interiority conditions for a nontrivial competitive equilibrium path with any feasible exogenous tax rates.

[21] Notice that the growth rate is not written in terms of the parameter values since $z(t)$ and $x(t)$ are not explicitly solvable in Eqs. (14a) and (14b).

decentralized competitive economy with Joneses habit preferences and distortionary fiscal policies:

Proposition 2: *Given any feasible fiscal policies* $\{\tau, g(t)\}$*, the unique balanced growth path* $\{\tilde{c}, \tilde{h}, \tilde{k}\}$ *exists when the interior allocation* $\{\tilde{z}, \tilde{x}\}$ *satisfies the system of dynamic Eqs. (14a)–(14b). Moreover, the long-run growth rate* $\tilde{\Gamma}_{CE}$ *is equal to* $\xi^{-1}[r(\tilde{z};\tau)-\rho]\geq 0$ *with* $r(\tilde{z};\tau)=\alpha A\tau^{1-\alpha}\tilde{z}^{1-\alpha}$ *in the decentralized competitive economy.*

Now, I examine the long-run growth property for the balanced growth path $\{\tilde{c}, \tilde{h}, \tilde{k}\}$. The long-run growth rate $\xi^{-1}[r(\tilde{z};\tau)-\rho]$ is positive under the conditions $\xi^{-1}>0$ and $r(\tilde{z};\tau)-\rho>0$. For persistent economic growth, under a sufficiently high effective elasticity ξ^{-1} of intertemporal substitution, the realized rate of interest is greater than the rate of time preferences. The long-run rate of growth also depends on the consumption/capital ratio $\tilde{z}(t)$ and the usual fundamental parameters of preferences and technology, α, A, σ, and ρ with the weight of habits γ on felicity and the consumption tax rate τ. The habit stocks influence a long-run growth rate through the habit parameter γ, in combination with σ. In a specific case that no habit exists in preferences (i.e., $\gamma=0$), the parameter ξ collapses to σ, thus its long-run growth rate is $\sigma^{-1}[r(\tilde{z};\tau)-\rho]$.

Ceteris paribus,[22] the next lemma summarizes the growth comparison between endogenously growing economies with and without habit persistence.

Lemma 2: *When the effective elasticity of intertemporal substitution* ξ^{-1} *is higher than the elasticity of intertemporal substitution* σ^{-1}*, the balanced growth path with Joneses habit stocks grows faster than the balanced growth path with no habit stocks in the decentralized competitive economy with any feasible fiscal policies.*

Moreover, in the Joneses habit economies with $\xi^{-1}\geq\sigma^{-1}$ (i.e., fast-growing competitive economies with external habits), habit persistence is associated with catching-up-with-the-Joneses preferences or keeping-up-with-the-Joneses preferences, whereas the Joneses habit economies with $\xi^{-1}\leq\sigma^{-1}$ (i.e., slow-growing competitive economies) are associated with running-away-from-the-Joneses preferences. Notably, competitive economies with catching/keeping-up-with-the-Joneses preferences result from altruism/admiration (*resp*. envy/jealousy) $\gamma<0$ (*resp*. $0<\gamma<1$) along with a high (*resp*. low) elasticity of intertemporal

[22] Here, I suppose that the consumption–capital ratio $\tilde{z}(t)$ is identical in both economies. In general, the ratio $\tilde{z}(t)$ is not necessarily the same. Nevertheless, this ceteris paribus condition helps to illustrate the effect of habits when the equilibrium conditions are not explicitly solvable.

substitution σ^{-1}, whereas competitive economies with running-away-from-the-Joneses preferences result from altruism/admiration (*resp*. envy/jealousy) along with a low (*resp*. high) elasticity of intertemporal substitution. Therefore, the long-run economic performance in the presence of habits confirms the joint condition of a type of habits in preferences and an elasticity of intertemporal substitution.

The rate of returns $r(\tilde{z};\tau)=\alpha A\tau^{1-\alpha}\tilde{z}^{1-\alpha}$ captures the property of the public finance growth model with consumption taxation and productive public spending under balanced budget constraints. This property implies that external habits and public capital services generate permanent distortions in the competitive economy, whereas it induces a proper fiscal policy to correct for such distortions in a planning economy. Contrary to the habit literature (Carroll, Overland, and Weil, 2000) where only internal habits along with a sufficiently high elasticity of intertemporal substitution have a long-run effect, the present model shows that external habits with no restriction on the elasticity of intertemporal substitution σ^{-1} have a long-run effect.[23]

To examine the role of productive fiscal policy on long-run economic growth, I consider that in the decentralized competitive economy, public capital services are not productive in private production. That is, in the case that $\alpha=1$, the realized rate of returns becomes $r(\tilde{z};\tau)=A$ in Eqs. (13a) and (13b). Thus, this public finance growth model becomes an AK–growth model with habit persistence in preferences. From Eqs. (14a) and (14b), the new balanced growth path satisfies that $\tilde{z}=[(1+\tau)\xi]^{-1}[(\xi-1)A+\rho]$ and $\tilde{x}=\varphi\xi[A-\rho+\delta\xi]^{-1}$. Thus, Eq. (12a) implies that the long-run growth rate becomes $\xi^{-1}[A-\rho]$. Except the generalized preference parameter $\xi^{-1}>0$, the positive long-run growth condition coincides with the usual growth condition $A-\rho>0$ in an AK–growth model (Rebelo, 1991). This model suggests that habit persistence continuously affects the long-run growth rate through the effective elasticity of intertemporal substitution ξ^{-1} in an AK–growth model without productive fiscal policies. The long-run growth properties in Proposition 2 and Lemma 2 are generalized to the usual growth condition in an AK–growth model with habit persistence in preferences.

IV. Equilibrium Cycles in the Competitive Economy

In this section, I investigate the uniqueness and stability property of a transitional competitive equilibrium path. I establish the local indeterminacy of the balanced

[23] Turnovsky and Monteiro (2007) show that external habits do not affect the long-run growth rate of a balanced growth path in an endogenous growth model with capital externalities. In addition, Alonso-Carrera, Cabellé, and Raurich (2005) introduce an asymptotic linear technology for nontrivial transitional dynamics of competitive allocations in a discrete-time growth model with Joneses habit persistence.

growth path and the equilibrium cycles of the transitional dynamics around a balanced growth path. I now linearize the reduced system of the dynamic equations of $\{z, x\}$ given exogenous fiscal policies $\{\tau, g\}$ corresponding to the competitive dynamic equations of $\{c, h, k\}$ in a growing competitive economy with habit stocks. As in the previous section, suppose that $\{z, x\}$ corresponds to $\{c, h, k\}$. The control variable $c(t)$, together with the two state variables $h(t)$ and $k(t)$ in the dynamic Eqs. (12a)–(12d), implies that either $z(t)$ or $x(t)$ is a non-predetermined jump variable.[24]

When all eigenvalues of the linearized system of the continuous dynamic equations of $\{z, x\}$ are positive or unstable near the balanced growth path $\{\tilde{z}, \tilde{x}\}$, the transitional equilibrium path vanishes; thus, the balanced growth path is locally determinate. This result occurs because one of the initial values for $\{z, x\}$ is free to jump to the balanced growth path; otherwise, the transitional equilibrium path violates the transversality condition. When one eigenvalue is negative or stable and the other eigenvalue is positive or unstable around the balanced growth path, the transitional equilibrium paths are saddle stable, in which the unique transitional equilibrium path asymptotically converges to the unique balanced growth path; thus, the balanced growth path is locally determinate. No sunspot equilibrium exists, so no self-fulfilling equilibrium cycles emerge in either case of the determinate balanced growth path. Most interestingly, when the two eigenvalues are negative around the balanced growth path, the balanced growth path is absolutely stable. A continuum of transitional equilibrium paths exists in the same fundamentals, each of which asymptotically converges to the unique balanced growth path; thus, the balanced growth path is locally indeterminate from the initial capital and habit stocks. Hence, equilibrium cycles emerge in the self-fulfilling rational expectation equilibrium in a decentralized competitive economy with external habits and exogenous fiscal policies.

Applying the usual practice to determine the sign of eigenvalues in the dynamic system, linearizing the dynamic Eqs. (13a) and (13b), and evaluating them at the steady state $\{\tilde{z}, \tilde{x}\}$ in Eqs. (14a) and (14b) yields a 2×2 matrix such that

$$\begin{bmatrix} \dot{z} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} (1+\tau)\tilde{z} - \left[\dfrac{\sigma-\alpha}{\sigma}\right]\left[\dfrac{1-\alpha}{\alpha}\right] r(\tilde{z};\tau) & \left[\dfrac{\varphi(\sigma-\xi)}{\sigma}\right]\dfrac{\tilde{z}}{\tilde{x}^2} \\ -\left[\dfrac{1-\alpha}{\sigma}\right]\dfrac{\tilde{x}r(\tilde{z};\tau)}{\tilde{z}} & -\left[\dfrac{\varphi\xi}{\sigma}\right]\dfrac{1}{\tilde{x}} \end{bmatrix} \begin{bmatrix} z-\tilde{z} \\ x-\tilde{x} \end{bmatrix},$$

[24] Alternative transformation of variables, $p \equiv c/k$ and $q \equiv h/k$, verifies that one non-predetermined p and one predetermined variable q exist in the model with one control and two state variables $\{c, h, k\}$.

where $\xi \equiv \gamma + \sigma(1-\gamma)$ and $\eta(\tilde{z};\tau) \equiv \alpha A \tau^{1-\alpha} \tilde{z}^{1-\alpha}$.

To determine the sign of the two eigenvalues of the Jacobian matrix, called Θ, I compute the determinant of Θ as

$$det\Theta = -\frac{\varphi}{\sigma\tilde{x}}\left[\rho + \left[\frac{\xi-\alpha}{\alpha}\right] r(\tilde{z};\tau)\right].$$

The sign of $\xi-\alpha$ determines the sign of $det\Theta$, given the other positive parameters $\{\alpha,\rho,\varphi,\sigma\}$ and the positive variables $\tilde{x}$ and $r(\tilde{z};\tau)$. In particular, the positive value of $\xi-\alpha$ ensures that $det\Theta < 0$.[25] Hence, the matrix Θ has one positive and one negative eigenvalue, so the equilibrium paths $\{z,x\}$ are saddle stable. Therefore, the transitional equilibrium path is unique and converges asymptotically to the unique balanced growth path $\{\tilde{z},\tilde{x}\}$.

The following proposition summarizes the above analysis.

Proposition 3: *Suppose the effective elasticity of intertemporal elasticity* ξ^{-1} *is smaller than the inverse of a share of private capital* α^{-1}*, the transitional equilibrium path* $\{c(t),h(t),k(t)\}$ *is unique from the initial capital stock* k_0 *and the initial habit stock* h_0*. Hence, the balanced growth path is locally determinate, thereby no self-fulfilling equilibrium cycle arises in the decentralized competitive economy with altruistic or envious habit preferences and a time-invariant consumption tax rate* τ *and productive public spending* $g(t)$.

However, when $\xi-\alpha<0$ is negative,[26] $det\Theta > 0$ if and only if

$$r(\tilde{z};\tau) > -\frac{\alpha\rho}{\xi-\alpha}.$$

Under the positive value of $det\Theta$, the eigenvalues of the matrix Θ can be either positive or negative. To assign the sign of each of the eigenvalues of the matrix Θ, I must assign the sign of the trace of the matrix Θ. Specifically, under $det\Theta > 0$, the two eigenvalues are negative whenever the trace of the matrix Θ is negative. Otherwise, the two eigenvalues are positive.

A simple computation yields the trace of the matrix Θ as

[25] When $\sigma^{-1} < 1$, $\xi \equiv \gamma + \sigma(1-\gamma)$ is always greater than 1. Thus, $\xi-\alpha$ is always positive. Alonso-Carrera, Cabellé, and Raurich (2005) restrict their model to this range of the parameter values, so that their equilibrium is determinate in the presence of internal and external habits.

[26] If the elasticity of intertemporal substitution σ^{-1} is greater than 1, $\xi-\alpha$ can be less than 0, and thus $det\Theta > 0$.

$$tr\Theta = \frac{1}{\sigma\xi}[(-\sigma + \xi(\sigma - \alpha))r(\tilde{z};\tau) - \delta\xi^2 + \rho(\sigma + \xi)].$$

Hence, $tr\Theta < 0$ when $r(\tilde{z};\tau) > [\delta\xi^2 - \rho(\sigma + \xi)]/[-\sigma + \xi(\sigma - \alpha)]$. Then, the condition that $tr\Theta < 0$ under $det\Theta > 0$ is equivalent to

$$r(\tilde{z};\tau) > \max\left\{-\frac{\alpha\rho}{\xi - \alpha}, \frac{\delta\xi^2 - \rho(\sigma + \xi)}{-\sigma + \xi(\sigma - \alpha)}\right\}.$$

Therefore, when $tr\Theta < 0$ under $det\Theta > 0$, the two eigenvalues of the matrix Θ are negative, the balanced growth path $\{\tilde{x}, \tilde{z}\}$ is absolutely stable, and a continuum of transitional competitive paths $\{x, z\}$ exists, each of which converges to the unique balanced growth path $\{\tilde{x}, \tilde{z}\}$.

Alternatively, $tr\Theta > 0$ when $r(\tilde{z};\tau) < [\delta\xi^2 - \rho(\sigma + \xi)]/[-\sigma + \xi(\sigma - \alpha)]$, Therefore, $tr\Theta > 0$ under $det\Theta > 0$ is equivalent to

$$-\frac{\alpha\rho}{\xi - \alpha} < r(\tilde{z};\tau) < \frac{\delta\xi^2 - \rho(\sigma + \xi)}{-\sigma + \xi(\sigma - \alpha)},$$

so that the two eigenvalues of the matrix Θ are positive. This result implies that the transitional path vanishes into the unique balanced growth path. Otherwise, it violates the transversality condition, which, in effect, makes it locally determinate.

I summarize the indeterminacy property and thus emergence of equilibrium cycles in the balanced growth path in the decentralized competitive economy with external habit stocks and exogenous fiscal policies in the following proposition.

Proposition 4: *Under the conditions in Propositions 1 and 2, the continuum of transitional equilibrium paths* $\{c(t), h(t), k(t)\}$ *exists from the initial capital stock* k_0 *and the initial habit stock* h_0*, providing that (i) the effective elasticity of intertemporal substitution* ξ^{-1} *is larger than the inverse of a share of private capital* α^{-1}*, and (ii) the long-run realized interest rate* $\eta(\tilde{z};\tau)$ *is sufficiently large; that is,* $r(\tilde{z};\tau) > \max\{-\frac{\alpha\rho}{\xi-\alpha}, \frac{\delta\xi^2 - \rho(\sigma+\xi)}{-\sigma+\xi(\sigma-\alpha)}\}$. *Therefore, the balance growth path* $\{\tilde{c}, \tilde{h}, \tilde{k}\}$ *is locally indeterminate, whereby self-fulfilling equilibrium cycles arise in the decentralized competitive economy with altruistic or envious habit preferences and the time-invariant consumption tax rate* τ *and productive public spending* $g(t)$.

Along with the endogenous growth mechanism with exogenous fiscal policies, which include the productive government expenditure and distortive consumption tax, the presence of external habits—either altruism/admiration or envy/jealousy—

acts as a destabilizer in a growing economy. The indeterminacy of competitive equilibria emerges because each agent makes consumption and investment decisions without taking into account other agents' level of habit stocks and the government's public capital services, which, in effect, rely on all other agents' consumption and investment decisions in a decentralized competitive economy. In the presence of both external habit stocks and exogenous public capital services, the lack of coordination among agents causes indeterminacy around the unique balanced growth path, and equilibrium cycles are driven by self-fulfilling rational expectations in a decentralized competitive economy.

The emergence of indeterminacy is intuitive in a growing competitive economy. Starting with an arbitrary competitive equilibrium path, consider constructing another competitive equilibrium path by increasing the savings rate and accelerating the accumulation of private capital. For this path to be an equilibrium path, the realized rate of returns to private capital must increase to justify the higher accumulation rate of private capital. This increase can be accomplished by reallocating the current consumption and the private capital accumulation in the presence of habit stocks and positive public capital services. On the one hand, the increase in private capital, ceteris paribus, decreases at least temporarily but maintains its level of future rate of returns due to production externalities. This effect results from the productive public capital services provided by the consumption taxation. On the other hand, an increase in private capital reduces current consumption at least temporarily. The reduction of current consumption is allowed in equilibrium whenever the effective elasticity of intertemporal substitution is high along with habit persistence in preferences.[27] The high effective elasticity of intertemporal substitution is accompanied by a decline of the absolute value of shadow prices of habit stocks—either with a positive shadow price of altruism/admiration as positive habit externalities or a negative shadow price of envy/jealousy as negative habit externalities. Therefore, indeterminacy arises under the high interest rate with public capital services as production externalities and a high effective elasticity of intertemporal substitution with habit stocks incorporating altruistic and envious habit persistence in preferences.

I now check the robustness of equilibrium cycles of the balanced growth path in a competitive economy with external habits and exogenous fiscal policies. First, consider a growing competitive economy with public capital services but no habit stocks. When no habit stocks exist (i.e., $\gamma = 0$, $\varphi = 0$, and $\delta = 0$), the 2×2 matrix Θ collapses to a 1×1 matrix whose element consists of the first row and the first column of the matrix. This resulting element, $(1+\tau)\tilde{z} - [(\sigma-\alpha)(1-\alpha)/$

[27] In the absence of habits, the value of investment continuously increases. Thus, the alternative equilibrium eventually violates the transversality condition. Hence, an agent's belief regarding another equilibrium path cannot be fulfilled without habit persistence. The balanced growth path is determinate, and the transitional path is uniquely determined.

$\alpha\sigma]r(\tilde{z};\tau)$, is always positive, as indicated in Eq. (14a). Because $z(t)$ is the only non-predetermined variable in this extreme case, a transitional equilibrium path from the initial habit and capital stocks vanishes into the unique balanced growth path. As expected in a usual growth model without habits, no transitional dynamic path exists, and the balanced growth path is locally determinate, thereby displaying no equilibrium cycles. This determinacy in an endogenous growth model with consumption taxation is an extension of the aggregate stability property in a model with capital taxation and productive public services (Barro, 1990). Importantly, this analysis suggests that in the absence of habit persistence in preferences, productive public services as production externalities are not sufficient for the existence of equilibrium cycles. The main feature of habit persistence shifts the main element of indeterminacy from production externalities to consumption externalities in preferences in a general rational expectations economy.

Second, consider a perpetually growing AK–economy with no government policy. That is, $\alpha = 1$ and thus $det\Theta < 0$. The balanced growth path is saddle stable and displays no indeterminacy. No growth fluctuation occurs in habit economies with no productive spending. I, however, consider an asymptotic AK–production function with productive public capital services. In the case that $\alpha \to 1$, $r(\tilde{z};\tau) \equiv \alpha A \tau^{1-\alpha} \tilde{z}^{1-\alpha} \to A$. Therefore, the indeterminacy condition is satisfied in Proposition 4: $\xi - 1 < 0$ and $r(\tilde{z};\tau) = A > \max\{-\frac{\rho}{\xi-\alpha}, \frac{\delta\xi^2 - \rho(\sigma+\xi)}{-\sigma+\xi(\sigma-1)}\}$ when the total factor productivity A is sufficiently large. An asymptotic AK–technology with productive government policy generates indeterminacy in a growing economy with habit persistence as long as the total factor productivity is sufficiently large. The governmental role in productive public services is essential for the emergence of indeterminacy. This analysis suggests that in the presence of habit persistence, equilibrium cycles occur in a competitive economy with productive public services.

Third, a theoretical parameter value of the marginal productivity of public capital (services) takes a 1 minus a share α of private capital in the AK-production function. First, the usual share of private capital is close to 0.34, so that the marginal productivity of public capital is $1-\alpha = 0.66$. This value seems too high. Benhabib and Gali (1995) suggest the marginal productivity of public capital can be approximately 0.15 by assuming that the share of private capital is around 0.85 in the context of the public-finance endogenous growth model. Nevertheless, the estimates of the marginal productivity of public capital are far from any consensus in the literature. Aschauer (1989) finds that a 1 percent increase in the public capital stock increases a 0.39 percent increase in the final output. Recently, Bom and Ligthart (2014) have estimated that the short-run marginal productivity of public capital is 0.083, which increases to 0.122 in the long run. Hence, such a broad range of those estimates provides a room for satisfying the indeterminacy condition on the effective elasticity of intertemporal substitution and the realized rate of interest in the decentralized competitive economy with Joneses habit preferences and productive public capital.

Fourth, equilibrium cycles also prevail under the condition that $\varphi = \delta$ in the habit evolution function (see Eq. (2)). That is, aggregate growth instability relies neither on the specification of habit formation nor on a magnitude of coefficient of habit formation. This result differs from Chen, Hsu, and Mino's (2010) indeterminacy, which requires a sufficiently large coefficient of the current consumption for habit formation in keeping-up-with-the-Joneses preferences. Also different from the literature, indeterminacy in my model can emerge in the presence of keeping-up-with-the-Joneses (i.e., $\varphi \neq 0$, $\delta > 0$, and $\gamma(1-\sigma) > 0$), catching-up-with-the-Joneses (i.e., $\varphi \neq 0$, $\delta \to \infty$, and $\gamma(1-\sigma) > 0$), or running-away-from-the-Joneses preferences (i.e., $\gamma(1-\sigma) < 0$).

Finally, regardless of multiplicative or subtractive habit preferences, the habit formation function in the present study is widely introduced in the literature. As in Carroll, Overland, and Weil (2000) and Turnovsky and Monterio (2007), this habit formation function has an arithmetic weighted habit evolution function as in Eq. (2) along with multiplicative habit preferences as in Eq. (1). Along with subtractive utility function, Constantinides (1990) and Lee and Park (2015) adopt the same arithmetic weighted habit evolution function. Campbell and Cochran (1999) and Alvarez-Cuadrado, Monteiro, and Turnovsky (2004) extend to a generalized habit formation function:

$$h(t) = \left\{ [h(0)e^{-\delta t}]^{\mu} + \varphi \int_{-\infty}^{t} [e^{-\delta(t-s)} \overline{c}(s)]^{1-\mu} ds \right\}^{\frac{1}{\mu}},$$

where $0 < \mu < 1$. This generalized function induces to a geometric weighted average habit evolution function: $\dot{h}(t) = \varphi[\overline{c}(t)]^{\mu}[h(t)]^{1-\mu} - \delta h(t)$. The generalized habit formation function captures the intertemporal complementarity/substitutability between the current consumption and the habit stocks. Using this habit formation function, Chen (2007) shows the global (not local) indeterminacy when the intertemporal complementarity (not substitutability) reinforces the internal (not external) habit formation. However, my conjecture is that such a generalization would not enrich the indeterminacy property in this paper. In contrast with Chen (2007), this study shows that the local indeterminacy depends not on the intertemporal complementarity but on the (effective) elasticity of intertemporal substitution in the utility function.

V. Social Optimum Allocation with Distortionary Fiscal Policy

My main concern in the next two sections is to study the role of fiscal policies as a selection mechanism among indeterminate competitive equilibria under altruistic or envious habit preferences. I set up a social planning problem, corresponding to a competitive economy with external habit stocks and exogenous fiscal policies, in which a benevolent planner internalizes external habit stocks $h(t)$ and chooses fiscal policies to finance public capital services $g(t)$ by imposing a time-variant consumption tax $\tau(t)$.[28] The consumption tax and public capital services are endogenous in the social optimum allocation. The social planner intends to restore social efficiency under distortive consumption taxes and productive public services to each private agent's decision in the decentralized competitive economy. This fiscal policy is not first-best policy. Particularly, the social planner possesses the distortionary fiscal policy instrument to correct a market failure from habit externalities and to provide public capital services efficiently to the private firm's production decision by imposing distortive consumption taxes.[29]

Formally, the social planner maximizes the lifetime utility of Eq. (4) for the representative household subject to the evolution function of habit stocks in Eq. (12c), economy-wide feasible sets in Eq. (12b), and balanced budget constraints in Eq. (10). The current-value Hamiltonian equation $\hat{H}(c,h,k,\tau,\lambda_h,\lambda_k)$ for the planner's problem is defined as

$$\hat{H}(c,h,k,\tau,\lambda_h,\lambda_k) \equiv \frac{1}{1-\sigma}\left[\frac{c}{h^{\gamma}}\right]^{1-\sigma} + \lambda_h[\varphi c - \delta h] + \lambda_k[A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha} - (1+\tau)c],$$

where λ_h is a multiplier for the habit evolution equation, and λ_k is a multiplier for the resource feasibility constraints under the balanced budget constraints $g(t) = \tau(t)c(t)$, incorporating distortionary fiscal policies $g(t)$. The first-order

[28] Noticeably, the social optimal solution for external habits can be interpreted as internal habits because a social planner takes into account consumption externalities in the decentralized competitive economy and attempts to internalize external habit stocks in a household's decision. Moreover, the distortionary fiscal policies in the social planning economy are also determined by taking into account an agent's decision in social optimum.

[29] The optimal allocation of this planning problem is in contrast with those for the first-best efficient allocation in the literature. For example, Alonso-Carrera, Cabellé, and Raurich (2005) and Turnovsky and Monteiro (2007) solve for the two allocations—one for an imperfectly competitive equilibrium due to consumption externalities and the other for a social optimum by internalizing consumption externalities—and choose an implementable optimum tax/subsidy rate that is equal to the wedge between the competitive and social allocations. However, I cannot apply their method because, unlike their models, my model does not have a policy instrument for a lump-sum transfer of tax revenue.

necessary conditions for an interior solution for $\tau(t)$, $c(t)$, $k(t)$, $\lambda_k(t)$, $h(t)$, and $\lambda_h(t)$, including the transversality conditions, are, respectively, as follows:

$$\lambda_k[(1-\alpha)A[c^{1-\alpha}\tau^{-\alpha}]k^{\alpha}-c]=0\,, \tag{15a}$$

$$\frac{c^{-\sigma}}{h^{\gamma(1-\sigma)}}+\lambda_h\varphi+\lambda_k[(1-\alpha)A[c^{-\alpha}\tau^{1-\alpha}]k^{\alpha}-(1+\tau)]=0\,, \tag{15b}$$

$$\dot{\lambda}_k=\rho\lambda_k-\lambda_k\alpha A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha-1}\,, \tag{15c}$$

$$\dot{k}=A[c^{1-\alpha}\tau^{1-\alpha}]k^{\alpha}-(1+\tau)c\,, \tag{15d}$$

$$\dot{\lambda}_h=(\rho+\delta)\lambda_h+\gamma\left[\frac{c^{1-\sigma}}{h^{1+\gamma(1-\sigma)}}\right], \tag{15e}$$

$$\dot{h}=\varphi c-\delta h\,. \tag{15f}$$

I suppose that $\hat{H}^*(h,k;\lambda_h,\lambda_k)=\max_{\{c,\tau\}}\hat{H}(c,h,k,\tau,\lambda_h,\lambda_k)$. To ensure that an interior solution exists for the social planner's problem, I assume that $\hat{H}^*(h,k;\lambda_h,\lambda_k)$ is convex in $\{h,k\}$ given $\{\lambda_h,\lambda_k\}$. In addition to the underlining assumptions on the felicity, production, and habit evolution function as in a competitive economy, the convexity assumption on $\hat{H}^*(h,k;\lambda_h,\lambda_k)$ ensures the Arrow–Mangasarian conditions for the planner's optimization problem. I am unable to specify explicitly the parameter values for the convexity condition in this model because the dynamic functions in Eqs. (15a)–(15f) are an implicit and nonlinear function of $\{c,h,k,\tau,\lambda_h,\lambda_k\}$.[30] Later in the section, I establish the uniqueness and stability of the social optimum, thereby justifying the convexity of the planning problem and characterizing its optimum interior solution to the planner's problem for consumption taxation and productive expenditure policies, corresponding to a decentralized competitive economy with Joneses habit persistence.

As shown in Eq. (A.6) in Appendix I, the dynamic of a consumption path in $\{z,x,\tau,q\}$ is dictated by

$$\frac{\dot{c}}{c}=\frac{1}{\sigma}\left[\left[\frac{q}{q-\varphi}\right]\hat{r}-\rho+\frac{\varphi\gamma}{x}+\delta\left[\xi-\sigma+\frac{\varphi}{q-\varphi}\right]\right],$$

[30] To the best of my knowledge, the literature does not clearly specify the necessary and sufficient condition for a solution to internal habit models of a multiplicative felicity function. Alonso-Carrera, Cabellé, and Raurich (2005) show that $\sigma>1$ and $0<\gamma<1$ ensure an interior solution but are not sufficient for optimization in a discrete time model. In a continuous time model with habit persistence, Chen (2007) imposes a restriction on the parameter values such that the Hamiltonian equation is convex in all state variables in an AK–production function. His multiplicity requires an inelastic rate of intertemporal substitution.

where $\hat{r} = r(z;\tau) = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}$ is the realized rate of interest in the social optimum allocations.[31] A simple observation shows that the rate of consumption growth clearly relies on the parameters φ and δ of habit persistence. This property suggests that the growth rates are different between keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences. Through the effective elasticity of intertemporal substitution ξ^{-1}, altruism/admiration with negative γ or envy/jealousy with positive γ also influences the economic growth rates of the social optimum allocations. In addition, the rate of consumption induces the transversality condition for the social optimum allocation:

$$\frac{1}{\sigma}\left[\left[\frac{q}{q-\varphi}\right]\hat{r} - \rho + \frac{\varphi\gamma}{x} + \delta\left[\xi - \sigma + \frac{\varphi}{q-\varphi}\right]\right] < \rho . \tag{15g}$$

The following proposition shows the existence conditions for a social optimum allocation with habit stocks and fiscal policies:

Proposition 5: *Suppose $\hat{H}^*(h,k;\lambda_h,\lambda_k)$ is convex in $\{h,k\}$ with endogenous fiscal policies $\{\tau(t),g(t)\}$, the social optimum allocation $\{c(t),h(t),k(t)\}$ with Joneses habit preferences exists if and only if the sequence of $\{c(t),h(t),k(t),\tau(t),\lambda_h(t),\lambda_k(t)\}$ satisfies the system of dynamic Eqs. (15a)–(15g) from the initial capital stock k_0 and the initial habit stock h_0.*

Now, I investigate the economic growth condition of the planner's optimal allocation and its associated distortionary fiscal policies. As in previous sections for the competitive equilibrium, I introduce the variables $z(t) \equiv c/k$ and $x(t) \equiv h/c$ in a perpetually growing economy. I also introduce an additional auxiliary variable $q(t) \equiv \lambda_k / \lambda_h$, the ratio of the shadow value of capital stocks with respect to the shadow price of habit stocks. By applying the usual practice for a growing economy, I reduce the six-dimensional dynamic system of $\{c,h,k,\tau,\lambda_h,\lambda_k\}$ to a four-dimensional dynamic system of $\{z,x,\tau,q\}$. Appendix I provides the simple but tedious manipulation on Eqs. (15a)–(15f) that yields the following system of the dynamic equations for $\{z,x,\tau,q\}$:

$$\dot{z} = z\left[z + \frac{\varphi\gamma}{x} + \frac{1}{\sigma}\left[\left[\frac{q}{q-\varphi} - \sigma\right]\hat{r} - \rho + \delta\left[\xi - \sigma + \frac{\varphi}{q-\varphi}\right]\right]\right], \tag{16a}$$

$$\dot{x} = (1-\gamma)\varphi - \frac{x}{\sigma}\left[\left[\frac{q}{q-\varphi}\right]\hat{r} - \rho + \delta\left[\xi + \frac{\varphi}{q-\varphi}\right]\right], \tag{16b}$$

[31] See the derivation for Eq. (A.2) in Appendix I.

$$\dot{q} = -q\left[\hat{r} + \frac{\gamma[q-\varphi]}{x} + \delta\right], \tag{16c}$$

where ξ^{-1}, $\xi \equiv \gamma + \sigma(1-\gamma)$, is again the effective elasticity of intertemporal substitution. The optimal tax rate (see Eq. (A.1) in Appendix I) is such that

$$\tau = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{z}. \tag{16d}$$

The socially optimal consumption tax rates are time variant, state contingent, and inversely related to the consumption–capital ratio z. Ceteris paribus, the optimal consumption tax rate increases in capital but decreases in consumption.[32]

This dynamic equation also suggests that the social optimum allocation enjoys nontrivial transitional dynamics with habit persistence. However, in an economy with no habit formation (e.g., $\gamma = 0$, $\varphi = 0$, and $\delta = 0$), this rate of growth is equivalent to the one in common public finance endogenous growth models with a social optimum tax rate $\tau(t)$ and public capital services $g(t)$ under balanced budget conditions; that is, the realized rate of returns $\hat{r} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}$ is constant over time and the economic growth rate is $\dot{c}/c = \dot{k}/k = \sigma^{-1}[\hat{r} - \rho]$. Transitional dynamics do not exist without habit persistence. Hence, habit persistence induces nontrivial transitional dynamics in the social optimum allocation with distortionary fiscal policies.

Before characterizing a transitional path, I focus on the long-run properties of a socially optimum allocation with distortionary fiscal policies. The balanced growth path for a social planning allocation satisfies that (i) the allocations $c(t)$, $k(t)$, and $h(t)$ grow at a constant rate over time; (ii) the shadow prices of habit stocks and capital stocks change at a same constant rate; and (iii) the consumption tax rate $\tau(t)$ remains constant over time. Specifically, in the balanced growth path $\{\tilde{c}, \tilde{k}, \tilde{h}, \tilde{\tau}\}$, the corresponding system of equations $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$ in Eqs. (16a)–(16d) satisfies that $\dot{z}/z = \dot{x}/x = \dot{\tau}/\tau = \dot{q}/q = 0$. As before, a tilde over a variable denotes a stationary variable. By a simple manipulation of Eqs. (16a)–(16d), the balanced growth path $\tilde{z}, \tilde{x}, \tilde{\tau}$ and $\tilde{q}$ satisfies, respectively,

$$\tilde{z} = \frac{1}{\xi}[-(1-\xi)\hat{r} + \rho], \tag{17a}$$

[32] Gómez (2006) and Turnovsky and Monteiro (2007) provide a similar property for a (first-best) social optimum consumption tax rate. Unlike my social optimum policy, their method is based on the implementation of a social optimum allocation. In addition, their policy instruments ignore the distortive effects in agents' decisions.

$$\tilde{x} = \frac{\varphi\xi}{\hat{r} - \rho + \delta\xi}, \tag{17b}$$

$$\tilde{\tau} = \frac{(1-\alpha)\xi\hat{r}}{\alpha[-(1-\xi)\hat{r} + \rho]}, \tag{17c}$$

$$\tilde{q} = \varphi\left[1 - \frac{\xi[\hat{r} + \delta]}{\gamma[\hat{r} - \rho + \delta\xi]}\right], \tag{17d}$$

where $\hat{r} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}$. Appendix II provides the derivations of Eqs. (17a)–(17d). In addition, by substituting Eqs. (16d) and (17a) into Eq. (15d), I also determine that the long-run growth rate $\tilde{\Gamma}_{SO}$ of consumption, capital, and habit stocks is expressed as

$$\tilde{\Gamma}_{SO} = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \frac{\hat{r} - \rho}{\xi}. \tag{17e}$$

The condition $\hat{r} - \rho > 0$ with $\xi^{-1} > 0$ assures a positive balanced growth; that is, the realized rate of interest should be larger than the rate of time preferences. Moreover, in Eqs. (17a)–(17d), I impose the interior condition for a strictly positive value for $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$ such that $\hat{r} - \rho + \delta\xi > 0$ for Eqs. (17b) and (17c) and $\rho > (1-\xi)\hat{r}$ for Eqs. (17a) and (17c). In the perpetually growing social optimum in Eq. (17e), $\hat{r} - \rho + \delta\xi > 0$ is not binding, and $\rho > (1-\xi)\hat{r}$ is binding only when ξ is between zero and 1. Hence, I impose the inequality condition, $\rho > (1-\xi)\hat{r} > (1-\xi)\rho$, to ensure the existence of an interior balanced growth path in the social optimum. The second inequality is from the transversality condition. Under this interior condition, the social optimum balanced growth path exists in the presence of habit persistence with a consumption tax and public capital services. In fact, the balanced growth path is also the unique solution to Eqs. (17a)–(17d).

The following proposition summarizes the long-run properties for the social optimum allocation in a planning economy with habit persistence and optimal fiscal policies.

Proposition 6: *Under the interiority condition that $\rho > (1-\xi)\hat{r} > (1-\xi)\rho$ with Joneses habit preferences, the unique and stationary social optimum allocation $\{\tilde{z}, \tilde{x}, \tilde{q}\}$ with the endogenous fiscal policies $\{\tilde{\tau}, \tilde{g}\}$ exists if and only if the sequence of $\{\tilde{z}, \tilde{x}, \tilde{q}\}$ satisfies Eqs. (17a)–(17d). Moreover, the long-run growth rate $\tilde{\Gamma}_{SO}$ is equal to $\xi^{-1}[\hat{r} - \rho] > 0$ where $\hat{r} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}$.*

Similar to the decentralized competitive economy, habit persistence alters the elasticity of intertemporal substitution σ^{-1} to the effective elasticity of intertemporal substitution ξ^{-1} in household decisions. The long-run growth rate is affected by habit persistence in preferences through the effective elasticity of intertemporal substitution ξ^{-1}, but the realized rate of returns $\hat{r}$ is independent of the specification of habit formation in the long run (see also Turnovsky and Monteiro, 2007). Thus, habit persistence in preferences influences not only the short-run but also the long-run dynamics in the social optimum. The realized long-run rate of returns to capital $\hat{r}$ is identical to a long-run interest rate as in a standard public finance endogenous growth model without habit persistence (e.g., Park and Philippopoulos, 2004). The stationary consumption tax rate in Eq. (17c) is also affected by habit preferences through the effective elasticity of intertemporal substitution ξ^{-1} and technology with the productive public spending $1-\alpha$.[33] This result also confirms that the consumption tax has a long-run distortionary effect under balanced budget conditions with productive public spending.

VI. Aggregate Stability in Social Optimum Allocation

In this section, I examine the aggregate stability of a transitional optimum path and the determinacy property for the associated balanced growth path. Again, by applying the usual method for stability analysis, I first linearize the system of equations of $\{z, x, \tau, q\}$ in Eqs. (16a)–(16d) and then evaluate it in the balanced growth path $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$ in Eqs. (17a)–(17d). As in the previous sections, for the decentralized competitive economy, one of the two variables $z(t) \equiv c/k$ and $x(t) \equiv h/c$ is a predetermined variable because consumption $c(t)$ is a choice variable, and both habit stocks $h(t)$ and capital $k(t)$ are state variables. The ratio of the shadow values of the capital and habit stocks $q(t) \equiv \lambda_k / \lambda_h$ is a non-predetermined jump variable because the shadow prices λ_k and λ_h are co-state variables. I reduce one more dimension because $\tau(t)$ and $z(t)$ contain the same dynamic information as in Eq. (16d).

Hence, if all three eigenvalues are positive around the balanced growth path in the linearized system of Eqs. (16a)–(16c), the unique path $\{z, x, \tau, q\}$ exists and immediately converges to the balanced growth path $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$. Thus, no transitional dynamic path exists. If one eigenvalue is negative and two eigenvalues are positive near the balanced growth path $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$, the transitional optimum path $\{z, x, \tau, q\}$ is unique and saddle stable, and it converges to the unique balanced growth path. If more than two eigenvalues are negative, an infinite

[33] Gómez (2006) focuses on the unique transitional and steady-state equilibrium path in an endogenous growth model of external habits and consumption tax. Nevertheless, his optimal consumption tax rates are multiple under an implementable fiscal policy. This tax policy indicates the emergence of global multiplicity and sunspots of socially optimal allocations.

number of transitional optimum paths $\{z, x, \tau, q\}$ exist, with each one converging to the unique balanced growth path $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$. That is, a continuum of transitional optimal paths exists from the same initial habit stocks and capital stocks along with the same fundamentals of preferences and technology in a social optimum economy. The sign of the eigenvalues provides the sufficient condition for determining the uniqueness or multiplicity of transitional dynamic paths and the existence of equilibrium cycles around the unique optimum balanced growth path.

Formally, I linearize Eqs. (16a)–(16c) at the balanced growth path of Eqs. (17a)–(17d). As a result, I obtain the following linearized system of the three-dimensional dynamic equations of $\{z, x, q\}$:

$$\begin{bmatrix} \dot{z} \\ \dot{x} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \tilde{z} & -\dfrac{\gamma\varphi\tilde{z}}{\tilde{x}^2} & -\dfrac{\varphi(\tilde{r}+\delta)\tilde{z}}{\sigma(\tilde{q}-\varphi)^2} \\ 0 & -\dfrac{(1-\gamma)\varphi}{\tilde{x}} & \dfrac{\varphi(\tilde{r}+\delta)\tilde{x}}{\sigma(\tilde{q}-\varphi)^2} \\ 0 & \dfrac{\gamma\tilde{q}(\tilde{q}-\varphi)}{\tilde{x}^2} & -\dfrac{\gamma\tilde{q}}{\tilde{x}} \end{bmatrix} \begin{bmatrix} z-\tilde{z} \\ x-\tilde{x} \\ q-\tilde{q} \end{bmatrix}.$$

Let Φ be the Jacobian matrix of the linearized dynamic system of Eqs. (16a)–(16c). To determine the sign of eigenvalues of the matrix Φ, I take advantage of the sign of the determinant and trace of the matrix Φ. The sign of the determinant of the matrix Φ contains all the information necessary to determine the sign of each eigenvalue of the Jacobian matrix Φ.

By a simple computation, the determinant of the matrix Φ is

$$det\Phi = \frac{\varphi\xi}{\sigma}\left[\frac{\gamma\tilde{q}\tilde{z}}{\tilde{x}^2}\right].$$

Given the positive parameters φ and σ with the interior solution $\tilde{z} > 0$ and $\tilde{x} > 0$, the positive value of ξ in the growing economy implies that the sign of $det\Phi$ is determined only by the sign of $\tilde{q}$ and γ. Appendix III shows that the sign of $\tilde{q}$ is opposite to the sign of γ (i.e., $\gamma\tilde{q} < 0$); thus, $det\Phi$ is always negative in a socially optimal allocation. Hence, matrix Φ has either one or three negative eigenvalues. By simple observation, the matrix Φ can be divided into two submatrices: a 1×1 submatrix with the element $\tilde{z}$ at the upper and left-hand corner of matrix Φ and a 2×2 submatrix with the four elements at the lower and right-hand corner of matrix Φ. The 1×1 submatrix yields one positive eigenvalue of Φ so that the negative $det\Phi$ means that the 2×2 submatrix of Φ has one positive and one negative eigenvalue. The matrix Φ has two positive and one negative eigenvalues; thus, the system of the dynamic equations of $\{z, x, \tau, q\}$ has one stable and two unstable manifolds. Therefore, the transitional optimum path $\{z, x, \tau, q\}$ is saddle stable and converges to the unique balanced growth path $\{\tilde{z}, \tilde{x}, \tilde{\tau}, \tilde{q}\}$. The transitional dynamic path is unique from the initial capital stock k_0 and the initial habit stock h_0. The balanced growth path is locally determinate, and no sunspot occurs in the social optimum.

The following proposition summarizes the uniqueness and aggregate stability of the social optimum with endogenous fiscal policies.

Proposition 7: *With Joneses habit preferences and the endogenous fiscal policies* $\{\tilde{\tau}, \tilde{g}\}$, *the sequence of the social optimum balanced growth path* $\{\tilde{z}, \tilde{x}, \tilde{q}\}$ *is saddle stable and thus locally determinate. Hence, the socially optimal transitional path* $\{z(t), x(t), q(t)\}$ *is unique and converges to the unique social optimum balanced growth path* $\{\tilde{z}, \tilde{x}, \tilde{q}\}$.

The determinacy and stability result in the social planning economy has a few noticeable characteristics. The uniqueness of social optimum with the unique optimal fiscal policy is obtained under the condition of the effective elasticity of intertemporal substitution $\xi^{-1} > 0$, which contains the range of the habit parameter $\gamma < 1$ and the elasticity of intertemporal substitution $\sigma^{-1} > 0$ in a decentralized competitive economy. This set of the parameters includes the parameter values for indeterminacy of the decentralized competitive equilibrium (i.e., $0 < \sigma^{-1} < 1$ with $\gamma < 0$ or $0 < \gamma < 1$). Therefore, uniqueness is restored in the presence of altruistic and envious preferences with a high rate of elasticity of intertemporal substitution. The distortionary fiscal policy acts as a stabilizer for indeterminate competitive equilibrium paths. That is, the social planning program resolves the market failure of habit and production externalities and selects a unique equilibrium among many decentralized competitive equilibria. A distortionary fiscal policy serves as a selection mechanism. No short- and long-run sunspot exists, and no self-fulfilling equilibrium cycle emerges in the corresponding social planning economy.

Consumption taxation as an automatic stabilizer is regressive in consumption but progressive in capital. This feature of time-variant consumption tax rates is complementary to that in Guo and Lansing (1998) and Christiano and Harrison (1999). Their output and labor tax rates, respectively, are progressive for stabilizing sunspot business fluctuations in a growing economy with production externalities.[34] Hence, although a first-best policy instrument is not available, a planner can successfully choose a unique short- and long-run dynamic allocation for a perpetually growing economy. This analysis is in contrast with previous studies in the literature (e.g., Gómez, 2006; Palivos, Yip, and Zhang, 2003), where

[34] Recently, Koehne and Kuhn (2015) have also suggested subsidies on savings and labor to correct habit externalities.

implementable policies for social optimum should be a first-best fiscal policy.

VII. Concluding Remarks

This study shows the equilibrium cycles of a dynamic competitive economy in which preferences capture altruism/admiration and envy/jealousy, and the government provides productive public services in private production by imposing exogenous consumption taxation. The study generalizes the joint condition of the elasticity of intertemporal substitution and the nature of altruistic and envious preferences on persistent economic growth in the presence of habit persistence. Altruism/admiration and envy/jealousy yield the opposite effects on short- and long-run economic growth depending on the effective elasticity of intertemporal substitution and the degree of habit persistence in preferences.

In a decentralized competitive economy, habit persistence in preferences is a source of multiple transitional equilibrium paths, thereby triggering equilibrium cycles in a perpetually growing economy. I also solve for social planning allocations and demonstrate that a distortionary fiscal policy ensures the existence of a unique social optimum dynamic path in a perpetually growing economy. The implementation of these distortionary fiscal policies selects the unique dynamic competitive equilibrium, eradicating sunspot-driven equilibrium cycles of a decentralized competitive economy and potentially improving social welfare in the presence of Joneses habit preferences.

This Joneses habit model can be extended to models with endogenous labor supply for indeterminacy and designed for an alternative policy scheme of taxation and expenditure in fiscal policy which acts as a selection device to pin down a unique transitional dynamic and balanced growth path in a growing economy. A second-best Ramsey fiscal policy can be introduced, where a social planner takes into account each agent's reactions to a government policy. Such a model can examine the validity of implementations of stabilization policies on equilibrium cycles in a decentralized competitive habit economy. In addition, a government policy can be designed to deal with aggregate instability with an alternative form of the felicity function or with a hybrid form of habit stocks that combines internal and external habits. Future research should explain endogenous propagation mechanisms including autocorrelations, impulse responses, and cyclical frequencies of the quantities of real variables and their associated prices in equilibrium cycles.

Appendices

Appendix I: Derivations for Eqs. (16a)～(16c)

First, given that $\lambda_k \neq 0$, Eq. (15a) implies that

$$\tau = [(1-\alpha)A]^{\frac{1}{\alpha}} \frac{1}{z}. \tag{A.1}$$

Second, as in the previous sections (except that $\tau(t)$ is no longer exogenous), I define $r(z;\tau) = \alpha A \tau^{1-\alpha} z^{1-\alpha}$, so that from Eq. (A.1),

$$\hat{r} = r(z;\tau) = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}, \tag{A.2}$$

for all $z(t)$ and $\tau(t)$. Third, Eq. (A.1) also simplifies Eqs. (15c) and (15e), respectively, as

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \hat{r}, \tag{A.3}$$

$$\frac{\dot{\lambda}_h}{\lambda_h} = \rho + \delta + \frac{\gamma(q-\varphi)}{x}. \tag{A.4}$$

Again, Eqs. (A.1) and (15d) show that

$$\frac{\dot{k}}{k} = \frac{z\tau}{1-\alpha} - (1+\tau)z. \tag{A.5}$$

Fourth, combining Eqs. (15a) and (15b) yields $c^{-\sigma} h^{-\gamma(1-\sigma)} + \lambda_h \varphi - \lambda_k = 0$. Thus, differentiating this expression with respect to time provides the dynamic consumption equation as

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[-\left[\frac{q}{q-\varphi}\right]\frac{\dot{\lambda}_k}{\lambda_k} + [\sigma - \xi]\frac{\dot{h}}{h} + \left[\frac{\varphi}{q-\varphi}\right]\frac{\dot{\lambda}_h}{\lambda_h}\right].$$

Now, substituting Eqs. (15f), (A.3), and (A.4) into the previous equation yields

$$\frac{\dot{c}}{c}=\frac{1}{\sigma}\left[\left[\frac{q}{q-\varphi}\right]\hat{r}-\rho+\frac{\varphi\gamma}{x}+\delta\left[\xi-\sigma+\frac{\varphi}{q-\varphi}\right]\right]. \tag{A.6}$$

Fifth, by combining all dynamic equations Eqs. (A.1)–(A.6), I derive $\dot{z}/z\equiv \dot{c}/c-\dot{k}/k$, $\dot{x}/x\equiv\dot{h}/h-\dot{c}/c$, and $\dot{q}/q\equiv\dot{\lambda}_k/\lambda_k-\dot{\lambda}_h/\lambda_h$ which are summarized in Eqs. (16a), (16b), and (16c), respectively.

Appendix II: Derivations for Eqs. (17a)∼(17d)

From Eq. (16c), $\dot{q}/q=0$ implies that $\tilde{q}=\varphi-\gamma^{-1}[\hat{r}+\delta]\tilde{x}$. Substituting this expression $\tilde{q}$ into Eq. (16b) yields $\tilde{x}=\varphi\xi[\hat{r}-\rho+\delta\xi]^{-1}$ in Eq. (17b), along with $\dot{x}/x=0$ in the balanced growth path. In turn, I obtain Eq. (17d) for $\tilde{q}$. Again, substituting Eqs. (17d) and (17b) into Eq. (16a) yields Eq. (17a) for $\tilde{z}$. Finally, $\tilde{z}$ in Eq. (17a) pins down $\tilde{\tau}$ as in Eq. (17c).

Appendix III: Proof of the Sign of $det\,\Phi$

Except γ, all parameter values are positive in $det\Phi$. In addition, the variables $\tilde{z}$ and $\tilde{x}$ are positive. Hence, under the condition for the positive long-run growth condition (i.e., $\xi>0$ in Eq. 18), the signs of γ and $\tilde{q}$ determine the sign of $det\Phi$. Recall $\tilde{q}=\varphi[1-\xi(\hat{r}+\delta)/\gamma[\hat{r}-\rho+\delta\xi]]$ from Eq. (17c). First, when $\gamma<0$, $\tilde{q}$ is always positive. Therefore, $\gamma\tilde{q}<0$. Second, when $0<\gamma<1$, $\tilde{q}$ is negative only when $\gamma/\xi<[\hat{r}+\delta]/[\hat{r}-\rho+\delta\xi]$. This inequality condition is always satisfied in the range of parameter values for indeterminacy in the competitive economy: First, recall $\xi-\gamma=\sigma(1-\gamma)>0$, which implies that $\xi-\gamma>0$. Second, $\delta>-\rho+\delta\xi$ because $\xi-\alpha<0$ in the parameter values for indeterminacy (Proposition 4). This result implies that the right-hand of the inequality is greater than 1. Therefore, $\gamma\tilde{q}<0$.

References

Abel, A. (1990), "Asset Prices under Habit Formation and Catching up with the Joneses," *American Economic Review (Papers and Proceedings)*, 80, 38–42.

Ascari, G., L. Phaneuf, and E. Sims (2015), "On the Welfare and Cyclical Implication of Moderate Trend Inflation," NBER Working Paper No. 21392.

Aschauer, D. A. (1989), "Does Public Capital Crowd Out the Private Capital?" *Journal of Monetary Economics*, 24, 171–188.

Alonso-Carrera, J., J. Cabellé, and X. Raurich (2005), "Growth, Habit Formation, and Catching up with the Joneses," *European Economic Review*, 49, 1665–1691.

Alvarez-Cuadrado, F., G. Monteiro, and S. Turnovsky (2004), "Habit Formation, Catching up with the Joneses, and Economic Growth," *Journal of Economic Growth*, 9, 47–80.

Azariadis, C., B. Chen, C. Lu, and Y. Wang (2013), "A Two-sector Model of Endogenous Growth with Leisure Externalities," *Journal of Economic Theory*, 148, 843–857.

Barro, R. (1990), "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, 98, S103–S125.

Benhabib, J., and R. Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, 63, 19–41.

Benhabib, J., and J. Gali (1995), "On Growth and Indeterminacy: Some Theory and Evidence," *Carnegie-Rochester Conference Series on Public Policy*, 43, 163–211.

Binsbergen, J. (2016), "Good-specific Habit Formation and the Cross-section of Expected Returns," *Journal of Finance*, 72, 1699–1732.

Boldrin, M., L. Christiano, and J. Fisher (2001), "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 91, 149–166.

Bom, P., and T. Ligthart (2014), "What have we Learned from Three Decades of Research on the Productivity of Public Capital?" *Journal of Economic Survey*, 28, 889–916.

Campbell, J., and J. Cochrane (1999), "By Force of Habit: A Consumption-based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.

Carroll, C., J. Overland, and D. Weil (2000), "Saving, Growth, Habit Formation," *American Economic Review*, 90, 341–355.

Cazzavillan, G. (1996), "Public Spending, Endogenous Growth and Endogenous Fluctuations," *Journal of Economic Theory*, 71, 394–415.

Chen, B. (2007), "Multiple BGP in a Growth Model with Habit Persistence," *Journal of Money, Credit and Banking*, 39, 25–48.

Chen, B., and M. Hsu (2009), "Consumption Externalities, Efficiency and Optimal Taxation in One-sector Growth Model," *Economic Modelling*, 26, 1328–1334.

Chen, B., Y. Hsu, and K. Mino (2010), "A Note on Catching up with the Joneses and Indeterminacy," Unpublished Manuscript.

Chilarescu, C. (2016), "A Closed-form Solution of a Two-sector Endogenous Growth Model with Habit Formation," *Australian Economic Papers*, 55, 112–127.

Christiano, L., and S. Harrison (1999), "Chaos, Sunspots and Automatic Stabilizers," *Journal of Monetary Economics*, 44, 3–31.

Constantinides, G. (1990), "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98, 519–543.

Dupor, B., and W. Liu (2003), "Jealousy and Equilibrium Overconsumption," *American Economic Review*, 93, 423–428.

Farmer, R., and J.-T. Guo (1996), "The Econometrics of Indeterminacy: An Applied Study," *Carnegie-Rochester Conference Series on Public Policy*, 43, 225–271.

Fuhrer, J. (2000), "Habit Formation in Consumption and its Implication for Monetary Policy Models," *American Economic Review*, 90, 367–390.

Gali, J. (1994), "Keeping up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices," *Journal of Money, Credit, and Banking*, 26, 1–8.

Giannitsarou, C. (2007), "Balanced Budget Rules and Aggregate Instability: The Role of Consumption Taxes," *Economic Journal*, 117, 1423–1435.

Gómez, M. (2006), "Optimal Consumption Taxation in a Model of Endogenous Growth with External Habit Formation," *Economic Letters*, 93, 427–435.

Guo, J.-T., and K. Lansing (1998), "Indeterminacy and Stabilization Policy," *Journal of Economic Theory*, 82, 481–490.

Jermann, U. (1998), "Asset Pricing in Production Economies," *Journal of Monetary Economy*, 41, 257–275.

Kamiguchi, A., and T. Tamai (2011), "Can Productive Government Spending be a Source of Equilibrium Indeterminacy?" *Economic Modelling*, 28, 1335–1340.

Koehne, S., and M. Kuhn (2015), "Optimal Taxation in a Habit Formation Economy," *Journal of Public Economics*, 122, 31–39.

Ljungqvist, L., and H. Uhlig (2000), "Tax Policy and Aggregate Demand Management under Catching up with the Joneses," *American Economic Review*, 90, 356–366.

Lee, H., and H. Park (2015), "Indeterminacy and Balanced Growth under Habit Persistence and Fiscal Policies," *International Economic Journal*, 29, 259–284.

Nourry, C., T. Seegmuller, and A. Venditti (2013), "Aggregate Instability under Balanced-budget Consumption Taxes: A Re-examination," *Journal of Economic Theory*, 148, 1977–2006.

Osborn, D. (1988), "Seasonality and Habit Persistence in a Life Cycle Model of Consumption," *Journal of Econometrics*, 3, 255–266.

Palivos, T., K. Yip, and J. Zhang (2003), "Transitional Dynamics and Indeterminacy of Equilibria in an Endogenous Growth Model with a Public Input," *Review of Development Economics*, 7, 86–98.

Park, H. (2013), "Do Habits Generate Endogenous Fluctuations in a Growing Economy," *International Review of Economics and Finance*, 27, 54–68.

Park, H., and A. Philippopoulos (2004), "Indeterminacy and Fiscal Policies in a Growing Economy," *Journal of Economic Dynamics and Control*, *28*, 645–660.

Rebelo, S. (1991), "Long-run Policy Analysis and Long-run Growth," *Journal of Political Economy*, 99, 500–521.

Schmitt-Grohe, S., and M. Uribe (1997), "Balanced Budget Rules, Distortionary Taxes, and

Aggregate Instability," *Journal of Political Economy*, 105, 976–1000.

Stadt, H., A. Kapteyn, and A. Geer (1985), "The Relativity of Utility: Evidence from Panel Data," *Review of Economic and Statistics*, 67, 179–187.

Thomas, J. (2004), "Do Sunspots Produce Business Cycles?" GSIA Working Paper No. 1999-E10, Carnegie Mellon University.

Turnovsky, S., and G. Monteiro (2007), "Consumption Externalities, Production Externalities, and Efficient Capital Accumulation under Time Non-separable Preferences," *European Economic Review*, 51, 479–504.