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Triple Regime Stochastic Volatility Model with Threshold and Leverage Effects*

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This study considers a new stochastic volatility model, in which the sign and magnitude of stock returns play roles in explaining a substantially detailed relationship between stock returns and volatility. The proposed model allows for threshold and leverage effects, and accommodates three regimes (i.e., large negative return; mid-range, including moderate negative and positive returns; and large positive return) to better capture the time-varying aspect of the leverage effect. Applications of the proposed model on the return series of the S&P 500 Index and Microsoft Corporation suggest that the relationship between stock returns and volatility depends on the magnitude of the returns and their signs. The comparison of the deviance information criterion for various stochastic volatility models reveals a good fit of the proposed model for the data.

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I. Introduction

The relationship between returns and volatility in equity markets is well known to be asymmetric. That is, negative returns are associated with higher volatility than positive returns. In the literature of autoregressive conditional heteroskedasticity (ARCH)-type volatility models, various models have been proposed to

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accommodate this stylized fact. These studies include Engle and Ng (1993), Glosten et al. (1993), Nelson (1991), and Pagan and Schwert (1990). Moreover, the research of stochastic volatility (SV) models has actively addressed an asymmetric relationship between returns and volatility. The SV models specify volatility as a separate random process and can have advantages over the ARCH-type models for modeling the dynamics of return series (Kim et al., 1998). Poon and Granger (2003) reported that SV models generally outperform ARCH-type models in out-of-sample volatility forecasting. Given the rapid development in the estimation methods of SV models, these models recently become more popular than they used to be.

In the SV model framework, one common approach to accommodate the asymmetric relationship between returns and volatility is to adopt a correlation coefficient between two innovations in lagged return and volatility process (Harvey and Shephard, 1996; Yu, 2005). If the correlation is negative, then a negative lagged return will be associated with high subsequent volatility. This asymmetry based on the correlation coefficient is typically referred to as the *leverage effect* in the stochastic volatility literature. The other approach to explain asymmetric relationship between returns and volatility is adopting the threshold effect considered by So et al. (2002), who defined two regimes based on the sign of stock returns and let the parameters in the SV model have different values in each regime. Studies have attempted to accommodate the threshold and leverage effects in the SV model (Smith, 2009; Wu and Zhou, 2014[1]; Xu, 2010).

The leverage effect was assumed to be constant. However, recent studies have found evidence against a constant parameter for the leverage effect. Empirical data have shown that the leverage parameter characterizing the correlation between innovations to return and innovations to variance vary with time. Daouk and Ng (2011) reported evidence of strong leverage effect when prices decrease. Christensen et al. (2015) used the daily US stock index return series from 1926 to 2010, and found a negative leverage effect throughout but a significant increase in magnitude during financial crises. Moreover, nonparametric or semiparametric modeling for time-varying leverage effect has been extensively studied. These studies include Ait-Sahalia et al. (2013), Bandi and Reno (2012), Linton et al. (2016), Wang and Mykland (2014), and Yu (2012).

The current study focused on the idea that the relationship between returns and volatility depends on the magnitude and sign of the former. One important common feature of the majority of existing SV models is that the relationship between returns and volatility is determined only by the sign of the former, regardless of its magnitude. For example, the moderate and large negative returns in

[1] Wu and Zhou (2014) designated their model a triple-threshold leverage SV model, not because they actually consider three different regimes in the model but they allow the state dependent leverage effects (i.e., two regime-specific correlation coefficients). Moreover, each regime is still determined by only the signs of returns.

the majority of prior studies have the same relationship with volatility. However, this result is not realistic and the natural expectation is that investors behave differently when stock prices drop (or rise) below (or above) a certain level. Consequently, the relationship between returns and volatility would be different. We propose a new stochastic volatility model, in which the sign and magnitude of returns play roles in explaining a considerably detailed relationship between returns and volatility. In particular, we accommodate three regimes in the model (i.e., large negative return; mid-range, including moderate negative and positive returns; and large positive return) to better capture the time-varying aspect of the leverage effect instead of the usual two regimes depending only on the sign. We let the parameter for the leverage effect have a different value for each regime, expecting that the behavior of investors would be different in each regime.

We applied our model on two stock return series from 3 January 2006 to 30 June 2015: the return series of the S&P 500 Index and the stock return of Microsoft Corporation (MSFT). We utilized the Markov chain Monte Carlo (MCMC) method to implement a practical Bayesian estimation approach for our model. Chib and Greenberg (1995) and Chib et al. (2002) provided extensive reviews on the method. This method has been successfully applied to estimate basic and extended stochastic volatility models (e.g., Jacquier et al., 1994; Kim et al., 1998; Chib et al., 2002). The MCMC method is a simulation technique that generates a sample from the target distribution. The simulation is conducted by specifying the transition density of an irreducible aperiodic Markov chain, the limiting invariant distribution of which is the target posterior distribution. Thereafter, the Markov chain is iterated numerous times in a computer-generated Monte Carlo simulation, and the draws generated from the simulation can be used to summarize the posterior distribution.

We present evidence that the relationship between returns and volatility depends on the magnitude and sign of the former. In the S&P 500 Index and MSFT cases, the estimated leverage effect differed in each of the three regimes. First, when the stock prices in both cases cross a certain threshold, the leverage effect becomes substantially stronger where the leverage effect of the index is generally stronger than that of MSFT. Second, although the conventional leverage effect appeared for the index in regime 1 (with generally negative returns), the relationship between stock returns and volatility was estimated to be positive for MSFT. Such a reverse leverage effect also appeared in the model of Yu (2012) when stock returns are negative. Third, when stock return was moderately negative or positive (regime 2), the conventional leverage effect appeared in the index and individual firm's stock. Lastly, a comparison of the deviance information criterion for various SV models showed that our model fit the data the best compared with various existing SV models.

The remainder of this paper is organized as follows. Section 2 introduces the model and explains the estimation method. Sections 3 provides the main results.

Section 4 concludes this research. The Appendix contains the tables and figures.

II. Proposed Model and Estimation Method

2.1. Proposed Model

We denote r_t as a demeaned stock return series and let θ be the vector of unknown parameters that will be specified in the next subsection. We define a sequence of random variables s_t^j as follows:

$$s_t^1 = \begin{cases} 1 & \text{if } r_t < \tau_1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Regime 1)},$$

$$s_t^2 (= 1 - s_t^1 - s_t^3) = \begin{cases} 1 & \text{if } \tau_1 \le r_t < \tau_2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Regime 2)},$$

$$s_t^3 = \begin{cases} 1 & \text{if } r_t \ge \tau_2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Regime 3)},$$

where τ_1 and τ_2 are the threshold levels that satisfy $\tau_1 < \tau_2$. We let $s_t = (s_t^1, s_t^2, s_t^3)'$ for $t = 1, \ldots, n$. The triple regime stochastic volatility (TRSV) model with threshold and leverage effects is defined as follows:

$$r_t = \sqrt{h_t} u_t ,$$
$$\log h_{t+1} - \mu_{s_t} = \beta_{s_t} (\log h_t - \mu_{s_t}) + \varepsilon_t , \quad \varepsilon_t = \sigma v_{t+1} ,$$

where

$$\left(\begin{matrix} u_t \\ v_{t+1} \end{matrix} \middle| s_t, \theta \right) \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_{s_t} \\ \rho_{s_t} & 1 \end{pmatrix} \right)$$

and

$$\mu_{s_t} = \mu_1 s_t^1 + \mu_2 s_t^2 + \mu_3 s_t^3$$
$$\beta_{s_t} = \beta_1 s_t^1 + \beta_2 s_t^2 + \beta_3 s_t^3$$
$$\rho_{s_t} = \rho_1 s_t^1 + \rho_2 s_t^2 + \rho_3 s_t^3 .$$

Therefore, we may rewrite

$$\begin{aligned}
&\log h_{t+1} - \mu_1 = \beta_1(\log h_t - \mu_1) + \varepsilon_t, \quad cor(u_t, v_{t+1}) = \rho_1 \quad \text{if} \quad r_t < \tau_1 \\
&\log h_{t+1} - \mu_2 = \beta_2(\log h_t - \mu_2) + \varepsilon_t, \quad cor(u_t, v_{t+1}) = \rho_2 \quad \text{if} \quad \tau_1 \le r_t < \tau_2 \\
&\log h_{t+1} - \mu_3 = \beta_3(\log h_t - \mu_3) + \varepsilon_t, \quad cor(u_t, v_{t+1}) = \rho_3 \quad \text{if} \quad r_t \ge \tau_2
\end{aligned}$$

Our TRSV model is a triple-regime model, in which each regime is determined by return. Note that the sign and magnitude of the returns determine the regime in the model. When the return is negative with a large magnitude, it belongs to regime 1. When the return is moderately negative or positive, it is in regime 2. When the return is positive with a large magnitude, it belongs to regime 3. In each regime, the leverage effect represented by the correlation coefficient ρ_{s_t} takes a different value. This model also allows for the threshold effect, which means that μ_{s_t} and β_{s_t} have different values in each regime.

Our reason for introducing a triple-regime model rather than the traditional two-regime models is that the empirical results of the relationship between volatility and returns for the periods of large negative return or financial crisis has been found to be mixed. Christensen et al. (2015) found that the risk–return tradeoff is significantly positive only during financial crises but insignificant during non-crisis periods. This result can be explained by the fact that a given increase in the debt/equity ratio leads to increased risk during crisis. Furthermore, an increase in risk increases the discount rate more during financial crisis than during normal periods following the volatility feedback interpretation. Christensen et al. (2015) used the daily US stock index return series from 1926 to 2010 and found that the magnitude of leverage effect changes drastically during financial crises. The current study shows that time-variant leverage effect can be substantially explained by our triple-regime model. The reason is that the strength of the leverage effect can be drastically changing for the periods of generally negative lagged return or financial crisis period.

Our model is related with recent studies on nonparametric or semiparametric modeling for time-varying leverage effect (Ait-Sahalia et al., 2013; Bandi and Reno, 2012; Linton et al., 2016; Wang and Mykland, 2014; Yu, 2012). Yu (2012) and Bandi and Reno (2012) obtained strong evidence for time-varying aspects for asymmetric relationships between lagged return and volatility. Linton et al. (2016) suggested a method of testing the leverage hypothesis nonparametrically using the concept of first order distributional dominance. They found that investors consider the level of volatility and the entire conditional distribution of volatility. Bandi and Reno (2012) and Patton and Sheppard (2015) considered the current volatility level as the main driving force or strength of the time-varying leverage effect. By contrast, Yu (2012) assumed that the driving factor for time-varying leverage was the lagged return.

Yu (2012) proposed a SV model that can allow for multiple regimes. He also considered a three-regime model even if his empirical applications support two-regime models instead of three-regime models. However, our model is different from his model in two main aspects. First, we estimate τ_i in our model, whereas it is predetermined in Yu (2012). In his three-regime model, τ_1 and τ_2 are chosen to enable each regime to have a nearly equal split of observations (34.5%, 31%, and 34.5% of returns). However, investors' behaviors would expectedly be different if stock prices dropped (or rose) below (or above) a certain level. Moreover, estimating τ_1 and τ_2 would be considerably desirable to accommodate the effects of such a behavior in the model. Second, we allow for the threshold effect in the volatility level parameter μ_{s_t} and volatility persistence parameter β_{s_t} but it is not allowed in Yu's model.

Danielsson (1994) and Asai and McAleer (2006) considered SV models that incorporate the sign and magnitude of return. However, their models are based on an EGARCH type representation and do not focus on the correlation coefficient between two innovations. Therefore, their models do not provide the detailed features of the leverage effect that depend on the sign and magnitude of a return series as our model does.

2.2. Bayesian Estimation Method

Estimating SV-type models is considerably challenging because these models lack closed form likelihood functions owing to their latent structure of the conditional variance. Therefore, maximum likelihood estimation cannot be directly used. Several estimation methods have been proposed in the literature, including quasi-maximum likelihood method (QML) (Harvey et al., 1994), simulated maximum likelihood method (Danielsson, 1994; Durbin and Koopman, 1997; Sandmann and Koopman, 1998), efficient method of moments (Gallant and Tauchen, 1996; Andersen et al., 1999), simulated method of moments (Duffie and Singleton, 1993), and generalized method of moments (GMM; Melino and Turnbull, 1990; Andersen and Sørensen, 1996; Sørensen, 2000). Apart from these methods, the Bayesian Markov chain Monte Carlo (MCMC) method has been used to estimate the parameters of the SV models. Compared with other estimation methods, the Bayesian method is explicitly suitable and has been proven to perform well and provide relatively accurate results (e.g., Jacquier et al., 1994; Andersen et al., 1999). Andersen et al. (1999) showed that MCMC is one of the most efficient methods. The first Bayesian approach was provided by Jacquier et al. (1994), in which the posterior distribution of the unknown parameters was sampled by the MCMC method. The aforementioned study also showed that in the SV framework, the MCMC method is superior to QML and GMM. Kim et al. (1998) and Chib et al. (2002) developed an alternative and markedly efficient MCMC algorithm for SV models.

Therefore, the current study uses the Bayesian approach to estimate our model. We define the vector of the observed samples $R=(r_1,r_2,\cdots,r_n)'$ with n sample size. We let $\theta=(\mu,\beta,\sigma^2,\rho,\tau,h_1)'$ be the vector of the unknown parameters with $\mu=(\mu_1,\mu_2,\ \mu_3)$, $\beta=(\beta_1,\beta_2,\beta_3)$, $\rho=(\rho_1,\rho_2,\rho_3)$, $\tau=(\tau_1,\tau_2)$, and $H=(h_1,h_2,\cdots,h_n)'$; and $S=(s_1,s_2,\cdots,s_n)'$ be the vectors of the latent variables. We followed Yu (2005) in rewriting our model as follows:

$$\log h_{t+1}\,|\log h_t,\theta,s_t \sim N(\mu_{s_t}+\beta_{s_t}(\log h_t-\mu_{s_t}),\sigma^2)$$

$$r_t\,|\log h_{t+1},\log h_t,\theta,s_t \sim N\left(\frac{\rho_{s_s}}{\sigma}\sqrt{h_t}(\log h_{t+1}-\mu_{s_t}-\beta_{s_t}(\log h_t-\mu_{s_t})),h_t(1-\rho_{s_t}^2)\right).$$

Using Bayes' theorem, we can construct the joint posterior distribution of the unobservables given the data in terms of the prior distribution $p(\theta)$. Moreover, the likelihood function is as follows:

$$p(\theta,H\,|\,R)\propto p(R,H\,|\,\theta)p(\theta)\,, \tag{1}$$

where

$$p(R,H\,|\,\theta)\propto p(\log h_1\,|\,\theta)\prod_{t=1}^{n-1}p(r_t,\log h_{t+1}\,|\log h_t,\theta,s_t)p(r_n\,|\log h_n,\theta)$$

$$=p(\log h_1\,|\,\theta)\prod_{t=1}^{n-1}p(r_t\,|\log h_{t+1},\log h_t,\theta,s_t)p(\log h_{t+1}\,|\log h_t,\theta,s_t)p(r_n\,|\log h_n,\theta)$$

$$p(\theta)=p(\mu_1)p(\mu_2)p(\mu_3)p(\beta_1)p(\beta_2)p(\beta_3)p(\sigma^2)p(\rho_1)p(\rho_2)p(\rho_3)p(\tau_1)p(\tau_2)p(\log h_1)\ .$$

We follow the literature for the prior distribution of θ. That is, all variables of θ are assumed to be independent. For parameters β and σ^2, we precisely follow the prior specification of Kim et al. (1998); $\sigma^2 \sim$ *Inverse* − *Gamma* (2.5, 0.025), which has a mean of 0.167 and standard deviation of 0.024. For β, Kim et al. (1998) specified a beta distribution with parameters 20 and 1.5, thereby implying a mean of 0.86 and standard deviation of 0.11. For parameter μ, we take a slightly informative prior, such as $\mu_j \sim N(-10,4)$ for all j. The correlation parameter ρ_j for all j is assumed to be uniformly distributed with support between −1 and 1, and is completely flat. Therefore, the prior distributions with different regimes are not informative. For the threshold level parameters τ, we assume that the threshold has a uniform prior for the first iteration, $U[\tau_{1i},\tau_{2i}]$ for τ_i to ensure that each regime has sufficient observations, in which the lower and upper bounds correspond to selected quantiles of r_t. We suggest that each regime must contain at least 10% of the samples and τ is constrained to satisfy $\tau_1 < \tau_2$. We also impose $\underline{\tau_1} < \tau_m$ and $\tau_2 > \tau_m$, where τ_m denotes the median of the samples. That is, the support for $p(\tau_1)$ is $[\underline{\tau_1}, \tau_m]$ and that for $p(\tau_2)$ is $[\tau_m, \overline{\tau_2}]$, where $\underline{\tau_1}$ and $\overline{\tau_2}$ are 10% and 90% quantile, respectively.[2] We use normal priors with posterior mean from the first iteration to accelerate the convergence.

For the usual Bayesian procedure, we implemented an MCMC method to sample the latent variables and unknown parameters from the joint posterior density $p(\theta, H \mid R)$ in (1). The MCMC algorithm repeatedly samples from the posterior distributions, thereby generating a Markov chain over (θ, H), and eventually converging to the equilibrium/stationary posterior distribution $p(\theta, H \mid R)$. For our MCMC procedure, we used the Gibbs sampler and Metropolis–Hastigns (MH) algorithm within the Gibbs sampler. These methods have had an extensive influence on the theory and practice of Bayesian inference. Chib and Greenberg (1995) provided a detailed account of the MH algorithm.

Let $\omega = (\theta, H, S)$ and ω_{-h_t} denotes ω excluding h_t. The Gibbs sampler, which is used to generate a Markov chain with stationary distribution as the joint posterior distribution (1), works as follows in the first step. Given the initialization (θ^0, H^0), we draw from each of the following distributions:

1. (a) Sample h_1 from $p(\log h_1 \mid \omega_{-h_1}, r_1) \propto p(r_1, \log h_2 \mid \log h_1, \theta, s_1) p(\log h_1)$.[3]

(b) Sample h_t from $p(\log h_t \mid \omega_{-h_t}, R) \propto p(r_t, \log h_{t+1} \mid \log h_t, \theta, s_t) p(r_{t-1}, \log h_t \mid \log h_{t-1}, \theta, s_{t-1})$ for $t = 2, \ldots, n-1$,

(c) Sample h_n from $p(\log h_n \mid \omega_{-h_n}, R) \propto p(r_n \mid \log h_n, \theta, s_n) p(r_{n-1}, \log h_n \mid \log h_{n-1}, \theta, s_{n-1})$

2. Sample (ρ_1, ρ_2, ρ_3) from $p(\rho_1, \rho_2, \rho_3 \mid w_{-\rho}, R) \propto \prod_t p(r_t \mid \log h_{t+1}, \theta, s_t) p(\rho_1, \rho_2, \rho_3)$

3. Sample σ from $p(\sigma \mid w_{-\sigma}, R) \propto \prod_t p(r_t, \log h_{t+1} \mid \log h_t, \theta, s_t) p(\sigma)$

4. Sample μ_j from $p(\mu_j \mid w_{-\mu_j}, R) \propto \prod_t p(r_t, \log h_{t+1} \mid \log h_t, \theta, s_t) p(\mu_j)$ $\quad j = 1,2,3$

5. Sample β_j from $p(\beta_j \mid w_{-\beta_j}, R) \propto \prod_t p(r_t, \log h_{t+1} \mid \log h_t, \theta, s_t) p(\beta_j)$ $\quad j = 1,2,3$

6. Sample τ_i from $p(\tau_i \mid w_{-\tau_i}, R) \propto \prod_t p(r_t, \log h_{t+1} \mid \log h_t, \theta, s_t) p(\tau_i)$ $\quad i = 1,2$

7. Sample s_t, $t = 1, \ldots, n$.

8. Go to 1.

The random walk chain MH algorithm is applied to sample parameter θ, and a common and convenient choice of density for the increment random variable is the normal. The scale parameter for increment random variable determines the precise form of the candidate-generating density. A suitable value of the scale parameter

[2] The similar estimates of τ_i are observed even if two threshold levels are allowed to be negative.

[3] $p(\log h_1) \sim N(\overline{\mu}, \frac{\overline{\sigma^2}}{1-\overline{\rho^2}})$, where $\overline{\mu}$, $\overline{\sigma^2}$ and $\overline{\rho^2}$ are obtained from the results of the SV model with leverage effect (SVL).

with good convergence properties can be selected by having an acceptance probability of 20% to 60%.

The most difficult part of this Gibbs sampler is to sample h_t from $p(h_t \mid \omega_{-h_t}, r_t)$. For sampling h_t, we used the grid-based chain suggested by Tierney (1994). Ritter and Tanner (1992) proposed the griddy Gibbs sampler for Gibbs sampling in problems where the conditional distributions cannot be sampled directly. This method was also described in Tierney (1994). Using this algorithm in its pure form may require quite a fine grid, thereby necessitating numerous posterior density evaluations to control the error in the approximation. To deal with this problem, Tierney (1994) proposed this algorithm in a Metropolis chain to ensure that the equilibrium distribution is precisely the target distribution even in a coarse grid.

The reliability of posterior inference based on the simulation algorithm depends on whether the Markov chain has reached convergence. Hence, the simulated sample is drawn from the stationary distribution. Accordingly, we use the convergence diagnostic proposed by Geweke (1992) to check the convergence for all parameters. Geweke (1992) showed that the convergence diagnostic converges to the standard normal distribution as the number of samples goes to infinity (i.e., if the sequence of the Gibbs samples for a parameter is stationary). All results reported in this research are based on samples after the burn-in period, which have passed the convergence diagnostic for all parameters.

III. Main Results

3.1. Data and Benchmark Models

We consider the daily stock return series of the S&P 500 Index and the MSFT. We use the period of stock price data with the global financial crisis to include the extremely volatile movements in stock markets. Hence, our data set spans from 3 January 2006 to 30 June 2015. The sample size was 2,388 in each case. Figure 1 shows the graphs of both return series. Each return series is demeaned by subtracting its sample mean.

For each return series, we estimate the following five models. We let

$$r_t = \sqrt{h_t} u_t, \quad u_t \sim N(0,1).$$

1. Basic SV model (SV_0):

$$\log h_{t+1} - \mu = \beta(\log h_t - \mu) + \sigma v_{t+1},$$

where $v_t \sim N(0,1)$.

2. SV model with (constant) leverage effect (SVL):

$$\log h_{t+1} - \mu = \beta(\log h_t - \mu) + \sigma v_{t+1},$$

where $cor(u_t, v_{t+1}) = \rho$.

3. SV model with time-varying leverage effects (SV2L):

$$\log h_{t+1} - \mu = \beta(\log h_t - \mu) + \sigma v_{t+1},$$

where $cor(u_t, v_{t+1}) = \rho_{s_t}$.

$$s_t = \begin{cases} 0 & \text{if } r_t < 0 \\ 1 & \text{if } r_t \geq 0 \end{cases}. \tag{2}$$

4. SV model with threshold effect (SVT):

$$\log h_{t+1} - \mu_{s_t} = \beta_{s_t}(\log h_t - \mu_{s_t}) + \sigma v_{t+1}$$

where s_t is defined as in (2).

5. SV model with threshold and (constant) leverage effects (SVTL):

$$\log h_{t+1} - \mu_{s_t} = \beta_{s_t}(\log h_t - \mu_{s_t}) + \sigma v_{t+1},$$

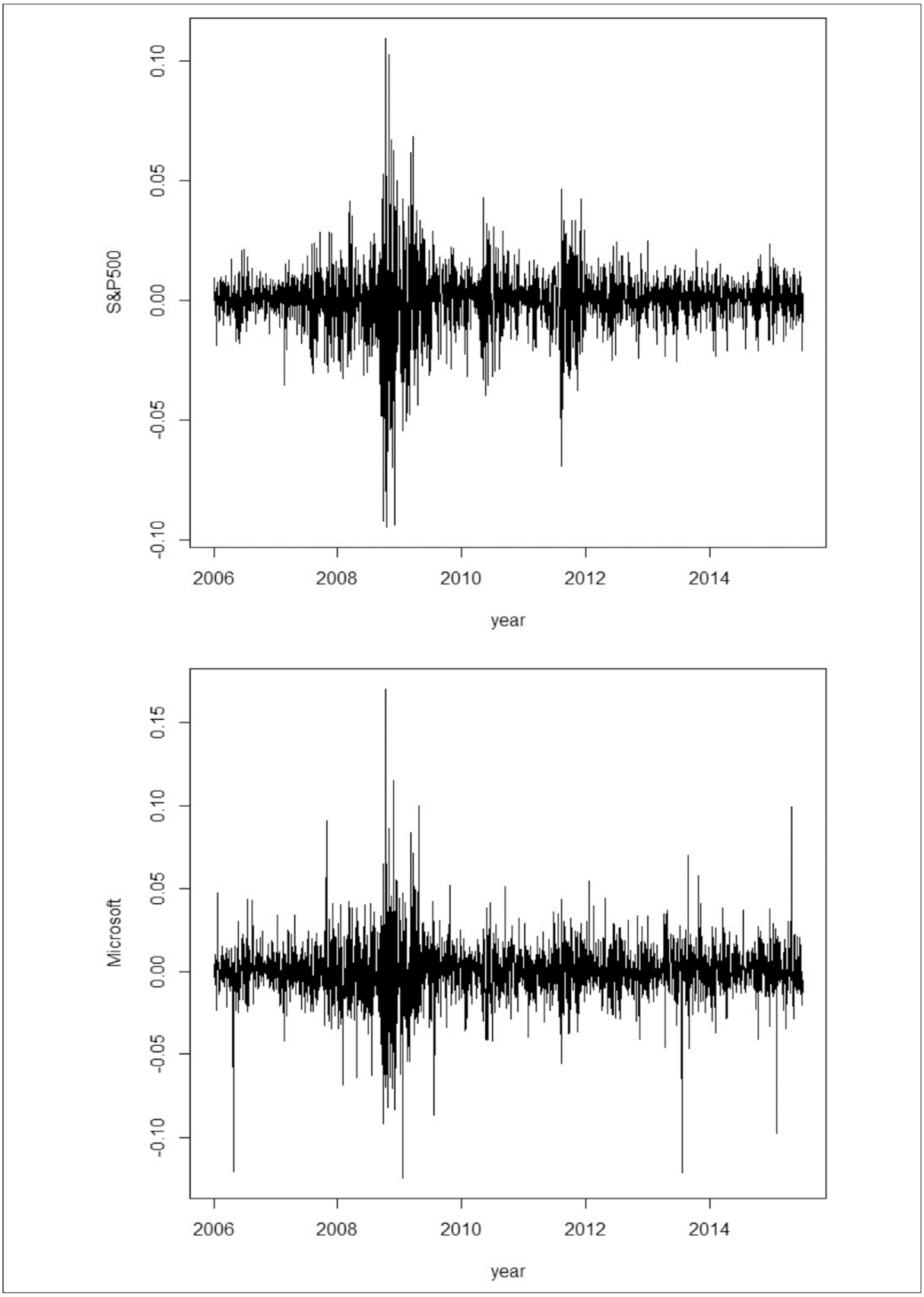
where $cor(u_t, v_{t+1}) = \rho$ and s_t is defined as in (2).

6. Triple regime SV model with threshold and leverage effects (TRSV), as defined in Section 2.

The basic SV model does not allow for any asymmetric relationship between return and volatility. The SVL model introduced by Harvey and Shephard (1996) allows for the leverage effect by incorporating the correlation between lagged return and volatility. Yu (2012) proposed a semiparametric SV model with time varying leverage effects by using the linear spline, and found the strong evidence to support the two-regime model rather than the three-regime model.[4] Therefore, we consider

[4] We also estimate the triple regime model by Yu (2012). Note that unlike our TRSV model,

[Figure 1] Plots of stock return series: S&P 500 Index and MSFT



parameters μ and β are not regime-dependent in his model. The estimation results indicate that our TRSV model fits the data better than the triple regime model in terms of the comparison of the deviation information criterion.

only his SV model with time-varying leverage in the two-regimes, which is denoted by the SV2L model in the current study. The threshold levels for each regime are predetermined and the volatility level and persistence parameters are assumed to be constant in Yu (2012), which is not the case in the present research. So et al. (2002) proposed the SVT model, which accommodates the threshold effect in the model. Each regime in the SVT model is determined by the sign of lagged return. Smith (2009) introduced the SVTL model by combining these two models.[5] Note that the correlation coefficient ρ in the SVTL model is constant and does not depend on any one regime. Although the three models (i.e., SVL, SV2L, and SVTL) explain the asymmetric relationship between return and volatility, only the sign of the lagged return determines the asymmetric relationship in these models. By contrast, our TRSV model incorporates the sign and magnitude of the lagged return in determining each regime. The explanation in Section 2 indicates that the TRSV model allows three regimes, depending on the sign and magnitude of lagged return.

All the SV models are estimated using the Bayesian method described in Section 2.2. For the prior distribution for the benchmark SV models, we use the same distributions as the TRSV model, which are specified in Section 2.2.

We use the deviance information criterion (DIC) proposed by Spiegelhalter et al. (2002) to compare the SV models. Berg et al. (2004) demonstrated that model selection can be easily done using DIC. This criterion combines a Bayesian measure of fit with a measure of model complexity, and can be expressed as follows:

$$DIC = \bar{D} +_{PD} = D(\bar{\theta}) + 2_{PD},$$

where $\bar{D}$, which is a Bayesian measure of model fit, is defined as the posterior expectation of the deviance, and $_{PD}$ is a measure of complexity (penalty term for increasing model complexity) defined as the difference between the posterior expectation of the deviance $\bar{D}$ and the deviance evaluated at the posterior mean of the parameters $D(\bar{\theta})$. Thus, $\bar{D}(= D(\bar{\theta}) +_{PD})$, which is also a Bayesian measure of model fit, already contains a penalty term for model complexity. Moreover, DIC can be divided into a pure measure of fit $D(\bar{\theta})$ and a measure of complexity 2_{PD}. Berg et al. (2004) provided a detailed explanation. For the latent variable models, Celeux et al. (2006) explained numerous alternative definitions of DIC, depending on the different concepts of likelihood. Although DIC based on the conditional likelihood by conditioning on the latent variables is widely used for comparing stochastic volatility models owing to its easy computation, recent studies have argued against its use based on theoretical and practical grounds. Chan and Grant (2016) showed via a Monte-Carlo study that the conditional DIC tends to favor

[5] Smith (2009) allowed σ_{s_t} but found that the likelihood improvement over the model with invariant σ is trivial.

overfitted models, whereas DIC based on the observed-data likelihood appears to perform well. To compare the SV models, we use DIC based on the observed-data likelihood using the importance sampling algorithms proposed by Chan and Grant (2016).

3.2. Results for the S&P 500 Index

We first consider the S&P 500 Index return series. In particular, we collected every 10th iteration for the basic SV model, after burn-in period of 30,000 iterations, and follow-up period of 70,000. A total of 200,000 iterations were drawn for the SVL model. We chose a burn-in period of 110,000 iterations and stored every 10th iteration. For the SV2L and SVTL models, we iterated 200,000 and collected every 20th iteration. We iterated 300,000 and stored every 20th iteration for SVT and 30th iteration for TRSV because posterior correlations among the parameters are relatively high and the convergence of the Gibbs samplers is relatively slow. In the SV2L, SVT, and SVTL models, the first 70,000, 20,000, and 20,000 samples, respectively, were discarded. For the TRSV model, the results were reported after the burn-in period of 30,000.

Table 1 presents the estimation result of the basic SV model, and reports the posterior mean, posterior standard deviations, 5% quantile, and 95% quantile of all the parameters. The convergence diagnostics by Geweke (1992) is also provided in the table. The autoregressive coefficient β represents volatility persistence, which is estimated to be extremely close to unity ($\hat{\beta} = 0.984$). This result is no longer surprising because the sample period contains the financial crisis in 2008. During a crisis period, volatility is much higher than the rest period, and such a persistency makes the logarithm of volatility estimated to be a near unit root process.

[Table 1] Estimation results of the basic SV model for S&P 500

Parameters	Mean	Std errors	5%	95%	Convergence Diagnostics
	Posterior				
μ	-9.3851	0.2652	−9.8277	−8.9485	−0.3970
β	0.9838	0.0047	0.9754	0.9910	−0.6349
σ^2	0.0377	0.0078	0.0271	0.0527	0.3635

Note: Values in the fourth and the fifth columns are the 5th and 95th quantile, respectively. The last column indicates the convergence diagnostic by Geweke (1992).

Table 2 presents the estimation result of the SVL model. This model includes the correlation coefficient ρ, which exhibits the relationship between return and future volatility. This relationship is estimated to be −0.775, which indicates that return and volatility has a negative relationship that is relatively strong. The negative value of $\hat{\rho}$ confirms what is known in the literature, including Harvey and Shephard (1996) and Yu (2005). The autoregressive coefficient β is estimated to be 0.971, which is slightly lower but continue to be similar to that in the basic SV model.

[Table 2] Estimation results of the SVL model for S&P 500

	Posterior				Convergence
Parameters	Mean	Std errors	5%	95%	Diagnostics
μ	−9.3354	0.1252	−9.5392	−9.1273	1.4670
β	0.9711	0.0044	0.9637	0.9781	0.8821
ρ	−0.7747	0.0355	−0.8277	−0.7106	−0.7078
σ^2	0.0660	0.0091	0.0512	0.0813	−0.2663

Note: Same as in Table 1.

Table 3 shows the estimation result of the SV2L model. Compared with the SVL model, parameters ρ_0 and ρ_1 take different values, depending on the sign of the return. When a return is negative, ρ_0 is estimated to be −0.629. When a return is non-negative, ρ_1 is estimated to be −0.863. This result implies that the leverage effect is stronger when a return is positive, which is also observed in Yu (2012) and Wu and Zhou (2014).

[Table 3] Estimation results of the SV2L model for S&P 500

	Posterior				Convergence
Parameters	Mean	Std errors	5%	95%	Diagnostics
μ	−8.4947	0.5954	−9.4311	−7.4743	−1.2160
β	0.9708	0.0051	0.9622	0.9786	0.1355
σ^2	0.0700	0.0130	0.0499	0.0916	0.1013
ρ_0	−0.6287	0.1120	−0.7976	−0.4151	−1.2870
ρ_1	−0.8625	0.0637	−0.9541	−0.7461	0.2275

Note: Same as in Table 1.

Table 4 presents the estimation result of the SVT model. In this model, parameters μ and β take different values depending on whether a return is negative or not. The level of volatility is represented by the value of μ. When a return is negative, μ_0 is estimated to be −5.200. When a return is non-negative, μ_1 is estimated to be −13.904. This result implies that the volatility level is considerably higher when a return is negative. Although $\hat{\mu}_0$ and $\hat{\mu}_1$ are estimated to be relatively different from each other, volatility persistence parameter β is estimated to be similar in both regimes ($\hat{\beta}_0 = 0.965$ and $\hat{\beta}_1 = 0.971$).

Table 5 provides the estimation result of the SVTL model. Compared with the SVT model, the model now includes a (constant) correlation coefficient ρ. Correlation coefficient ρ is estimated to be −0.750, which is similar to that in the

SVL model. Compared with the SVT model in Table 4, the estimates of β_0 and β_1 are similar, while the estimates of μ_0 and μ_1 are relatively different. This result shows that incorporating the leverage effect does not substantially change the volatility persistence estimates but substantially affects the volatility level estimates. Compared with the SVT model, the difference between $\hat{\mu}_0$ and $\hat{\mu}_1$ particularly becomes considerably minimal.

[Table 4] Estimation results of the SVT model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	−5.1997	0.9065	−6.5174	−3.5536	0.4153
μ_1	−13.9036	0.9300	−15.5326	−12.4276	−0.5469
β_0	0.9651	0.0078	0.9514	0.9768	0.1171
β_1	0.9711	0.0064	0.9595	0.9803	0.3598
σ^2	0.0429	0.0086	0.0306	0.0576	−0.3977

Note: Same as in Table 1.

[Table 5] Estimation results of the SVTL model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	−8.9397	0.7988	−10.1186	−7.6049	−0.0642
μ_1	−9.9655	1.0292	−11.7662	−8.3616	0.1148
β_0	0.9660	0.0126	0.9444	0.9862	−0.7465
β_1	0.9765	0.0111	0.9566	0.9926	−0.1092
ρ	−0.7491	0.0529	−0.8293	−0.6546	1.1260
σ^2	0.0617	0.0115	0.0444	0.0838	0.7423

Note: Same as in Table 1.

The estimation result of our TRSV model is provided in Table 6.[6] The convergence diagnostics by Geweke (1992) in Table 5 shows that the Markov chains converged well. Figures 2 to 4 provide the trace of the MCMC iterates after the burn-in period, autocorrelations of the draw sequences, and the estimated posterior densities of all parameters. From the trace and autocorrelation plots, we observed the high speed of convergence. The autocorrelations of the iterates decayed relatively quickly in all parameters. Figure 5 provides the plot of the posterior mean of the MCMC iterates for h_t. The thick line represents the estimated $\sqrt{h_t}$ and the dotted line indicates the absolute value of the demeaned S&P 500 returns. Figure 5 also shows that the estimated volatilities explain the absolute value of the

[6] We also investigate the posterior medians for all parameters, and find that the results are considerably similar.

demeaned S&P 500 returns relatively well, particularly in light of their trend behaviors.

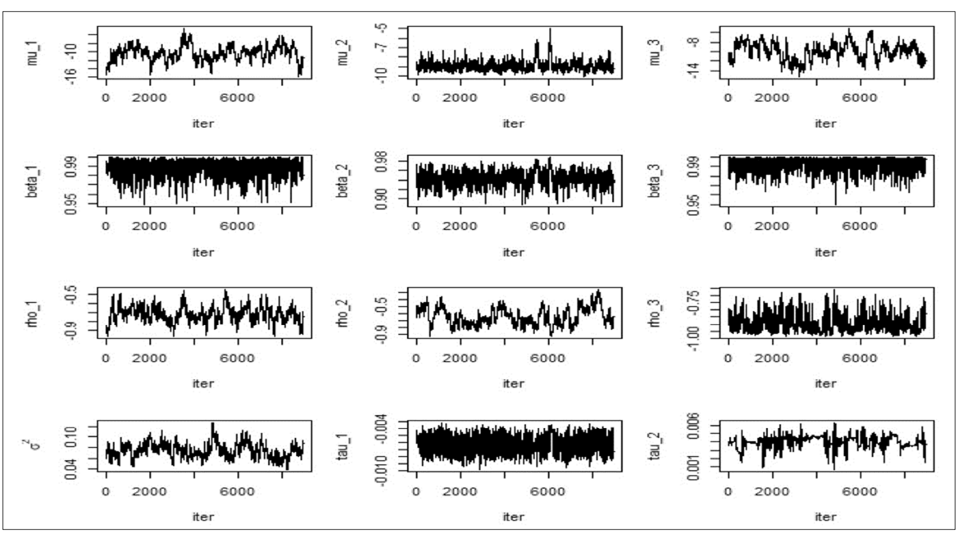
First, threshold parameters τ_1 and τ_2 are estimated to be −0.0062 and 0.004, which are the 21% and 68% quantiles, respectively, of the demeaned return series. When a return is below its 21% quantile, it indicates regime 1; and a return between its 21% and 68% quantiles belongs to regime 2. When a return is above its 68% quantile, it is in regime 3.

[Table 6] Estimation results of the TRSV model for S&P 500

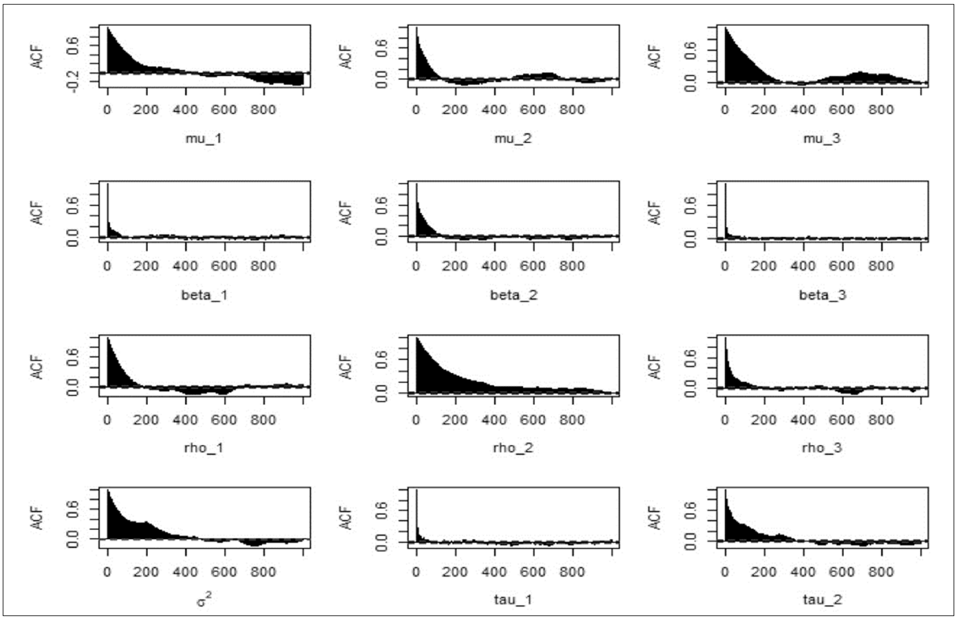
Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_1	−10.4208	1.7477	−13.3541	−7.5721	−1.0970
μ_2	−8.8825	0.5380	−9.5024	−7.8569	−0.2734
μ_3	−9.9058	1.8259	−12.9696	−7.0469	0.5169
β_1	0.9880	0.0072	0.9746	0.9975	1.0730
β_2	0.9437	0.0150	0.9188	0.9681	−0.1433
β_3	0.9923	0.0054	0.9820	0.9988	0.4069
ρ_1	−0.7210	0.0905	−0.8717	−0.5646	−0.1321
ρ_2	−0.6438	0.1280	−0.8248	−0.4253	0.5928
ρ_3	−0.9008	0.0556	−0.9703	−0.7922	0.3558
τ_1	−0.0062	0.0010	−0.0078	−0.0046	0.3749
τ_2	0.0040	0.0006	0.0029	0.0048	−1.0540
σ^2	0.0742	0.0133	0.0536	0.0966	−0.9142

Note: Same as in Table 1.

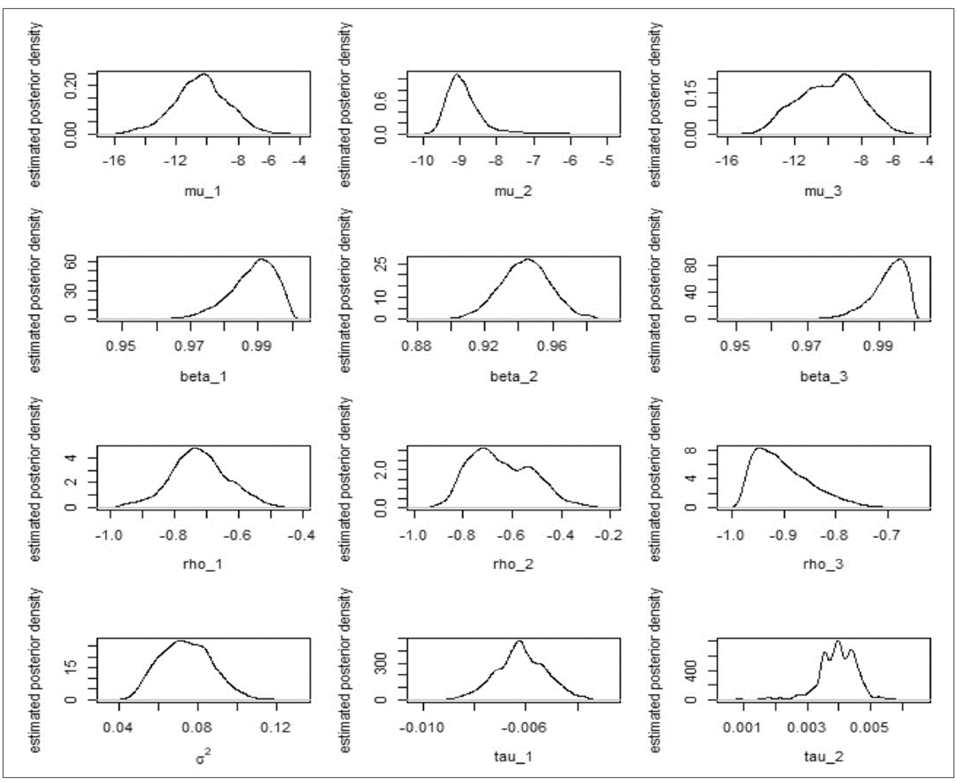
[Figure 2] Plots of the MCMC iterates obtained from the TRSV model fitting of S&P 500



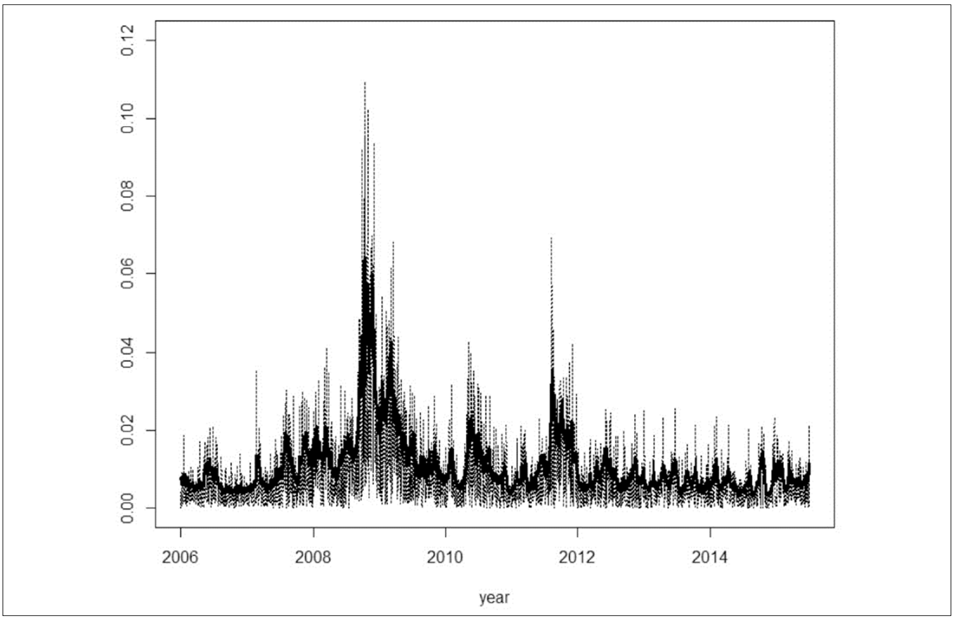
[Figure 3] Autocorrelation of the MCMC iterates for the TRSV model fitting of S&P 500



[Figure 4] Estimated posterior densities for the TRSV model fitting of S&P 500



[Figure 5] Estimated $\sqrt{h_t}$ with $|r_t|$ for the TRSV model fitting of S&P 500



Second, comparing the estimates of the correlation coefficient ρ is relatively interesting. Depending on each regime, the estimated values of ρ show different magnitudes of the leverage effect. In regime 1, it is estimated at −0.721, which is relatively similar to the values in the SVL and SVTL models. Meanwhile, ρ is estimated to be −0.6438 in regime 2. However, in regime 3, it is estimated to be −0.901, which implies that the leverage effect is considerably strong for the large positive return period.

Third, the volatility persistence coefficient β in regimes 1 and 3 is estimated to be similar. $\hat{\beta}_1$ and $\hat{\beta}_3$ are 0.988 and 0.992, respectively. Meanwhile, the volatility persistence coefficient β in regime 2 is estimated to be 0.944, which implies that the volatility in this regime is minimally persistent. The volatility level parameters $\hat{\mu}_1$, $\hat{\mu}_2$, and $\hat{\mu}_3$ are −10.421, −8.8825, and −9.906, respectively. They are not significantly different according to the posterior 90% confidence intervals.

Table 7 provides the comparison results, which imply that our TRSV model fit the data best in terms of DIC introduced in the previous section. DIC of the TRSV model is −15,498.3, which is the lowest. Therefore, the TRSV model performs best in terms of DIC.[7]

[7] We also estimate the two regime SV model with one unknown threshold level. The estimation results show that threshold τ is estimated to be negative. The estimates of τ in the two regime SV model are relatively similar to the estimates of τ_1 in our TRSV model for S&P500 and MSFT. Our estimation results are consistent with the empirical evidence in previous studies, such as So, Chen, and Chen (2005); and Chen and So (2006) in the GARCH frame. When we compare DIC of the model,

[Table 7] Comparison results of the SV models for S&P 500

Models	DIC		p_D		Rank
SV_0	−15356.2	(0.4123)	11.7	(0.4135)	6
SVL	−15483.4	(0.7605)	13.7	(0.7621)	2
SV2L	−15482.2	(0.5137)	14.6	(0.5154)	4
TSV	−15411.7	(0.6157)	12.6	(0.6153)	5
TSVL	−15482.8	(0.5040)	12.9	(0.5070)	3
TRSV	−15498.3	(0.8490)	19.0	(0.8521)	1

Note: DIC is deviance information criterion; p_D is a measure of complexity defined as the difference between $\overline{D}$ and $D(\overline{\theta})$, where $\overline{D}$ is the posterior expectation of the deviance; and $D(\overline{\theta})$ is the deviance evaluated at the posterior mean of the parameters. Numerical standard errors are in parentheses.

3.3. Result for Microsoft

We now consider the stock return series of Microsoft Corporation. For the SV, SVT, and SVTL models, we had 200,000 iterations, and the first 50,000 samples for SV and first 20,000 samples for SVT were discarded as a burn-in period. For SVTL, we chose a burn-in period of 40,000. For SVL, 250,000 iterations were drawn and the first 100,000 samples were discarded. For the SV2L and TRSV models, 300,000 iterations were drawn, and the first 40,000 and 130,000 samples, respectively, were discarded as a burn-in period. We collected every 10th samples for the SV model. For the SV2L and TRSV model, we stored every 30th iteration and collected every 20th iteration for the remainder of the models.

Table 8 provides the estimation results of the basic SV model. The volatility level μ is estimated to be −8.691, which is higher than that in the S&P 500 Index case. Autoregressive coefficient β is estimated to be similar to that in the S&P 500 Index case.

[Table 8] Estimation results of the basic SV model for MSFT

	Posterior				Convergence
Parameters	Mean	Std errors	5%	95%	Diagnostics
μ	−8.6907	0.1543	−8.9393	−8.4342	−1.1940
β	0.9746	0.0059	0.9647	0.9839	0.5896
σ^2	0.0325	0.0050	0.0250	0.0413	−0.5077

Note: Same as in Table 1.

Table 9 presents the estimation results of the SVL model. Correlation coefficient ρ is estimated to be −0.304. Similar to the stock index case, return and volatility has a negative relationship. However, the leverage effect is substantially weaker

the triple regime SV model is shown to fit better than the two regime SV model for S&P500 and MSFT.

compared with that in the stock index. Autoregressive coefficient β is estimated to be 0.970, which is similar to that in the basic SV model.

[Table 9] Estimation results of the SVL model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ	−8.6910	0.1352	−8.9141	−8.4666	1.2990
β	0.9700	0.0061	0.9596	0.9796	1.2610
ρ	−0.3041	0.0572	−0.3906	−0.2042	1.3240
σ^2	0.0376	0.0057	0.0288	0.0477	−1.0350

Note: Same as in Table 1.

Table 10 shows the estimation results of the SV2L model. When a stock return is negative, the correlation coefficient parameter ρ_0 is estimated to be positive at 0.7546. Finding a positive estimate for ρ_0 is relatively surprising, which was also observed in Xu (2010), Yu (2012), and Wu and Zhou (2014). One explanation for this result was suggested by Xu (2010). This outcome may be caused by the wait-and-see investing strategy by investors during turmoil periods. When stock price drops beyond a certain level, investors become cautious (more risk averse) and may choose a wait-and-see investing strategy. This strategy would lead to the reverse leverage effect. Meanwhile, when a stock return is non-negative, ρ_1 is estimated to be −0.9675. In addition, volatility level μ is estimated to be considerably higher than those in SV and SVL.

[Table 10] Estimation results of the SV2L model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ	−1.5721	0.7478	−2.8592	−0.3652	0.9373
β	0.9686	0.0036	0.9625	0.9742	0.4358
σ^2	0.0994	0.0164	0.0738	0.1278	0.0134
ρ_0	0.7546	0.0605	0.6431	0.8405	0.9878
ρ_1	−0.9675	0.0238	−0.9951	−0.9233	−1.5400

Note: Same as in Table 1.

Table 11 reports the estimation results of the SVT model. When a stock return is negative, the volatility level parameter μ_0 is estimated to be −7.843. When a stock return is non-negative, μ_1 is estimated to be −10.178. This result implies that the volatility level is substantially high when a return is negative. The volatility persistence parameter β_0 and β_1 are estimated to be 0.965 and 0.978, respectively.

[Table 11] Estimation results of the SVT model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_1	−7.8429	0.4817	−8.4998	−6.9563	−0.3615
μ_2	−10.1780	0.8415	−11.7750	−9.0710	1.0370
β_1	0.9649	0.0119	0.9446	0.9836	0.2324
β_2	0.9775	0.0105	0.9578	0.9920	−0.4136
σ^2	0.0347	0.0052	0.0269	0.0439	−0.2482

Note: Same as in Table 1.

Table 12 provides the estimation results of the SVTL model. Correlation coefficient ρ is estimated to be −0.330, which is similar to that in the SVL model. Volatility persistence parameters β_0 and β_1 are estimated to be 0.966 and 0.975, respectively, but their difference is not substantial.

[Table 12] Estimation results of the SVTL model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	−9.0072	0.6205	−10.1296	−8.2300	1.4030
μ_1	−8.3118	0.8297	−9.6212	-6.8313	-0.8689
β_0	0.9652	0.0136	0.9421	0.9868	-0.2463
β_1	0.9752	0.0128	0.9521	0.9936	0.6189
ρ	−0.3297	0.0756	−0.4500	-0.2043	0.5807
σ^2	0.0385	0.0058	0.0298	0.0486	-1.0830

Note: Same as in Table 1.

Compared with the SVT model, note that volatility level parameter μ is estimated to be high when a stock return is non-negative. When the model incorporates the threshold and leverage effects, the volatility level parameter is estimated to be higher when a stock return is non-negative, although the difference between $\hat{\mu}_0$ and $\hat{\mu}_1$ is not significant.

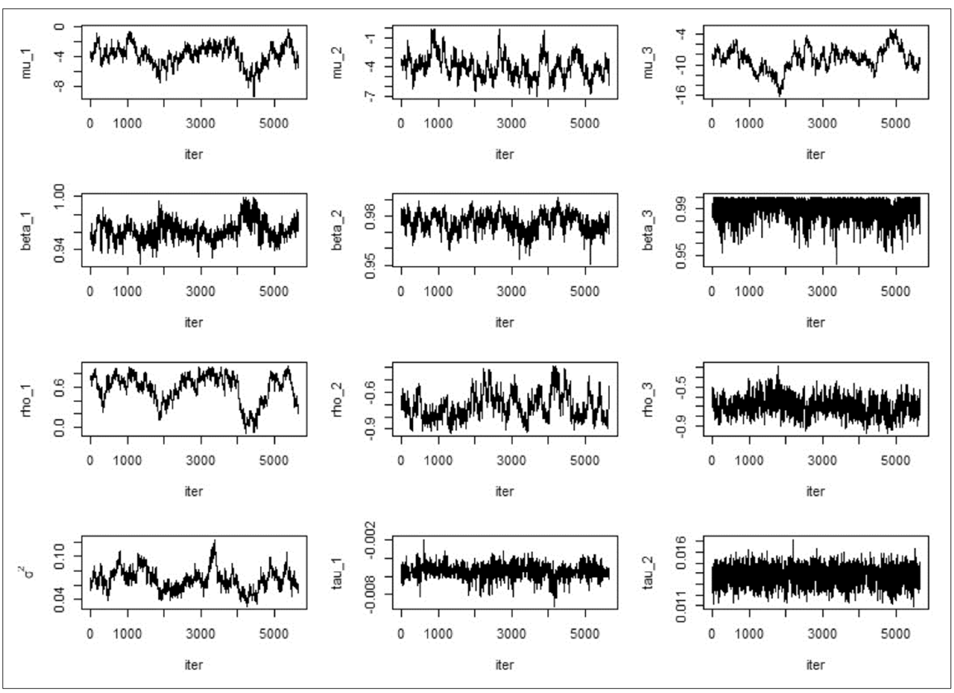
The estimation results of our triple regime SV model is provided in Table 13. The convergence diagnostics by Geweke (1992) in Table 13 shows that the Markov chains converged well. Figures 6 to 8 provide the trace of the MCMC iterates after a burn-in period, autocorrelations of the draw sequences, and estimated posterior densities of all parameters. Figure 5 provides the plot of the posterior mean of the MCMC iterates for h_t. The thick line represents the estimated $\sqrt{h_t}$ and the dotted line indicates the absolute value of the demeaned MSFT returns. Evidently, the estimated volatilities explain the absolute value of the demeaned MSFT returns rather well.

[Table 13] Estimation results of the TRSV model for MSFT

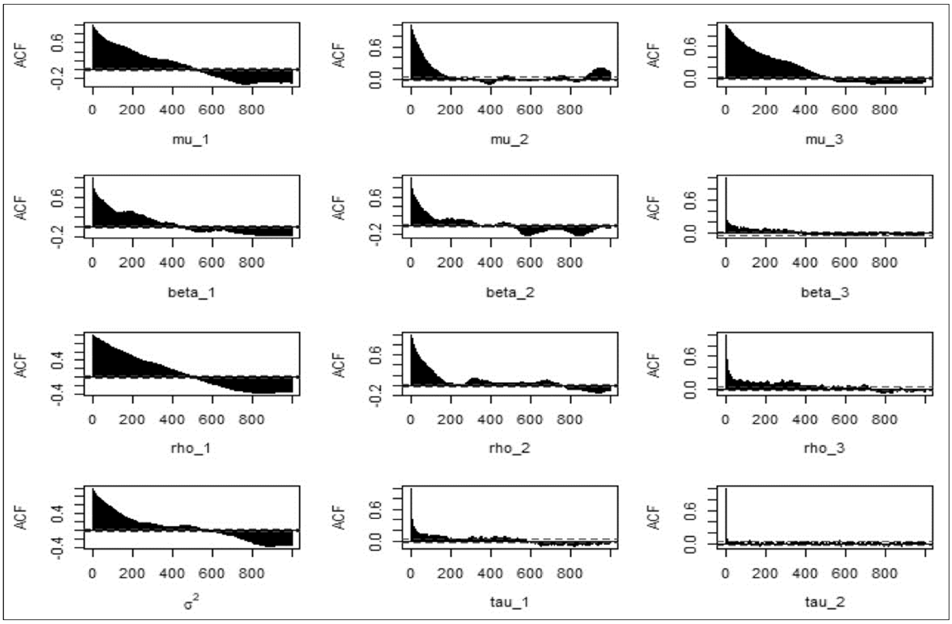
Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_1	−3.7631	1.3826	−6.3070	−1.7269	0.5950
μ_2	−3.9428	1.1566	−5.7137	−1.9253	1.3650
μ_3	−8.8866	2.1580	−12.4465	−5.2933	0.2560
β_1	0.9620	0.0108	0.9459	0.9823	−0.1534
β_2	0.9761	0.0053	0.9667	0.9842	1.2510
β_3	0.9906	0.0074	0.9761	0.9989	−0.2181
ρ_1	0.5839	0.2152	0.1254	0.8371	0.2914
ρ_2	−0.7128	0.1061	−0.8598	−0.5174	−0.4251
ρ_3	−0.7037	0.0906	−0.8512	−0.5537	0.3661
τ_1	−0.0055	0.0007	−0.0068	−0.0045	0.0849
τ_2	0.0139	0.0008	0.0124	0.0151	0.4944
σ^2	0.0677	0.0149	0.0446	0.0951	−0.3677

Note: Same as in Table 1.

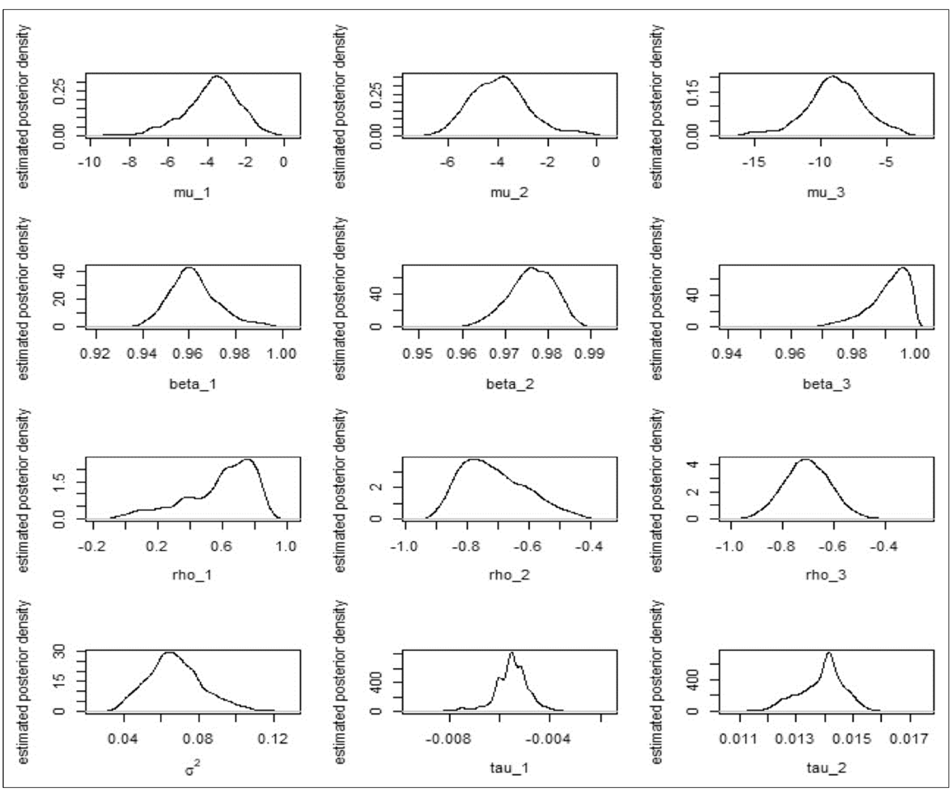
[Figure 6] Plots of the MCMC iterates obtained from the TRSV model fitting of MSFT



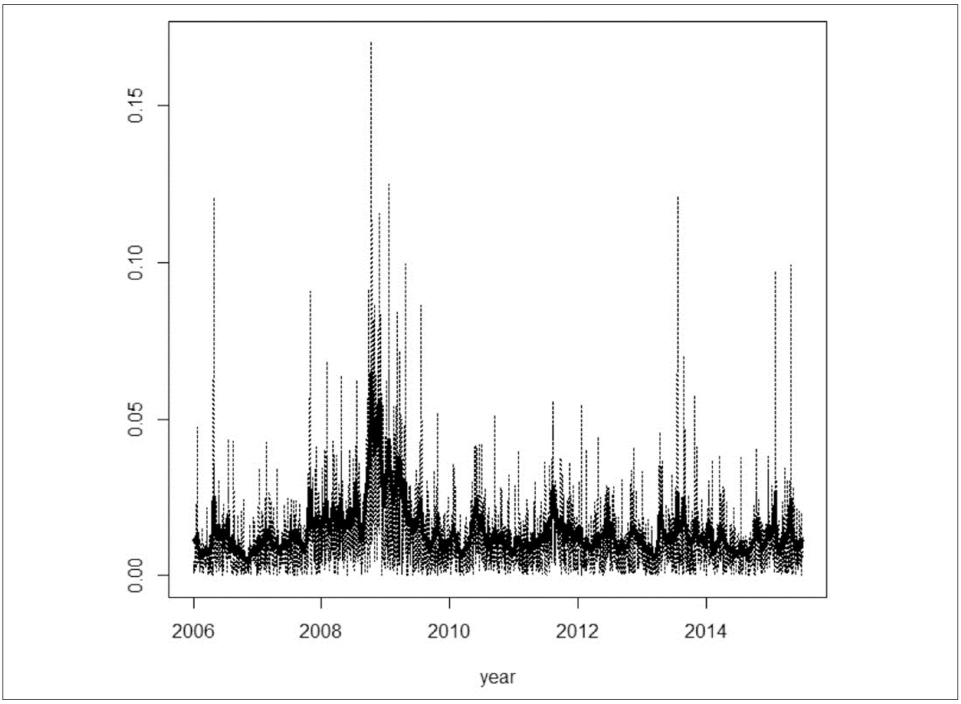
[Figure 7] Autocorrelation of the MCMC iterates for the TRSV model fitting of MSFT



[Figure 8] Estimated posterior densities for the TRSV model fitting of MSFT



[Figure 9] Estimated $\sqrt{h_t}$ with $|r_t|$ for the TRSV model fitting of MSFT



First, parameter τ_1 for our TRSV model is estimated to be −0.0055, which is the 33% quantile of the demeaned return series. When a return is lower than its 33% quantile, it belongs to regime 1. Parameter τ_2 is estimated to be 0.0139, which is 85% quantile of the return series. Hence, when a return is between its 33% and 85% quantiles, it belongs to regime 2. When a return is above its 85% quantile, it is in regime 3. Therefore, regime 1 contains 33% large negative returns, regime 2 includes 52% moderate negative/positive returns, and regime 3 has 15% large positive returns. Compared with the S&P 500 Index case, where regimes 1, 2 and 3 contain 21%, 47%, and 32%, respectively, of the return series, regime 1 includes more return series, while regime 3 contains less return series in MSFT.

Second, the correlation coefficient ρ in regimes 2 and 3 are estimated to be similar, and $\hat{\rho}_2$ and $\hat{\rho}_3$ are −0.713 and −0.704, respectively, which are below the estimates in SVL and SVTL models. This result indicates that when a return is moderately negative or positive or largely positive, the leverage effect is estimated to be stronger than that in the SVL and SVTL models. However, it is estimated to be positive (i.e., 0.584) in regime 1, thereby corresponding to the results in the SV2L model. When there is a largely negative lagged return, we can interpret the reverse leverage effect as follows. A large negative return leads to increased debt/equity ratio, and investors will expect increased future return. The reason is that blue chip stocks, such as Microsoft, have less risk compared with other stocks during periods of financial crisis. Investors expect that the stock price for MSFT will bounce, and the current price is substantially low, given all the financial aspects of Microsoft, including debt/equity ratio. This result may be caused by a wait-and-see investing strategy by investors during periods of turmoil. When stock price drops beyond a certain level, investors become cautious (more risk averse) and may choose a wait-and-see investing strategy. This strategy would lead to the reverse leverage effect. Accordingly, this aspect explains why triple regimes are introduced to differentiate the time-varying leverage effects that are dependent upon the sign and magnitude of lagged returns.

Third, volatility level parameter μ is estimated to be the highest and lowest in regimes 1 and 3, respectively. When the return is all positive (regime 3), volatility is the lowest. Volatility level parameter μ in regime 2 is estimated to be −3.943, which is similar to μ_1. If we compare regimes 1 and 3, then it appears that the volatility level and the leverage effect are substitutes for MSFT. In regime 1, the volatility level is markedly high, while there exists a reverse leverage effect. In regime 3, volatility level is low and there exists a conventional leverage effect.

The volatility persistence coefficient β is estimated to be the largest in regime 3 ($\hat{\beta}_1, \hat{\beta}_2$, and $\hat{\beta}_3$ are 0.962, 0.976, and 0.991, respectively).

Lastly, Table 14 shows that our TRSV model achieves the lowest DIC, which implies that our TRSV model fits the data best. DIC of our TRSV model is −13,346.6, which is substantially lower compared with those of the other SV models. The second best model in terms of DIC is the SV2L model, and its DIC is −13,311.6. The SVL model has a similar DIC as the SVTL model.

[Table 14] Comparison results of the SV models for MSFT

Models	DIC		PD		Rank
SV_0	−13297.9	(0.3910)	22.6	(0.3920)	6
SVL	−13310.5	(0.8043)	24.1	(0.8047)	3
SV2L	−13311.6	(1.2394)	31.4	(1.2380)	2
TSV	−13303.1	(0.7697)	24.2	(0.7711)	5
TSVL	−13310.3	(0.8350)	23.3	(0.8360)	4
TRSV	−13346.6	(1.6605)	24.7	(1.6592)	1

Note: Same as in Table 7.

IV. Conclusion

This study investigates a new stochastic volatility model that accommodates three regimes and the threshold and leverage effects. We find evidence that the relationship between stock return and volatility depends on the magnitude and sign of a return. In the S&P 500 Index and MSFT, the results show that the leverage effects are estimated to be different in each of the three regimes. In both cases, when stock price rises beyond a certain level (in regime 3), the conventional leverage effect becomes considerably strong. In regime 3, correlation coefficient ρ was estimated to be −0.901 and −0.704 for the index and MSFT, respectively. In regime 1 (large negative return), the reverse leverage effect appeared for MSFT, while the conventional leverage effect appeared for the index. In regime 2 (moderate negative or positive return), the conventional leverage effect appeared in the S&P 500 Index and MSFT. Comparing the leverage effects between the index and MSFT, the individual firm showed significantly weaker leverage effects than the index. Compared with the existing SV models, our model fits the data best in both stock cases, thereby supporting the idea of allowing three regimes in our model.

The current research provides empirical evidence that the relationship between return and volatility depends on the sign and magnitude of a return. We conjecture that investors behave differently when stock prices show rapid changes. It would be desirable to further investigate why the leverage effect becomes stronger when stock return is largely positive or the reverse leverage effect could appear when stock return is largely negative. Moreover, we extend our model to generalized n-regime stochastic volatility models, in which the number of regimes can be determined endogenously. We leave these aspects for future studies.

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