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Credit Constraint and Excess Return: The Case of Chonsei Leases in Korea

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Chonsei lease arrangements, in which an up-front deposit is paid at the start and returned at the end of a lease without any periodic payments, are a unique and dominant form of lease in Korea. This paper offers a simple model to explain the existence of the chonsei lease arrangement. The chonsei deposit can be thought of as a loan from the tenant to the landlord, and interest is paid in the form of housing consumption. From this perspective, a chonsei deposit is cheap because the calculated interest rate is higher than the market rate. The landlord should have a good investment opportunity to justify the use of chonsei. However, it is widely understood that chonsei deposits are used mostly as leverage to purchase a house. With credit constraints, this paper suggests excess return can exist in the housing market and that the chonsei lease arrangement is utilized to capture this return. The current demand for housing can be restricted by credit constraints and house prices can be undervalued. A credit-constrained agent may resort to chonsei to fund the purchase of a house. In contrast, the tenant will ask for high-interest payments.

JEL Classification: D15, G12
Keywords: Credit Constraint, Chonsei Lease, Housing Market

I. Introduction

Housing represents an important component of the asset portfolio of households in Korea. According to a report by the Korean Financial Investment Association, the proportion of non-financial assets was over 75% in 2012, though this number has been decreasing slowly. This figure contrasts with figures of around 30% in the US and 40% in Japan (both in 2012). Most non-financial assets are related to real estate, and housing assets are considered an important component of real estate assets.

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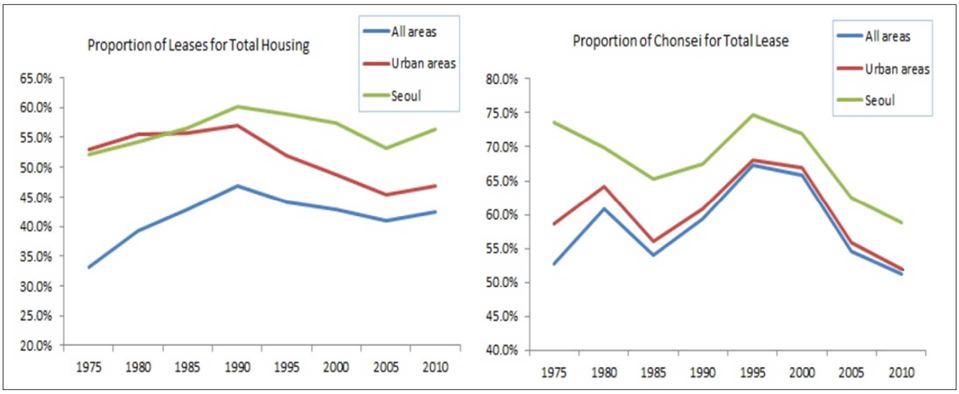
[Figure 1] Proportion of leases in the household housing and proportion of chonsei leases in the leasing market



Source: Population and Household Census, Korean National Statistical Office.

Korea has a special form of lease arrangement called chonsei.[1] In a chonsei lease, the tenant gives an up-front deposit to the landlord at the start of the lease, which will be returned to tenants at the end of the lease. There are no monthly payments in the pure form of chonsei.

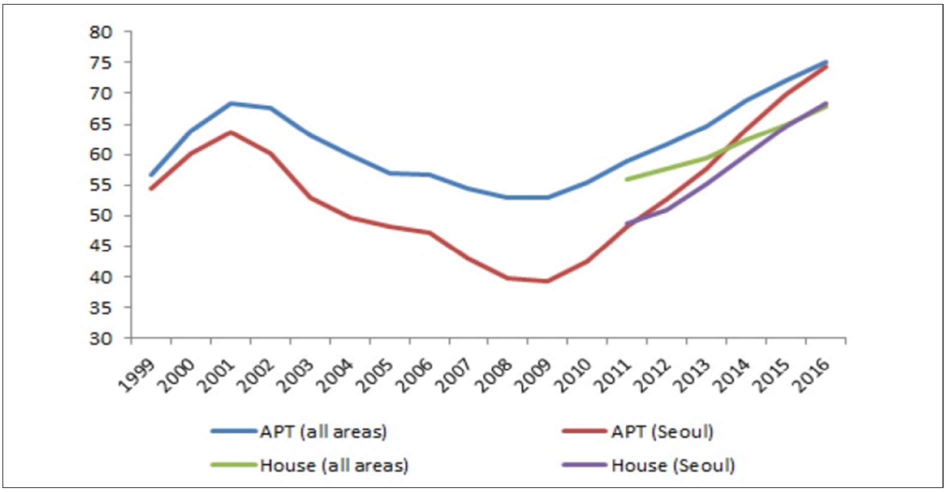
Chonsei is the dominant form of lease agreement in Korea. Figure 1 shows the proportion of leases in household housing and the percentage of chonsei lease contracts. Around 45% of households live in houses with a lease arrangement. This proportion is higher in urban areas, and in Seoul, the capital city of Korea, this figure is around 60%. Among these leases, chonsei accounted for more than 50% in 1975 and was also higher in urban areas and Seoul. Though the dominance of chonsei has decreased since 1995, it remains higher than 50% in all areas and 60% in Seoul in 2010.[2]

The interesting aspect of chonsei is that it is less expensive than periodic rent. The interest payments or investment return from the up-front deposit acts as the rental payment in a chonsei contract. However, given market interest rates, the deposit appears more reasonable than monthly rent. For the same reason, the ratio of the deposit to house price was quite low as shown in Fig. 2. It reached 70-75% in 2016 but was below 60% for apartments for most of the 2000s and even dropped below 50% for apartments in Seoul. Though official data are lacking, it is believed that the ratio of chonsei deposit to price was much lower in the 1970s. The rent-to-house-price ratio may fluctuate due to expectations of price increase, and it is

[1] According to Navaro and Turnbull (2010), this type of lease arrangement, which they call an antichresis lease, is also found in nearly all Latin American countries. A brief history of this form of lease is discussed there.

[2] Recently, a mixed form of chonsei and periodic rent has emerged where rent is paid in conjunction with a lower but still significant deposit. Here, chonsei is discussed only in its pure form.

[Figure 2] Ratio of chonsei deposit to price



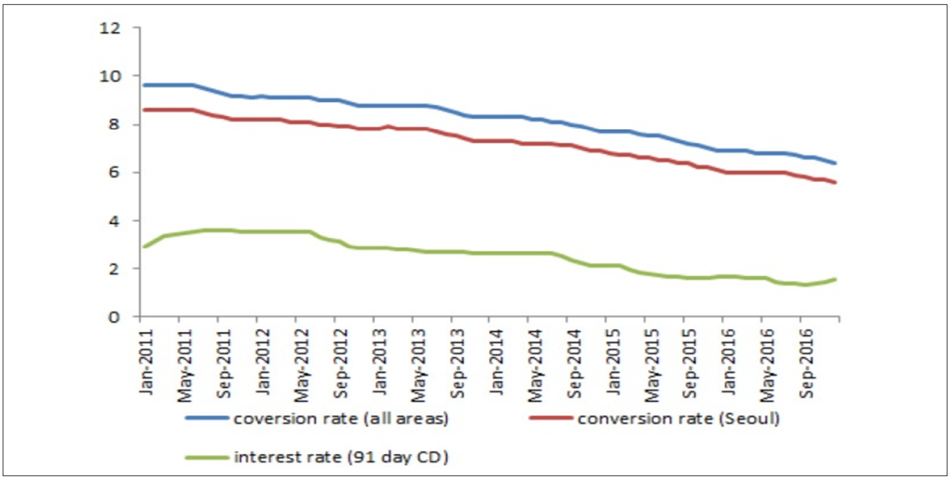
Source: Kookmin Bank.

arguable that expectations of inflation may justify the ratio of the chonsei deposit to a price below 50%.[3] However, if we calculate the interest rate for a chonsei deposit and the monthly rent for a similar house, the conversion rate is higher than the market interest rate. Figure 3 compares the deposit rent conversion rate and the 91-day CD rate. Even when the spread is considered, the deposit rent conversion rate is systematically higher than the borrowing rate based on the housing asset collateral. Though these data are very recent, they also reflect the period when chonsei was more dominant.

This relatively inexpensive deposit suggests that landlords must have some investment opportunities with a very high return. That is, landlords are willing to pay a higher interest rate than the market rate to take advantage of these investment opportunities. Indeed, past research has posited that chonsei contracts rely on these investment opportunities. For example, Kim and Shin (2011) assume that landlords/entrepreneurs have investment opportunities with potentially higher returns, but face an imperfect financial market. A chonsei contract may provide a source of capital for these investment opportunities. However, chonsei deposits are widely believed to have been used as leverage to buy other housing assets, especially in the 1970s and 1980s. Thus, the housing assets themselves provide excess returns over the average market returns, which requires an explanation of the chonsei lease in the housing market alone.

[3] Ambrose and Kim (2003) justifies the 50% deposit/price ratio using the option pricing model considering the default risk of the landlord. See the literature review.

[Figure 3] Deposit-rent conversion rate and interest rate



Source: Korea Appraisal Board.

This paper attempts to explain the existence and widespread use of the chonsei lease contract without pre-imposition of sources of excess returns. The key component is credit constraints or an imperfect financial market. The basic idea is outlined as follows. The value of housing is expected to grow rapidly during a period of strong income growth. When the financial market is perfect, the expectation of price increase will increase the current demand for that asset, and the asset price will appreciate until the expected returns of the asset is equal to the market interest rate. With credit constraints, however, the current demand is restricted because of limited funding, leading to housing assets being undervalued, which means a higher return from housing assets than the market rate could occur. This excess return of housing assets can be shared between the landlord and the tenant through a chonsei contract. The landlord can keep the house (or buy another house) by borrowing money from the tenant. The tenant can ask for higher interest payments in the form of housing consumption. This exchange makes the chonsei deposit less expensive than monthly rent given the market interest rate.

To illustrate this idea succinctly, this paper employs a simple two-period consumption choice model without uncertainty when a fixed volume of housing assets is provided exogenously. I initially assume an extreme form of financial market imperfection for simplicity: no borrowing is possible, which will then be relaxed in our extensions. If a financial market is perfect, the price is adjusted so that everyone is indifferent to selling (buying), leasing (renting) with chonsei, and leasing (renting) with periodic rent. However, if a financial market is imperfect, house prices in the first period can be undervalued. Some house owners with low first-period income may want to sell their houses for consumption smoothing even at an undervalued price, which creates room for the chonsei lease arrangement. If chonsei leases are available, house owners can lease houses with chonsei to achieve partial consumption smoothing and keep the excess return from their housing assets. Tenants will ask for a higher interest rate to share some of the excess return from the housing assets. Thus, a chonsei deposit is less expensive when considering the market interest rate and periodic rent. Because the same function cannot be achieved through a periodic rental contract, chonsei leases can become the dominant form of lease contract.

We also considered an alternative form of credit constraint and analyzed its effects. Suppose a certain portion of the house price can be borrowed with the housing asset as collateral. The relaxation of the credit constraint with a higher portion borrowed will increase current house prices and chonsei deposits. Eventually, the space for chonsei lease contracts may disappear. Fig. 1 shows that the use of chonsei leases has declined since 1995, even though it is still the dominant form of lease arrangement.[4] Recently, some landlords have looked into converting a portion of chonsei deposit into monthly rent, thus a mixed form of chonsei and periodic rent has become more prevalent. The development of the financial market, especially mortgage lending, may play an important role in this.

This paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the model and Section 3 analyses it. The implications of the model are briefly discussed in Section 4, while Section 5 outlines several extensions. The conclusion then follows.

1.1 Related Literature

Most Korean literature on chonsei has focused on the size of the chonsei deposit, taking the chonsei arrangements as given. These studies have tended to empirically test the relationship between price increase expectations and the ratio of the chonsei deposit to the house price. For example, Lee (2013) empirically tested the theoretically predicted relationship between the two using panel data. Similar studies focusing on the ratio of the chonsei deposit to the house price and other variables include Kim et al. (1998), Cho (2005), and Son et al. (2011). The ratio of chonsei deposit to house price is investigated from a different angle in Ambrose and Kim (2003). Based on an option pricing model, the equilibrium chonsei deposit is dependent on the landlord's default risk, which is the probability of the house price falling below the chonsei deposit by the end of the lease. In contrast, this paper is interested more in why the chonsei lease arrangement emerges and what economic environment facilitates its use.

[4] Once chonsei becomes the dominant form of lease, it may not easily disappear even when the economic environment changes. To change a lease arrangement from chonsei to monthly rent, the chonsei deposit has to be returned. Many landlords may have a liquidity constraint that prevents them from doing that.

To the best of my knowledge, only three papers have focused on the emergence of the chonsei lease arrangement. As mentioned earlier, Kim and Shin (2011) consider the chonsei lease arrangement as a conduit for shifting capital to profitable investment projects for landlords/entrepreneurs. They emphasize the efficiency-enhancing role of chonsei leases. Unlike Kim and Shin, this paper does not assume profitable investment projects outside the housing market. Navaro and Turnbull (2010) consider antichresis leases in Bolivia, which are the same arrangement as chonsei and explains its emergence as an incentive scheme to maintain the quality of the leased property. According to this argument, the optimal lease option, which is the periodic rent or antichresis, is determined by the relative importance of the efforts by the landlord and the tenant to maintain the property. If effort from the tenants is more important, antichresis is used, and the tenant will try to maintain the value of the property so that the deposit can be safely returned. Navaro and Turnbull focus on the variation in the contract depending on the type of property, while the present paper disregards the type of property and focuses on credit constraints as the driving force for chonsei leases. A similar reason is investigated by Kim (2013). He considers the contractual incentive of landlords and tenants when they choose between a chonsei lease and periodic rent. Based on the economic environment, including house prices, he searches for reasons why a chonsei lease would be selected over periodic rent. Kim lists excess return on the house and credit constraints on house purchases as the reasons for chonsei leases and also argues that the conversion rate between the chonsei deposit and monthly rent should be higher than the market interest rate. However, Kim's focus is on the contractual choice between landlords and tenants, and the excess return of the house is exogenously imposed as a condition of this contractual choice. The present paper considers an equilibrium model and focuses on the determination of the price and the chonsei deposit. I endogenously derive the excess return of housing assets, which provides a necessary condition for the emergence of chonsei leases.

II. Model

We consider a simple intertemporal consumption decision model with two periods, $t = 0,1$, without uncertainty.

Agents There is a continuum of agents of unit mass with income w_0^i and w_1 in periods 0 and 1 respectively, $i \in [0,1]$. The period 0 income w_0^i is distributed with distribution function $F(w)$ over $[0,\overline{w}]$, while the period 1 income is the same for every agent. For a given income profile, the agent i attempts to allocate their consumption over two periods to maximize the utility function

$$u(c_0^i)+\beta u(c_1^i),$$

where $u'>0$ and $u''<0$.

Imperfect Financial Market Agent i can reallocate their income from period 0 to period 1 by saving it at interest rate r. However, the financial market is imperfect in that borrowing is not possible. This describes a situation in which the retail lending market is not fully developed, such as in Korea until the 1990s. Thus, there is no way to reallocate their income from period 1 to period 0, and agents with low income in period 0 cannot smooth their consumption. We will consider alternative and less extreme assumptions for an imperfect financial market in our extensions.

Housing Assets There exist identical housing assets of mass s with $s<1$. Agents are randomly endowed with housing assets independent of their income levels, and I assume each agent can possess up to one housing asset unit, that is, a s portion of agents have one housing asset unit while a $1-s$ portion do not.

A housing asset in general has two characteristics. It is an object of housing consumption and a financial asset that can act as savings for future consumption. In this paper, we focus on the second aspect. Thus, the consumption of housing assets is the same as that of other consumption goods. Specifically, a housing asset unit has consumption value H, and we assume its period 1 price including its consumption value is fixed at p_1.

Housing assets can be traded at the start of period 0. House owners are willing to sell their houses for consumption-smoothing purposes if their period 0 income is low. Non-owners are willing to purchase a house as an alternative form of saving.

Housing Lease Contracts We consider two possible housing lease arrangements. One is a periodic rental payment scheme in which the tenant pays non-returnable rent for their use of the house. The other is a chonsei contract in which a tenant pays an up-front deposit that will be returned at the end of the contract. Interest income generated from the up-front deposit acts as periodic rent in the latter scheme.

Equilibrium We follow the textbook definition of equilibrium: a set of prices that clears the relevant markets. We can think of two or three types of markets related to housing assets: a market for ownership, a lease market for periodic rent, and possibly another lease market with chonsei if the arrangement exists. We call these the housing market, periodic rental market, and chonsei lease market, respectively. Therefore, an equilibrium is a pair or a triple of house price p_0, periodic rent R, and chonsei deposit p_r, which equilibrates the supply and demand for each market.

III. Analysis

We analyze the housing and lease market in period 0 and obtain the equilibrium prices as defined above.

3.1. Benchmark: Perfect Financial Market where Borrowing is Possible at Interest Rate r

As a benchmark, we analyze the market when borrowing is possible at the same interest rate r as saving. When the financial market is perfect, agent i will allocate their total wealth to consumption in each period to optimally smooth their consumption path. Agent i's optimization problem is

$$\max_{c_0^i, c_1^i} u(c_0^i) + \beta u(c_1^i)$$
$$s.t. \quad c_0^i + \frac{c_1^i}{1+r} = W^i$$

where W^i is the present value of wealth. Optimal consumption is obtained by

$$u'(c_0^i) = \beta(1+r)u'(c_1^i) \tag{1}$$

and constraint. The present value of wealth W^i depends on whether the agent has a house. The present value of wealth for non-owners, W_N^i, is

$$W_N^i = w_0^i + \frac{w_1}{1+r}, \tag{2}$$

and that for house owners, W_O^i, additionally contains the consumption value of housing in period 0 and its price in period 1:

$$W_O^i = w_o^i + H + \frac{w_1 + p_1}{1+r}. \tag{3}$$

The optimal consumption path is dependent on w_0^i, which varies by agent, but its characterization only depends on the ownership of a housing asset. From here on, we omit superscript i if it does not cause confusion. Let (c_0^N, c_1^N) and (c_0^O, c_1^O) be the optimal consumption paths for non-owners and owners, respectively.

Lemma 1 *Optimal consumption paths* (c_0^N, c_1^N) *and* (c_0^O, c_1^O) *are determined by (1) and the budget constraint with wealth defined by (2) and (3), respectively.*

If the financial market is perfect, the house price in period 0 will be adjusted so that the purchase or sale of a housing asset would not affect W. For example, if the price is too high and the sale of a housing asset increases W, house owners are willing to sell their houses and the price will fall. Therefore, the equilibrium house price p_0^* should be the present value of its current consumption value and its future price:

$$p_0^* = H + \frac{p_1}{1+r}. \tag{4}$$

By the same logic, rent R should be the consumption value of housing, and the interest income from chonsei deposit p_r should cover periodic rent:

$$\frac{r}{1+r} p_r^* = R^* = H. \tag{5}$$

Proposition 1 *When the financial market is perfect, the equilibrium house price, rent, and chonsei deposit are determined by (4) and (5).*

Note that no separate role for the chonsei lease arrangement can be observed in this situation and that a periodic rental contract can cover all lease arrangements. This is no longer true when we introduce imperfection to the financial market.

If we denote π as house price increase rate, $p_1 = (1+\pi)p_0$, the chonsei deposit and house price would satisfy

$$\frac{p_r^*}{p_0^*} = \frac{r-\pi}{r}.$$

Corollary 1 *When the financial market is perfect, the ratio of the chonsei deposit to the house price is negatively correlated with the expected house price inflation and is positively correlated with the interest rate.*[5] *When there is no expectation of an increase in house price, the chonsei deposit and the house price will be the same.*

This is an implication that has been tested quite often in the literature including

[5] Note that π cannot exceed r. If it does, no house owner will sell the house and π will decrease.

ones discussed in our literature review. In contrast to these studies, our interest is to explain the separate role of the chonsei lease arrangement.

3.2. Imperfect Financial Market: No Borrowing

If there is no borrowing, house owners whose incomes are low in the first period may be willing to sell a house at a lower price than p_0^*. We first discuss the possibility of an equilibrium in which the house price is lower than p_0^* without considering the chonsei lease market. Then, we discuss the chonsei lease market and its effect on the housing market.

3.2.1. Housing Market without Chonsei Leases

If periodic rent is the only possible arrangement in the lease market, rent will remain at $R^* = H$. Whether borrowing is possible, the consumption value of housing H is fairly traded in this market.

In the housing market, the price will be lower than p_0^*. Some house owners are willing to sell their house to increase their period 0 consumption even if the sale decreases their present value of wealth. Meanwhile, no buyer would be willing to buy a house if the purchase decreases the present value of wealth. Buyers can always save money to smooth their consumption instead of purchasing a house. Thus, if we denote the equilibrium price as p_0^I, then

$$p_0^I \leq p_0^*.$$

We discuss a house owner's sale decision and a non-owner's purchase decision in turn, and then determine the equilibrium price as a result of these decisions. First, consider a house owner's sale decision. If the owner keeps the house, the present value of wealth is W_O in (3). If optimal consumption is achievable without borrowing, they will choose (c_0^O, c_1^O). This is the case when the available income in period 0, $w_0 + H$, is greater than the optimal consumption c_0^O.[6] If period 0 income is lower than the optimal consumption, each period's income will be consumed. Thus, a house owner's utility when keeping a house, $U_O^0(w_0)$, is

$$U_O^0(w_0) = \begin{cases} u(c_0^O) + \beta u(c_1^O) \ \text{ if } \ w_0 + H \geq c_0^O \\ u(w_0 + H) + \beta u(w_1 + p_1) \ \text{ if } \ w_0 + H < c_0^O \end{cases}. \tag{6}$$

If the house owner sells the house, the present value of wealth W_O^S is

[6] It is possible that c_0^O is so small that an agent has to consume H without selling their house in period 0. I exclude this possibility by assuming that w_1 is large enough.

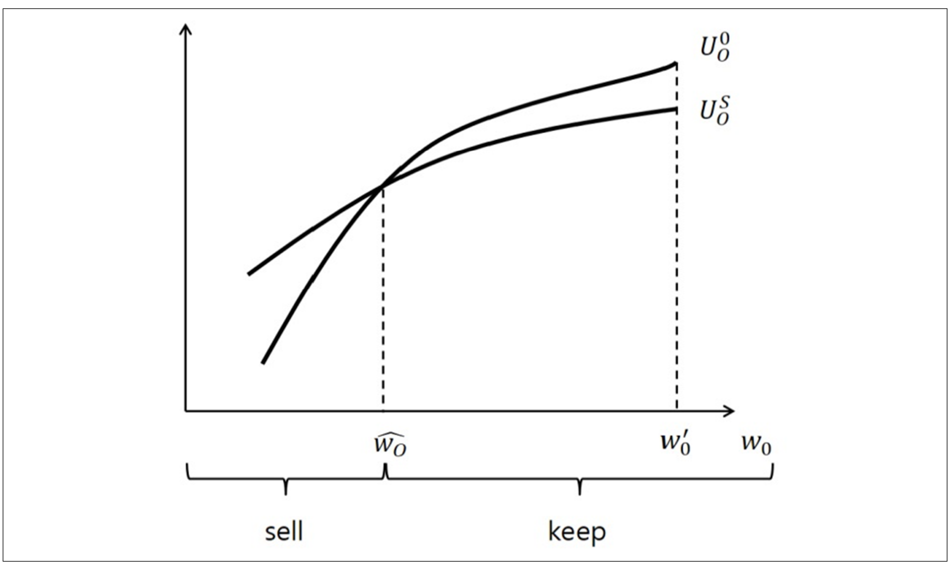
$$W_O^S = w_0 + p_0 + \frac{w_1}{1+r}. \tag{7}$$

Optimal consumption (c_0^{OS}, c_1^{OS}) is determined by (1) and budget constraints with the wealth of (7). If period 0 income $w_0 + p_0$ is greater than c_0^{OS}, optimal consumption can be achieved. Otherwise, all available income will be consumed in each period. Thus, a house owner's utility when selling a house, $U_O^S(w_0)$, is

$$U_O^S(w_0, p_0) = \begin{cases} u(c_0^{OS}) + \beta u(c_1^{OS}) \text{ if } w_0 + p_0 \geq c_0^{OS} \\ u(w_0 + p_0) + \beta u(w_1) \text{ if } w_0 + p_0 < c_0^{OS} \end{cases}. \tag{8}$$

A house owner will make a sale decision by comparing $U_O^0(w_0)$ and $U_O^S(w_0, p_0)$. Consider period 0 income level w_0', such that $w_0' + H = c_0^{OS} \leq c_0^O$. If $w_0 > w_0'$, keeping the house is better, i.e. $U_O^0(w_0) > U_O^S(w_0, p_0)$. The advantage of selling a house is higher consumption in period 0; this advantage disappears if the period 0 income is high enough. Moreover, $\frac{d}{dw_0}U_O^0 > \frac{d}{dw_0}U_O^S$ if $w_0 < w_0'$ as a credit-constrained consumer will receive more benefit from the increase in the current income. A unique threshold income level $\widehat{w_O}(p_0)$ for sales decision exists and the house owner is willing to sell a house if their income is lower than that level (Fig. 4).[7]

[Figure 4] Comparison of U_O^0 and U_O^S



[7] $\widehat{w_O}$ exists as long as p_0 is not too low, i.e. $u(p_0) + \beta u(w_1) > u(H) + \beta u(w_1 + p_1)$.

This decision will determine the supply in the housing market. Specifically, supply is given by $sF(\widehat{w_O}(p_0))$. Note that supply $sF(\widehat{w_O}(p_0))$ increases with p_0 because U_O^S increases as p_0 increases.

Lemma 2 *A threshold period* 0 *income level* $\widehat{w_O}(p_0)$ *exists, such that house owners are willing to sell their houses if* $w_0 \leq \widehat{w_O}(p_0)$. *The supply in the housing market is given by* $sF(\widehat{w_O}(p_0))$, *which increases with* p_0.

Intuitively, if a house owner's income is low in period 0, they are willing to sell their house to secure more liquidity.

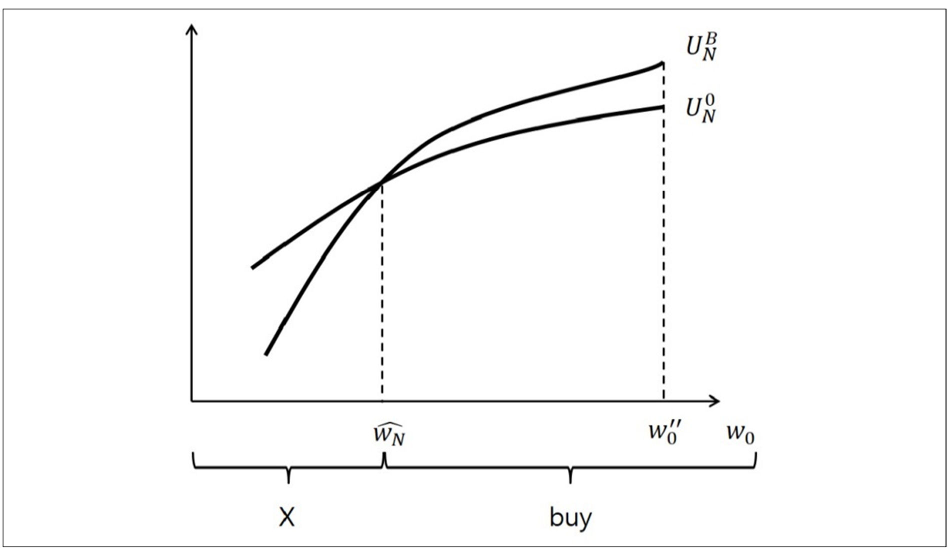
Similarly, we investigate the purchase decision of non-owners. A non-owner is willing to buy a house when they have enough liquidity in period 0. Thus, the following Lemma holds.

Lemma 3 *A threshold period* 0 *income level* $\widehat{w_N}(p_0)$ *exists, such that non-owners are willing to buy a house if* $w_0 \geq \widehat{w_N}(p_0)$. *The demand for houses is given by* $(1-s)\{1-F(\widehat{w_N}(p_0))\}$, *which decreases with* p_0.

Proof. In the appendix. ■

The above Lemma is illustrated in Fig. 5, wherein U_N^0 and U_N^B denotes a non-owners utility when not buying or buying a house, respectively.

[Figure 5] Comparison of U_N^0 and U_N^B



Combining the decisions of house owners and non-owners, the equilibrium price for a house in an imperfect financial market p_0^I satisfies

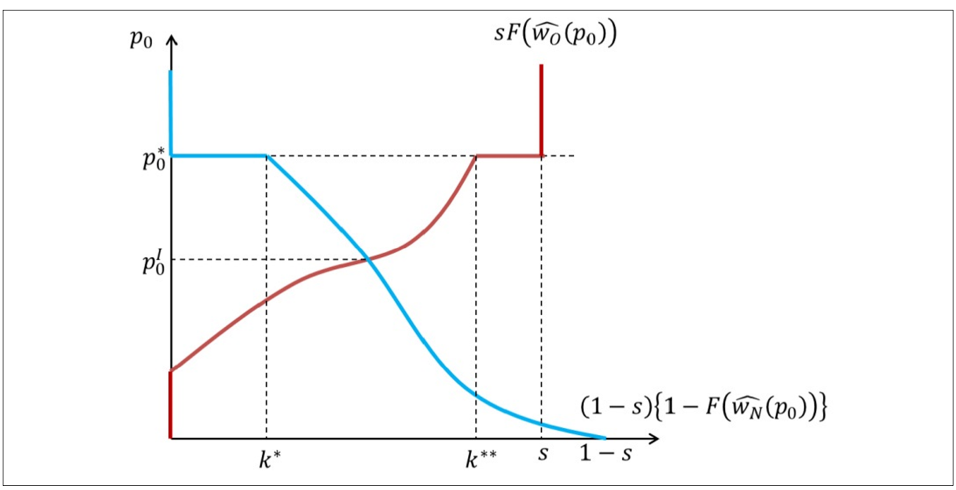
$$sF(\widehat{w_O}(p_0^I)) = (1-s)\{1-F(\widehat{w_N}(p_0^I))\}.$$

Figure 6 illustrates the equilibrium price in this market. As indicated, a unique equilibrium price exists because supply (demand) is increasing (decreasing) monotonically in p_0. The equilibrium price should be strictly lower than p_0^* as long as k^{**} is bigger than k^*. Some non-owners with sufficiently high income in period 0 are indifferent between buying and not buying a house when their wealth level remains the same. This mass is denoted by k^* in Fig. 6. Some house owners with sufficiently high income in period 0 will never sell their houses if doing so decreases their wealth level. The remaining mass of owners is willing to sell their house even if p_0 is lower than p_0^*. This mass is k^{**} in Fig. 6. If k^* is strictly smaller than k^{**}, demand falls short of supply at p_0^*, and thus the equilibrium price is strictly lower than p_0^*.[8]

Proposition 2 *Equilibrium price* p_0^I *exists and is unique. It is lower than the equilibrium price when the financial market is perfect,* $p_0^I \le p_0^*$.

The difference between p_0^I and p_0^* can be viewed as 'illiquidity penalty' for a house asset. An asset that can provide liquidity when it is demanded is

[Figure 6] Equilibrium of House Market



[8] Specifically, $k^* = (1-s)\{1-F(w_0^*)\}$ where $w_0^* - p_0^* + H = c_0^N$ and $k^{**} = sF(w_0^{**})$ where $w_0^{**} + H = c_0^O$. Thus a sufficiently large housing stock s or income distribution F with more low-income earners will guarantee $k^{**} > k^*$.

overvalued, which is called the liquidity premium (Holmstrom and Tirole, 2001). A house asset in this model cannot provide liquidity when it is demanded in period 0, i.e. its value cannot be translated instantly into consumption. Credit constraints encourage low-income house owners to sell their houses while restricting demand. Thus houses in period 0 are undervalued considering their consumption value H and future price p_1. This undervaluation of house assets creates excess returns. In this paper, excess return, we refer to a higher return than the market interest rate r. An agent with sufficient liquidity (or a high income in period 0) can enjoy a higher present value of wealth by keeping or buying a house.

We apply several comparative statics, which is summarized in the following proposition.

Proposition 3 *Equilibrium price* p_0^I *is affected by the parameters of the model as follows.*
i) If s *increases,* p_0^I *decreases with the same* p_0^*.
ii) If p_1 *increases, both* p_0^* *and* p_0^I *increase.*
iii) If r *increases,* p_0^* *decreases and* p_0^I *weakly decreases.*
iv) If w_1 *increases,* p_0^I *decreases with the same* p_0^*.
v) If w_0 *follows a distribution function* G *first-order stochastically dominated by* F, p_0^I *decreases.*

Proof. In the appendix. ■

The price of an asset goes down in i) if an asset is more abundant. A higher price in the future increases the current price in ii). If r is higher, the opportunity cost of buying a house increases, while that of selling house decreases, which will reduce the price. Both iv) and v) show the same qualitative results. If the current income is lower in v) or the future income is higher in iv), there will be more demand for liquidity in period 0 for consumption smoothing purposes. Thus, the illiquidity penalty will increase or housing assets will become more undervalued.

3.2.2. Housing Market with Chonsei Leases

The excess return on a housing asset creates room for another lease arrangement: chonsei. A chonsei lease can be viewed as a lending agreement from the tenant to the landlord. Interest payments are made in the form of housing consumption.[9] With this contract, the excess return on housing assets can be shared. Landlords can keep their houses and enjoy the excess returns while obtaining a certain level of

[9] This characteristic of chonsei contract, or house repo contract, is pointed out by Kim and Shin (2011). The difference is that they presupposed on excess return from other sectors, while this paper creates an excess return in the housing sector itself.

consumption smoothing by borrowing from tenants. Tenants, who provide this valuable credit, will ask for higher interest payments and gain a certain share of the excess return. Thus, chonsei deposits will be inexpensive considering the market interest rate and housing consumption value. This arrangement may drive out periodic rent leases, which do not provide the opportunity to share the excess return. We will investigate this possibility in this section.

A periodic rent market can still be in operation and its rent is $R^* = H$ as in a perfect financial market. Periodic rent is simply the fair exchange of housing consumption value H and rent R.[10]

We consider the necessary conditions for the chonsei deposit p_r for the chonsei lease market to be facilitated.[11] First, chonsei deposits should be low enough to attract tenants. Renting through chonsei reduces liquidity in period 0. To compensate for this loss of liquidity, the present value of wealth should increase:

$$p_r < p_r^* = \frac{1+r}{r} H . \tag{9}$$

Second, a chonsei deposit should be high enough to attract landlords. Selling a house would generate more liquidity in period 0 than leasing through chonsei. Thus, leasing through chonsei should generate a higher present value of wealth than selling a house:

$$p_0 - \frac{1}{1+r} p_1 < \frac{r}{1+r} p_r . \tag{10}$$

By combining (9) and (10), we have the necessary condition for the facilitation of chonsei leases.

Lemma 4 *If chonsei leases are to be used, then the chonsei deposit* p_r *should satisfy*

$$\frac{1+r}{r} p_0 - \frac{1}{r} p_1 < p_r < \frac{1+r}{r} H .$$

Note that $\frac{1+r}{r} p_0^I - \frac{1}{r} p_1 < \frac{1+r}{r} H$ as $p_0^I < p_0^*$. Thus, the room for chonsei leases is

[10] Note that no strictly positive gains from trade can be found in a periodic rent lease because we model a housing asset as a financial asset rather than a housing consumption good. If we have a separate utility function for housing consumption, the same housing consumption will have a different value for general consumption depending on the level of a consumption. Gains from trade can be achieved because the tenants will value housing consumption higher than the landlords.

[11] Though a chonsei lease can be used when the financial market is perfect, it does not provide a different role to that of a periodic rent arrangement. By facilitating the chonsei lease market, we mean that a chonsei lease plays a meaningfully different role from that of periodic rent.

created by the undervaluation of houses in period 0 because of the imperfect financial market. We now consider the decision of house owners and non-owners to lease and rent using chonsei.

We first consider the house owner's choice. Leasing a house using chonsei is another option that can be chosen. If they keep or sell their house, their maximized utility is as described in (6) and (8). If they lease a house through chonsei, their present value of wealth is

$$W_O^L = w_0 + p_r + \frac{w_1 - p_r + p_1}{1+r}. \tag{11}$$

If wealth can be allocated freely, their optimal consumption (c_0^{OL}, c_1^{OL}) is determined by condition (1) and budget constraints with the wealth level (11). Their liquidity in period 0 is $w_0 + p_r$. If it exceeds c_0^{OL}, optimal consumption (c_0^{OL}, c_1^{OL}) will be chosen. Otherwise, all available liquidity is consumed in each period. Thus, their utility when leasing a house using chonsei, U_O^L, is

$$U_O^L(w_0, p_r) = \begin{cases} u(c_0^{OL}) + \beta u(c_1^{OL}) \text{ if } w_0 + p_r \geq c_0^{OL} \\ u(w_0 + p_r) + \beta u(w_1 + p_1 - p_r) \text{ if } w_0 + p_r < c_0^{OL} \end{cases}.$$

If the necessary condition in Lemma 4 holds, the wealth level is higher in the order of keeping, leasing, or selling the house,

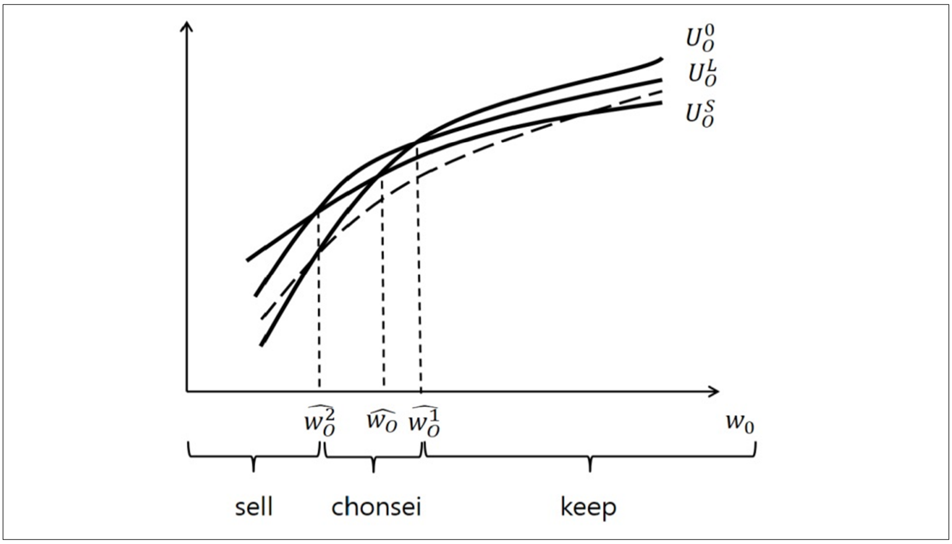
$$W_O > W_O^L > W_O^S.$$

The advantage of selling or leasing a house is to secure more liquidity in period 0. Because securing liquidity is more beneficial if w_0 is lower, then threshold income levels $\widehat{w_O^1}(p_r)$ and $\widehat{w_O^2}(p_0, p_r)$ exist, such that keeping the house is better than leasing it if $w_0 > \widehat{w_O^1}(p_r)$ and leasing the house is better than selling it if $w_0 > \widehat{w_O^2}(p_0, p_r)$.

If there is to be any supply in the chonsei market, $\widehat{w_O^1}(p_r)$ should be greater than $\widehat{w_O^2}(p_0, p_r)$ so that house owners in the income interval $[\widehat{w_O^2}(p_0, p_r), \widehat{w_O^1}(p_r)]$ choose to lease their houses.

This is illustrated in Fig. 7. Note that U_O^L, a house owner's utility when leasing a house through chonsei, increases with chonsei deposit p_r. Thus, if p_r is too low, there will be no supply of chonsei leases, as represented by the dashed line for U_O^L. If there is to be any supply of chonsei leases, the chonsei deposit p_r should be sufficiently high so that U_O^L is represented by the solid line in the figure. Period 0 income is then divided into three intervals, with house owners selling their houses if

[Figure 7] House owners' decisions - supply of chonsei leases when p_r is sufficiently high



w_0 is low, leasing them if w_0 is in the middle range, and keeping them if w_0 is high.

Lemma 5 *Let* $\underline{p_r}(p_0^I)$ *be defined in*

$$U_O^L(\widehat{w_O}(p_0^I), \underline{p_r}(p_0^I)) = U_O^0(\widehat{w_O}(p_0^I)) = U_O^S(\widehat{w_O}(p_0^I), p_0^I) .$$

If $p_r > \underline{p_r}(p_0^I)$*, there is a supply of chonsei leases because* $\widehat{w_O^1}(p_r) > \widehat{w_O^2}(\underline{p_0}, p_r)$*. A house owner sells their house if* $w_0 < \widehat{w_O^2}(p_0, p_r)$*, leases it using chonsei if* $\widehat{w_O^2}(p_0, p_r) \le w_0 < \widehat{w_O^1}(p_r)$*, and keeps it if* $w_0 \ge \widehat{w_O^1}(p_r)$*.*

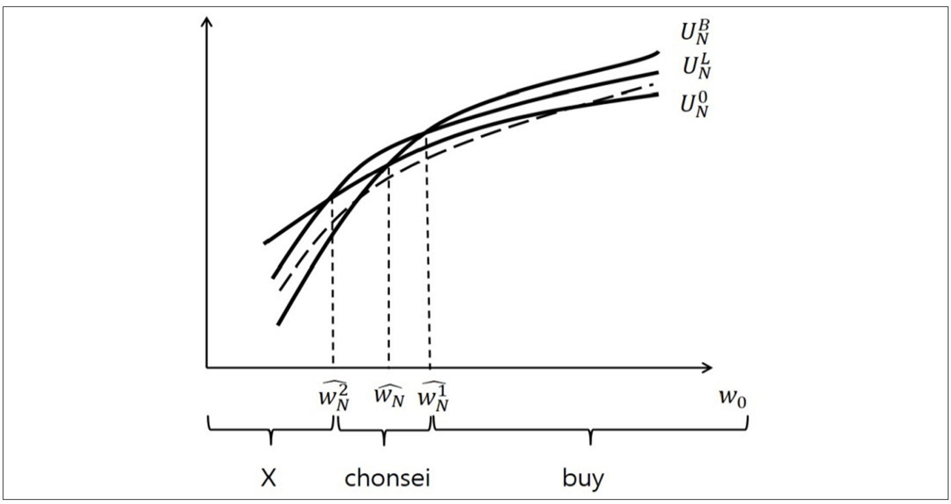
In terms of wealth, keeping the house is the best option, leasing the house using chonsei is the next best, and selling the house is the worst option. However, this order is reversed in terms of liquidity in period 0. Therefore, the most liquidity-thirsty house owners, who have a low income in period 0, will sell their house. The second most liquidity-thirsty house owners whose income is in the middle will lease them using chonsei.

The supply for the housing market and the chonsei market is $sF(\widehat{w_O^2}(p_0, p_r))$ and $s[F(\widehat{w_O^1}(p_r)) - F(\widehat{w_O^2}(p_0, p_r))]$, respectively. The law of supply holds in both markets and that the two are substitutes (i.e. an increase in the price in one market decreases the supply in the other market).

A similar argument can be applied to non-owners' choices. For non-owners, in terms of wealth, buying a house is the best option, renting a house using chonsei is the next best, and doing nothing is the worst option. In terms of liquidity, this order is reversed. Thus, liquidity-richest non-owners whose period 0 income is high will buy a house, and the second liquidity-richest non-owners whose income is in the middle will rent a house using chonsei, as is illustrated in Fig. 8.

The demand for chonsei leases may not exist if the chonsei deposit is too high. If p_r is too high, a non-owner's utility when renting a house using chonsei U_N^L may be represented by the dashed line in Fig. 8, and renting a house using chonsei can never be the optimal option. Thus, p_r should be sufficiently low so that U_N^L is represented by the solid line in the figure. We thus have two threshold income levels which determine the non-house owners' decision.

[Figure 8] Non-owners' decisions - demand for chonsei leases when p_r is sufficiently low



Lemma 6 *Let* $\overline{p_r}(p_0^I)$ *be defined as in*

$$U_N^L(\widehat{w_N}(p_0^I), \overline{p_r}(p_0^I)) = U_N^0(\widehat{w_N}(p_0^I)) = U_N^B(\widehat{w_N}(p_0^I), p_0^I)\,.$$

If $p_r < \overline{p_r}(p_0^I)$, *two threshold income levels* $\widehat{w_N^2}(p_r)$ *and* $\widehat{w_N^1}(p_0, p_r)$ *exist, such that a non-owner is inactive if* $w_0 < \widehat{w_N^2}(p_r)$, *rents a house using chonsei if* $\widehat{w_N^2}(p_r) \le w_0 < \widehat{w_N^1}(p_0, p_r)$, *and buys a house if* $w_0 \ge \widehat{w_N^1}(p_0, p_r)$.

Proof. In the appendix. ■

The demand in the housing market and the chonsei market is $(1-s)[1-F(\widehat{w_N^1}(p_0, p_r))]$ and $(1-s)[F(\widehat{w_N^1}(p_0, p_r)) - F(\widehat{w_N^2}(p_r))]$, respectively. As before, it is easy to confirm that the law of demand holds in both markets and that the two are substitutes.

If we combine Lemmas 5 and 6, we obtain a sufficient condition for the existence of chonsei lease arrangements. If we have $\overline{p_r}(p_0^I) > \underline{p_r}(p_0^I)$, there will be non-zero demand and supply in the chonsei market given house price p_0^I.

Summarizing the above argument, an equilibrium can be described as a pair of prices (p_0^I, p_r^I) satisfying

i) housing market clearing

$$sF(\widehat{w_O^2}(p_0^I, p_r^I)) = (1-s)[1 - F(\widehat{w_N^1}(p_0^I, p_r^I))]$$

ii) chonsei market clearing

$$s[F(\widehat{w_O^1}(p_r^I)) - F(\widehat{w_O^2}(p_0^I, p_r^I))] = (1-s)[F(\widehat{w_N^1}(p_0^I, p_r^I)) - F(\widehat{w_N^2}(p_r^I))].$$

The effect of the chonsei market on house prices is ambiguous. Fig. 7 shows that some landlords would sell their house if a chonsei lease was not available. Thus, chonsei leases will reduce the supply in the housing market. Likewise, some tenants would buy a house, which means the demand in the housing market is also reduced. As a result, we cannot predict the direction of the price change.

The above equilibrium does not always exist because the chonsei lease market in the aforementioned sense may not exist in some environments. However, we can illustrate such equilibrium in some economic environments. A parametric example of an equilibrium is shown below.[12]

Example 1 *Let* $u(c) = \ln c$, $r = 0.1$, $\beta = 0.95$, $w_0 \sim U[0, 100]$, $w_1 = 200$, $H = 10$, $p_1 = 250$, *and* $s = 0.5$. *When the financial market is perfect and borrowing is possible,* $p_0^* = 237.3$ *and* $p_r^* = 100$. *If borrowing is not possible and chonsei leases are not available,* $p_0^I = 154.6$ *with threshold income levels* $\widehat{w_O} = 122.7$ *and* $\widehat{w_N} = 277.3$. *If chonsei leases become available,* $p_0^I = 156.7$ *and* $p_r^I = 25.0$ *with threshold income levels* $\widehat{w_O^2} = 107.3$, $\widehat{w_O^1} = 258.7$, $\widehat{w_N^2} = 141.3$, *and* $\widehat{w_N^1} = 292.7$. *In this example, house prices are quite similar with or without chonsei leases.*

We can apply the same comparative static analysis when chonsei leases are also

[12] A sufficient condition for the existence of chonsei market, $\overline{p_r}(p_0^I) > \underline{p_r}(p_0^I)$, is defined implicitly by model parameters. However, it would be desirable if we can provide a sufficient condition in an explicit and interpretable form of the model parameters. Unfortunately, it is difficult to do that because $\overline{p_r}(p_0^I)$ and $\underline{p_r}(p_0^I)$ are also affected by the utility function $u(c)$ and distribution function $F(w)$.

available. The results are summarized in the following proposition.

Proposition 4 *Equilibrium prices* p_0^I *and* p_r^I *are affected by the parameters of the model as follows.*
i) If s *increases, both* p_0^I *and* p_r^I *decrease.*
ii) If p_1 *increases, the effect on* p_0^I *is not certain while* p_r^I *decreases.*
iii) If r *increases, both* p_0^I *and* p_r^I *weakly decrease.*
iv) If w_1 *increases, both* p_0^I *and* p_r^I *decrease.*
v) If income follows a distribution function G *that is first-order stochastically dominated by* F*, both* p_0^I *and* p_r^I *decrease.*

Proof. In the appendix. ■

If houses are more abundant, their price and the chonsei deposit will decrease. The housing market and the chonsei lease market (house price and chonsei deposit) interact with each other. Specifically, the change in excess demand due to a price change in the housing market is offset by the change in excess demand in the chonsei lease market. It turns out that the change in house prices and chonsei deposits is more dependent on the total supply and demand in the housing market and chonsei lease market combined, rather than their compositions. For example, if income in period 0 decreases as in v), for consumption smoothing purposes, more house owners are willing to lease or sell their house while fewer non-owners are willing to rent or buy. Thus, house prices and chonsei deposits will fall. The same logic applies if period 1 income w_1 increases in iv). If the interest rate increases in iii), the incentive for saving is usually determined by the relative size of the income and the substitution effect. However, a house owner at the margin who is indifferent between selling or leasing a house, for example, is credit-constrained when leasing a house. If this house owner sells their house instead, they may be able to enjoy the benefits of a higher interest rate from saving. Therefore, more house owners want to lease or sell their houses while fewer non-owners want to rent or buy. As a result, house prices and chonsei deposits will fall. If the future price of houses increases in ii), house owners have more incentive to smooth consumption either by leasing or selling, thereby creating a downward pressure on house prices and chonsei deposits. Nevertheless, to enjoy price appreciation, house owners are more likely to lease the house than sell. More non-owners want to buy rather than rent it. Thus, the chonsei deposit will fall but the effect on house prices is ambiguous. The change in the composition of supply (or demand) in two markets is likely to increase house prices while the fall in the chonsei deposit may weaken the price increase.

IV. Discussion

The explanation of chonsei provided by this paper differs from that by existing literature in that chonsei is explained completely within the housing market itself. Rather than simply assuming that chonsei is the dominant form of lease in the market and analyzing its deposit level (as in Cho, 2005; Kim et al., 1998; Lee, 2013; Son et al., 2011 among others), our model attempts to explain the prevalence of the chonsei lease itself. The model does not assume an excess return for other investment opportunities as in Kim and Shin (2011). In addition, the excess return for housing assets is not superimposed as in Kim (2013) but endogenously explained in the model.

The key factor affecting the presence of chonsei in our model is credit constraints. Credit constraints play two key roles in facilitating the establishment of the chonsei market. First, it causes house prices to be undervalued and creates an excess return for housing assets, which can be shared through a chonsei contract. Second, chonsei leases as a lending contract through housing service are possible because of credit constraints. Landlords can achieve consumption smoothing using the up-front deposit and are willing to pay high interest. Thus, some of the excess returns will go to the tenants via a low chonsei deposit. A limited credit supply for housing in Korea was presumed until the 1990s. Housing finance was virtually monopolized until 1990 by the Housing and Commercial Bank, a public bank established in the late 1960s. Even after deregulation, financial institutions were not particularly interested in lending to households because of the high demand from the corporate sectors.[13] It was only after the Asian financial crisis in 1997 and the ensuing deregulation that private financial institutions began to provide housing finance for households. Thus, the credit constraints assumed in the model would have been relevant until at least 1997.

According to the model, the excess return on housing assets is a precondition of chonsei lease arrangements, which is related to the expectation of house price inflation. In most literature, the chonsei lease is taken as an established arrangement and the focus is on the relationship between price increase expectations and the chonsei deposit to house price ratio. However, these arguments implicitly assume a perfect financial market in which chonsei leases in their current form (that is, with a low deposit) do not exist according to our model. Rather, the monthly rent to price ratio can be used as the same measure even when chonsei leases are not available. If we take the model seriously, the relationship between the ratio of the chonsei deposit to the house price and the house price increase is not straightforward. For example, if future income w_1 increases in the model, greater expectations of an

[13] In 1990, credit from financial institutions accounted for only 40% of total housing finance (Korean Housing Finance Corporation 2016).

increase in house prices would arise as the current house price p_0^I decreases. However, the ratio $\frac{p_r^I}{p_0^I}$ is not guaranteed to decrease because p_0^I and p_r^I decrease at the same time.[14] The proportion of chonsei leases may be related to the extent of credit constraints. The recent fall in popularity of chonsei lease arrangements may be related to the development of mortgage lending. This view is more formally argued in the extension of the model where the extent of credit constraints can be modified. In the next section, it is shown that relaxed credit constraints will reduce the trade of chonsei leases and may eventually eliminate the space for chonsei leases.

Though my model deals only with housing assets, the possibility of excess return for an asset may not be restricted to housing assets. Note that a housing asset in our model can be replaced with any asset. Any asset or investment opportunity which requires sizable funding may suffer from credit constraints. Therefore, these asset classes may have low valuation considering their fundamentals and have excess returns. If the credit constraints are proportional to the funding size, we may observe a higher return for an asset with a higher investment size. This possibility is left as a subject for future investigation.

V. Extension: Different Types of Credit Constraint

Many types of financial market imperfection can be observed. Our model assumes one extreme form in which no borrowing exists. In this subsection, we relax this assumption. We will consider two types of credit constraints. First, borrowing is possible but limited to a certain proportion of house price, which is called the loan-to-value (LTV) ratio credit constraint. Second, borrowing is limited to a certain multiple of current income, which is called the debt-to-income (DTI) ratio credit constraint.

5.0.3. Loan-to-value Ratio Credit Constraint (LTV)

Under the LTV credit constraint, lending requires a housing asset as collateral and only a certain proportion of the house price can be borrowed. Let ν be the proportion of the house price p_0 for which a loan can be offered. The possibility of borrowing does not affect the present value of wealth but affects the liquidity in period 0 of those who currently own a house. That is, house owners who keep it or non-owners who newly purchase a house can increase their liquidity in period 0 by up to νp_0.[15] As before, the chonsei deposit cannot be too high or too low, $\frac{1+r}{r} p_0 -$

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[14] This is a simplification of the model. Because p_1 is also determined in the future house market, change in w_1 would affect p_1 as well.

[15] For simplicity, we assume that owners leasing their houses through chonsei cannot borrow using

$\frac{1}{r}p_1 < p_r < \frac{1+r}{r}H$, which guarantees the order of wealth level $W_O > W_O^L > W_O^S$ and $W_N^B > W_N^L > W_N^0$. In addition, the proportion ν cannot be too high. If it is too high, house owners would keep their house and borrow rather than lease them and non-owners would rather buy a house and borrow than rent a house. That is, $p_r > H + \nu p_0$ and $(1-\nu)p_0 > p_r$, or $\nu < \min[\frac{p_r - H}{p_0}, 1 - \frac{p_r}{p_0}]$.

Compared with the main analysis without borrowing, the LTV credit constraint will increase the utility level of keeping a house for house owners, U_O^0, and that of buying a house for non-owners, U_N^B, as follows:

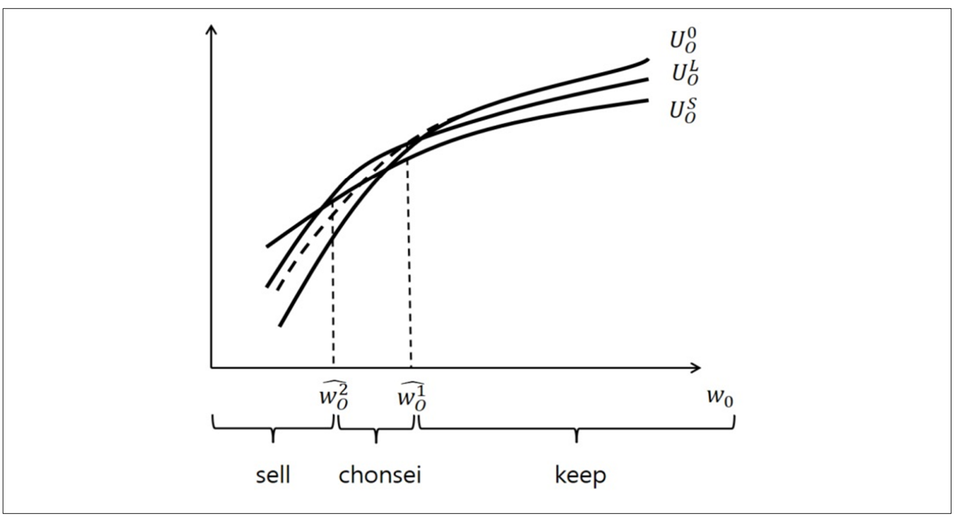
$$U_O^0(w_0, p_r) = \begin{cases} u(c_0^O) + \beta u(c_1^O) \text{ if } w_0 + H + \nu p_0 \geq c_0^O \\ u(w_0 + H + \nu p_0) + \beta u(w_1 + p_1 - (1+r)\nu p_0) \text{ if } w_0 + H + \nu p_0 < c_0^O \end{cases}$$

$$U_N^B(w_0, p_r) = \begin{cases} u(c_0^{NB}) + \beta u(c_1^{NB}) \text{ if } w_0 - (1-\nu)p_0 + H \geq c_0^{NB} \\ u(w_0 - (1-\nu)p_0 + H) + \beta u(w_1 + p_1 - (1+r)\nu p_0) \text{ if } w_0 - (1-\nu)p_0 + H < c_0^{NB} \end{cases}$$

Except for these changes, the decisions of house owners and non-owners are the same as before. Figure 9 shows the change in a house owner's decision with the LTV credit constraint. Only the utility of keeping the house U_O^0 changes to the dashed line because owners can secure more liquidity in period 0.

We note the relaxed credit constraint will reduce the trade in chonsei leases and may eventually eliminate them. As ν increases, more house owners will keep their house rather than leasing it through chonsei (or decreasing $\widehat{w_O^1}$) and more non-owners will buy a house rather than renting it through chonsei (or decreasing $\widehat{w_N^1}$).

[Figure 9] House Owners' Decision with LTV



their house as collateral.

Thus, the proportion of chonsei leases will reduce. In other words, as v increases, the existence of a chonsei lease market is less likely. Recall the condition for the existence of a chonsei lease market, $\overline{p_r}(p_0) > \underline{p_r}(p_0)$. Given the current house price p_0, higher v increases $\underline{p_r}(p_0)$ while $\overline{p_r}(p_0)$ decreases. With relaxed credit constraints, house owners can borrow on their house rather than lease it for a low deposit. Thus, they would ask for a higher chonsei deposit. Non-owners may buy a house and borrow on it rather than rent it for a high deposit. Thus, they would ask for a lower deposit. This means chonsei leases may not be sustainable

If a chonsei lease market exists, a similar equilibrium condition as before exists. The threshold income level $\widehat{w_N^1}(p_0^I, p_r^I)$ and $\widehat{w_O^1}(p_0^I, p_r^I)$ would change, but we abuse the notation. We can write the equilibrium condition as

$$sF(\widehat{w_O^2}(p_0^I, p_r^I)) = (1-s)[1 - F(\widehat{w_N^1}(p_0^I, p_r^I))]$$
$$s[F(\widehat{w_O^1}(p_0^I, p_r^I)) - F(\widehat{w_O^2}(p_0^I, p_r^I))] = (1-s)[F(\widehat{w_N^1}(p_0^I, p_r^I)) - F(\widehat{w_N^2}(p_r^I))]$$

A notable difference is that the owner's threshold income level $\widehat{w_O^1}$ is also dependent on house price p_0. If the house price increases, a house owner who keeps their house can borrow more and secure more liquidity in period 0. Thus, house owners may choose to keep their houses rather than lease them, or $\widehat{w_O^1}$ decreases in Fig. 9. This difference leads to a slight change in the comparative statics results, which are summarized in the following proposition.

Proposition 5 *Equilibrium prices* p_0^I *and* p_r^I *are affected by the parameters of the model as follows when equilibrium prices are stable.*
i) If s *increases, both* p_0^I *and* p_r^I *decrease.*
ii) If p_1 *increases, the effect on* p_0^I *and* p_r^I *is not certain.*
iii) If r *increases, both* p_0^I *and* p_r^I *decrease.*
iv) If w_1 *increases, both* p_0^I *and* p_r^I *decrease.*
v) If income follows a distribution function G *that is first-order stochastically dominated by* F*, both* p_0^I *and* p_r^I *decrease.*
vi) If v *increases, both* p_0^I *and* p_r^I *increase.*

Proof. See appendix. ■

As shown in the proof, the equilibrium may not be stable with the LTV credit constraint. Compared with the main analysis, the change in house price p_0 has a more sizable effect on the chonsei lease market. Without borrowing, the rise in p_0 causes an increase in the excess demand in the chonsei lease market only because some lessors decide to sell their house instead. With the LTV constraint, the excess demand for chonsei leases also increases because some house owners may borrow more money on the increasing collateral value rather than lease their house. This increase in excess demand in the chonsei market may also cause p_r to increase too much, which can also increase the excess demand in the housing market. Combining this indirect effect, a rise in p_0 may not decrease the excess demand in the housing market. Therefore, the equilibrium price may not be stable. As long as the equilibrium is stable, most qualitative results carry over from Proposition 4 except for ii). The intuition behind the result is virtually the same.

For ii), the main model dictates that an increase in future house price p_1 decreases chonsei deposit p_r^I, but its effect on p_r^I becomes ambiguous in this extension. The increase in p_1 has two countervailing effects. House owners have a greater incentive for consumption smoothing, but they also want to keep the house to enjoy the future price. Thus, they will want to lease their house instead and the chonsei deposit decreases. However, if house prices increase, house owners can borrow more, thereby decreasing the supply in the chonsei lease market. Therefore, the effect on chonsei deposits becomes ambiguous.

We have an additional comparative statics result with a change in ν. If the LTV ratio ν increases, more house owners want to keep their house rather than leasing it while more non-owners want to buy a house. Thus, house prices and chonsei deposits will increase. It is usually argued that LTV credit constraint may increase chonsei deposits because housing demand is replaced with chonsei demand. However, according to our model, tightening the LTV constraint will also increase the chonsei supply and decrease chonsei deposits.

5.0.4. Debt-to-income Ratio Credit Constraint (DTI)

Under the DTI credit constraint, debt is restricted so that debt servicing (interest and principal payments) relative to income is limited. In our model, this simply means a constant proportion of period 0 income can be borrowed. Let μ be that constant portion. Whether an agent has a house or not, they can borrow up to μw_0, which will weakly increase the utility levels of all choices. For example, U_N^0, the utility for non-owners who neither rent nor buy a house, changes to

$$U_N^0(w_0) = \begin{cases} u(c_0^N) + \beta u(c_1^N) \text{ if } (1+\mu)w_0 \geq c_0^N \\ u((1+\mu)w_0) + \beta u(w_1 - (1+r)\mu w_0) \text{ if } (1+\mu)w_0 < c_0^N \end{cases},$$

which is weakly greater than the utility level in the main analysis.

As with the LTV constraint, relaxed credit constraint will cause the trade of chonsei leases less likely to occur because a higher μ increases $\underline{p_r}(p_0)$ while decreasing $\overline{p_r}(p_0)$. House owners can borrow to secure liquidity in period 0 and do not lease their house if the chonsei deposit is not sufficiently high. Non-owners can buy a house and secure more liquidity by borrowing, and thus would ask for a lower deposit to rent a house.

The structure of the decision is the same and the equilibrium condition is similar to the main analysis. The results of the comparative static analysis are also similar except for the effect of interest rate in iii).

Proposition 6 *Equilibrium prices* p_0^I *and* p_r^I *are affected by the parameters of the model as follows.*
i) If s *increases, both* p_0^I *and* p_r^I *decrease.*
ii) If p_1 *increases, the effect on* p_0^I *is not certain while* p_r^I *decreases.*
iii) If r *increases, the effect on* p_0^I *and* p_r^I *is not certain.*
iv) If w_1 *increases, both* p_0^I *and* p_r^I *decrease.*
v) If income follows a distribution function G *that is first-order stochastically dominated by* F*, both* p_0^I *and* p_r^I *decrease.*
vi) If μ *increases, both* p_0^I *and* p_r^I *increase.*

Proof. See appendix. ■

The effect of an increase in the interest rate on house prices and chonsei deposits becomes ambiguous in this extension, while it weakly decreases house prices and chonsei deposits in the main model. Consider a house owner who is indifferent between selling and leasing their house. If this house owner leases their house rather than sells it, they can still be indebted and will suffer from the negative effects of an interest rate increase. We cannot exclude the possibility that the decrease in utility when selling their house is greater than that when leasing it for this marginal owner. Marginal house owners may want to lease rather than sell their house. By the same token, they may want to keep their house rather than lease it. Thus, both house prices and chonsei deposits may increase.

With a relaxed credit constraint, house owners are less likely to sell or lease their house, while non-owners are more likely to buy or rent their house. Thus, both house prices and chonsei deposits will increase.

VI. Conclusion

This paper explains the existence of chonsei leases without pre-imposing outside investment opportunities. With credit constraints, the model generates an undervaluation of house prices because of the illiquidity penalty. Thus, purchasing a house becomes a better saving apparatus with a higher return. Some agents would want to borrow to purchase a house to exploit the excess return. Chonsei leases will provide a borrowing opportunity but with a higher interest rate. The excess return from a house purchase will be shared with a lender (or a renter of a house).

Existing literature has implicitly assumed a perfect financial market to understand the relationship between house prices and chonsei deposits. However, this assumption may not be able to explain the existence of chonsei leases. This paper provides a theoretical framework for the chonsei market by assuming an imperfect financial market. It has different predictions of the behavior of house prices and chonsei deposits, but a quantitative evaluation of these predictions may require incorporating this paper's idea into a general equilibrium model. This step remains as a future research agenda.

Appendix

Proof of Lemma 3 If a non-owner does not buy a house, their present value of wealth is W_N in (2). If their period 0 income exceeds period 0 optimal consumption, optimal consumption will be chosen. Otherwise, all available income will be consumed in each period. If a non-owner buys a house, their present value of wealth W_N^B is

$$W_N^B = w_0 - p_0 + H + \frac{w_1 + p_1}{1+r}, \tag{12}$$

and optimal consumption (c_0^{NB}, c_1^{NB}) is determined by (1) and budget constraints with the wealth of (12). If their period 0 income $w_0 - p_0 + H$ exceeds c_0^{NB}, optimal consumption will be chosen. Otherwise, all available income will be consumed in each period. Let U_N^0 and U_N^B be their maximized utility with and without the purchase of a house, respectively. They are expressed as follows.

$$U_N^0(w_0) = \begin{cases} u(c_0^N) + \beta u(c_1^N) \text{ if } w_0 \geq c_0^N \\ u(w_0) + \beta u(w_1) \text{ if } w_0 < c_0^N \end{cases}$$

$$U_N^B(w_0, p_0) = \begin{cases} u(c_0^{NB}) + \beta u(c_1^{NB}) \text{ if } w_0 - p_0 + H \geq c_0^{NB} \\ u(w_0 - p_0 + H) + \beta u(w_1 + p_1) \text{ if } w_0 - p_0 + H < c_0^{NB} \end{cases}$$

Non-owners make a purchase decision by comparing U_N^0 and U_N^B. Consider a period 0 income level w_0'' such that $w_0'' - p_0 + H = c_0^N \leq c_0^{NB}$. If $w_0 > w_0''$, buying a house is preferable, $U_N^B(w_0, p_0) > U_N^0(w_0)$. The disadvantage of buying a house is the loss of consumption in period 0, and this disadvantage disappears if period 0 income is sufficiently high. Moreover, $\frac{d}{dw_0} U_N^B > \frac{d}{dw_0} U_N^0$ if $w_0 < w_0''$ because a credit-constrained consumer will gain more benefit from an increase in their current income. Thus, a unique threshold income $\widehat{w_N}(p_0)$ exists, such that a non-owner is willing to buy a house if $w_0 \geq \widehat{w_N}(p_0)$ as in Fig. 5.

Proof of Proposition 3 i) is trivial. A change in the distribution function will affect supply and demand without any change in threshold income, and v) follows. An increase in p_1 increases U_O^0 and U_N^B, thereby increasing the supply and decreasing the demand in the housing market and thus iii) follows.

At $\widehat{w_O}(p_0)$, we have either

$$u(\widehat{w_O}(p_0) + H) + \beta u(w_1 + p_1) = u(c_0^{OS}) + \beta u(c_1^{OS})$$

or

$$u(\widehat{w_O}(p_0)+H)+\beta u(w_1+p_1)=u(\widehat{w_O}(p_0)+p_0)+\beta u(w_1).$$

An increase in r does not change $\widehat{w_O}(p_0)$ in the second case but increases $\widehat{w_O}(p_0)$ in the first case. Agents are net savers when the consumption path (c_0^{OS}, c_1^{OS}) is chosen and an interest rate increase means an expansion of the budget set. Thus, an increase in r increases supply weakly. An increase in w_1 will increase $\widehat{w_O}(p_0)$ because the RHS increases more than the LHS in both cases because w_1+p_1 is greater than c_1^{OS} or w_1. Thus, an increase in w_1 increases the supply.

Similarly, at $\widehat{w_N}(p_0)$, we have either

$$u(\widehat{w_N}(p_0)-p_0+H)+\beta u(w_1+p_1)=u(c_0^N)+\beta u(c_1^N)$$

or

$$u(\widehat{w_N}(p_0)-p_0+H)+\beta u(w_1+p_1)=u(\widehat{w_N}(p_0))+\beta u(w_1).$$

An increase in r does not change $\widehat{w_N}(p_0)$ in the second case but increases $\widehat{w_N}(p_0)$ in the first case. Thus, an increase in r weakly decreases demand. An increase in w_1 will increased $\widehat{w_N}(p_0)$ because the RHS increases more than the LHS in both cases because w_1+p_1 is greater than c_1^N or w_1. Thus, an increase in w_1 decreases demand.

Combining both changes, p_0^I decreases if r increases or w_1 increases.

Proof of Lemma 6 A non-owner can now rent a house through chonsei, and their present value of wealth will be

$$W_N^L = w_0 - p_r + H + \frac{w_1+p_r}{1+r}. \tag{13}$$

Let (c_0^{NL}, c_1^{NL}) be the optimal consumption determined by (1) and budget constraints with the wealth level (13). A non-owners' utility when renting a house through chonsei is defined by comparing their liquidity w_0-p_r+H and optimal consumption c_0^{NL} in period 0. Thus, their utility when renting a house through chonsei, U_N^L, is

$$U_N^L(w_0,p_r)=\begin{cases} u(c_0^{NL})+\beta u(c_1^{NL}) \text{ if } w_0-p_r+H\geq c_0^{NL} \\ u(w_0-p_r+H)+\beta u(w_1+p_r) \text{ if } w_0-p_r+H< c_0^{NL} \end{cases}.$$

If the necessary condition in Lemma 4 holds, their wealth level is higher in the order of buying, renting, and not buying,

$$W_N^B > W_N^L > W_N^0 .$$

An advantage of not buying or renting is that more liquidity can be secured in period 0. Consider an income level w_0'''', such that $w_0'''' - p_0 + H = c_0^{NL}$. If $w_0 \geq w_0''''$, buying is preferable to renting. Moreover, $\frac{d}{dw_0} U_N^B(w_0, p_0) > \frac{d}{dw_0} U_N^L(w_0, p_r)$ if $w_0 < w_0''''$, as credit-constrained consumers can benefit more from an increase in their current income. A threshold income level $\widehat{w_N^1}(p_0, p_r)$ exists, such that buying is better than renting if $w_0 > \widehat{w_N^1}(p_0, p_r)$. Similarly, a threshold income $\widehat{w_N^2}(p_r)$ also exists such that renting is better than not buying if $w_0 > \widehat{w_N^2}(p_r)$.

If there is to be any demand for chonsei, $\widehat{w_N^1}(p_0, p_r)$ should be greater than $\widehat{w_N^2}(p_r)$ so that house owners in the income interval $[\widehat{w_N^2}(p_r), \widehat{w_N^1}(p_0, p_r)]$ choose to rent a house through chonsei as illustrated in Figure 8. Recall threshold income level $\widehat{w_N}(p_0^I)$ where $U_N^B = U_N^0$. As U_N^L is decreasing in p_r, p_r should be lower than $\overline{p_r}(p_0^I)$ defined in the Lemma so that $U_N^L > U_B^B = U_N^0$ at $\widehat{w_N}(p_0^I)$ if demand for chonsei leases is to exist. The period 0 income level is thus divided into three intervals as is illustrated in Fig. 8.

Proof of Proposition 4 The results for comparative statics are obtained from equilibrium conditions through the implicit function theorem. Let two equilibrium conditions be written as the excess demand for the two markets is equal to 0,

$$\Psi_1 = (1-s)[1 - F(\widehat{w_N^1}(p_0^I, p_r^I))] - sF(\widehat{w_O^2}(p_0^I, p_r^I)) = 0$$
$$\Psi_1 = (1-s)[F(\widehat{w_N^1}(p_0^I, p_r^I)) - F(\widehat{w_N^2}(p_r^I))] - s[F(\widehat{w_O^1}(p_r^I)) - F(\widehat{w_O^2}(p_0^I, p_r^I))] = 0 .$$

Given any parameter δ, we have

$$\frac{\partial \Psi_1}{\partial p_0^I}\frac{\partial p_0^I}{\partial \delta} + \frac{\partial \Psi_1}{\partial p_r^I}\frac{\partial p_r^I}{\partial \delta} = -\frac{\partial \Psi_1}{\partial \delta}$$
$$\frac{\partial \Psi_2}{\partial p_0^I}\frac{\partial p_0^I}{\partial \delta} + \frac{\partial \Psi_2}{\partial p_r^I}\frac{\partial p_r^I}{\partial \delta} = -\frac{\partial \Psi_2}{\partial \delta}$$

or

$$\begin{bmatrix} \dfrac{\partial \Psi_1}{\partial p_0^I} & \dfrac{\partial \Psi_1}{\partial p_r^I} \\ \dfrac{\partial \Psi_2}{\partial p_0^I} & \dfrac{\partial \Psi_2}{\partial p_r^I} \end{bmatrix} \begin{bmatrix} \dfrac{\partial p_0^I}{\partial \delta} \\ \dfrac{\partial p_r^I}{\partial \delta} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial \Psi_1}{\partial \delta} \\ -\dfrac{\partial \Psi_2}{\partial \delta} \end{bmatrix}.$$

Using Cramer's rule, we have

$$\frac{\partial p_0^I}{\partial \delta} = \frac{\begin{vmatrix} -\dfrac{\partial \Psi_1}{\partial \delta} & \dfrac{\partial \Psi_1}{\partial p_r^I} \\ -\dfrac{\partial \Psi_2}{\partial \delta} & \dfrac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}}{\begin{vmatrix} \dfrac{\partial \Psi_1}{\partial p_0^I} & \dfrac{\partial \Psi_1}{\partial p_r^I} \\ \dfrac{\partial \Psi_2}{\partial p_0^I} & \dfrac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}}, \quad \frac{\partial p_r^I}{\partial \delta} = \frac{\begin{vmatrix} \dfrac{\partial \Psi_1}{\partial p_0^I} & -\dfrac{\partial \Psi_1}{\partial \delta} \\ \dfrac{\partial \Psi_2}{\partial p_0^I} & -\dfrac{\partial \Psi_2}{\partial \delta} \end{vmatrix}}{\begin{vmatrix} \dfrac{\partial \Psi_1}{\partial p_0^I} & \dfrac{\partial \Psi_1}{\partial p_r^I} \\ \dfrac{\partial \Psi_2}{\partial p_0^I} & \dfrac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}}.$$

Because the law of supply and demand holds in each market and sales and leases of houses are substitutes (as $\frac{\partial \widehat{w_O^2}}{\partial p_0^I}$, $\frac{\partial \widehat{w_N^1}}{\partial p_0^I}$, $\frac{\partial \widehat{w_O^1}}{\partial p_r^I}$, $\frac{\partial \widehat{w_N^2}}{\partial p_r^I} > 0$ and $\frac{\partial \widehat{w_O^2}}{\partial p_r^I}$, $\frac{\partial \widehat{w_N^1}}{\partial p_r^I} < 0$), we have

$$\frac{\partial \Psi_1}{\partial p_0^I} = -\left\{ (1-s) f(\widehat{w_N^1}) \frac{\partial \widehat{w_N^1}}{\partial p_0^I} + s f(\widehat{w_O^2}) \frac{\partial \widehat{w_O^2}}{\partial p_0^I} \right\} < 0$$

$$\frac{\partial \Psi_2}{\partial p_0^I} = -\frac{\partial \Psi_1}{\partial p_0^I} > 0$$

$$\frac{\partial \Psi_1}{\partial p_r^I} = -\left\{ (1-s) f(\widehat{w_N^1}) \frac{\partial \widehat{w_N^1}}{\partial p_r^I} + s f(\widehat{w_O^2}) \frac{\partial \widehat{w_O^2}}{\partial p_r^I} \right\} > 0$$

$$\frac{\partial \Psi_2}{\partial p_r^I} = -\left\{ (1-s) f(\widehat{w_N^2}) \frac{\partial \widehat{w_N^2}}{\partial p_r^I} + s f(\widehat{w_O^1}) \frac{\partial \widehat{w_O^1}}{\partial p_r^I} \right\} - \frac{\partial w_0^1}{\partial p_r^I} < 0 .$$

Moreover, if we postulate that price increases (or decreases) when excess demand is positive (or negative), then equilibrium prices are stable because the derivative of excess demand is negative definite. That is, $\frac{\partial \Psi_1}{\partial p_0^I} < 0$ and

$$\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & \frac{\partial \Psi_1}{\partial p_r^I} \\ \frac{\partial \Psi_2}{\partial p_0^I} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_0^I}\left[-\left\{ (1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I} \right\} - \frac{\partial \Psi_1}{\partial p_r^I} \right]$$

$$+\frac{\partial \Psi_1}{\partial p_0^I}\frac{\partial \Psi_1}{\partial p_r^I} = -\frac{\partial \Psi_1}{\partial p_0^I}\left\{ (1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I} \right\} > 0 .$$

Thus, the comparative statics are determined by the sign of $\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \delta} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \delta} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}$ and $\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \delta} \\ -\frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \delta} \end{vmatrix}$.

For i), as

$$\frac{\partial \Psi_1}{\partial s} = -\{ [1-F(\widehat{w_N^1})] + F(\widehat{w_O^2}) \} < 0$$

$$\frac{\partial \Psi_2}{\partial s} = -[F(\widehat{w_N^1}) - F(\widehat{w_N^2})] - [F(\widehat{w_O^1}) - F(\widehat{w_O^2})] < 0 ,$$

we have $\frac{\partial p_0^I}{\partial s}$, $\frac{\partial p_r^I}{\partial s} < 0$.

For v), let us assume another parameter ε such that $\frac{\partial F}{\partial \varepsilon} > 0$. The distribution with a higher ε is first-order stochastically dominated by the one with a lower ε. Because

$$\frac{\partial \Psi_1}{\partial \varepsilon} = -(1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^1}) - s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^2}) < 0$$

$$\frac{\partial \Psi_2}{\partial \varepsilon} = -\frac{\partial \Psi_1}{\partial \varepsilon} - (1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^2}) - s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^1}) ,$$

we have

$$\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \varepsilon} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \varepsilon} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial \varepsilon}\left\{ (1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I} \right\}$$

$$-\frac{\partial \Psi_1}{\partial p_r^I}\left\{ (1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^2}) + s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^1}) \right\} < 0$$

$$\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \varepsilon} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \varepsilon} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^2}) + s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^1})\right\} < 0 .$$

Thus, $\frac{\partial p_0^I}{\partial \varepsilon}$, $\frac{\partial p_r^I}{\partial \varepsilon} < 0$.

For other comparative statics, we first need to consider the change in the threshold income levels, $\widehat{w_O^1}$, $\widehat{w_O^2}$, $\widehat{w_N^1}$, and $\widehat{w_N^2}$. At $\widehat{w_O^1}$, for example, we have either

$$u(\widehat{w_O^1} + H) + \beta u(w_1 + p_1) = u(c_0^{OL}) + \beta u(c_1^{OL})$$

or

$$u(\widehat{w_O^1} + H) + \beta u(w_1 + p_1) = u(\widehat{w_O^1} + p_r) + \beta u(w_1 + p_1 - p_r) .$$

When p_1 increases, the RHS increases more than the LHS because $w_1 + p_1 > c_1^{OL}$ (or $w_1 + p_1 - p_r$). Thus, $\frac{\partial \widehat{w_O^1}}{\partial p_1} > 0$. When r increases, the RHS in the first case increases because agents are net savers when (c_0^{OL}, c_1^{OL}) is chosen. In the second case, it does not affect either the LHS or the RHS. Thus, $\frac{\partial \widehat{w_O^1}}{\partial r} \ge 0$. When w_1 increases, the RHS increases more than the LHS for the same reason as above, $\frac{\partial \widehat{w_O^1}}{\partial w_1} > 0$.

We can perform a similar exercise for the other threshold incomes, $\widehat{w_O^2}$, $\widehat{w_N^1}$, and $\widehat{w_N^2}$. For a change in p_1 , we have $\frac{\partial \widehat{w_O^2}}{\partial p_1} < 0$, $\frac{\partial \widehat{w_N^1}}{\partial p_1} < 0$, and $\frac{\partial \widehat{w_N^2}}{\partial p_1} = 0$. For a change in r , we have $\frac{\partial \widehat{w_O^2}}{\partial r} \ge 0$, $\frac{\partial \widehat{w_N^1}}{\partial r} \ge 0$, and $\frac{\partial \widehat{w_N^2}}{\partial r} \ge 0$. For a change in w_1 , $\frac{\partial \widehat{w_O^2}}{\partial w_1} > 0$, $\frac{\partial \widehat{w_N^1}}{\partial w_1} > 0$, and $\frac{\partial \widehat{w_N^2}}{\partial w_1} > 0$.

For ii), because

$$\frac{\partial \Psi_1}{\partial p_1} = -(1-s) f(\widehat{w_N^1}) \frac{\partial \widehat{w_N^1}}{\partial p_1} - sf(\widehat{w_O^2}) \frac{\partial \widehat{w_O^2}}{\partial p_1} > 0$$

$$\frac{\partial \Psi_2}{\partial p_1} = -sf(\widehat{w_O^1}) \frac{\partial \widehat{w_O^1}}{\partial p_1} - \frac{\partial \Psi_1}{\partial p_1} < 0 ,$$

we have

$$\begin{vmatrix} -\dfrac{\partial\Psi_1}{\partial p_1} & \dfrac{\partial\Psi_1}{\partial p_r^I} \\ -\dfrac{\partial\Psi_2}{\partial p_1} & \dfrac{\partial\Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial\Psi_1}{\partial p_1}\left\{ sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_r^I} + (1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial p_r^I}\right\}$$

$$-\frac{\partial\Psi_1}{\partial p_r^I} sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_1} \lesseqgtr 0$$

$$\begin{vmatrix} \dfrac{\partial\Psi_1}{\partial p_0^I} & -\dfrac{\partial\Psi_1}{\partial p_1} \\ \dfrac{\partial\Psi_2}{\partial p_0^I} & -\dfrac{\partial\Psi_2}{\partial p_1} \end{vmatrix} = \frac{\partial\Psi_1}{\partial p_0^I} sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_1} < 0 .$$

For iii), because

$$\frac{\partial\Psi_1}{\partial r} = -(1-s)f(\widehat{w_N^1})\frac{\partial\widehat{w_N^1}}{\partial r} - sf(\widehat{w_O^2})\frac{\partial\widehat{w_O^2}}{\partial r} \le 0$$
$$\frac{\partial\Psi_2}{\partial r} = -(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial r} - sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r} - \frac{\partial\Psi_1}{\partial r} \lesseqgtr 0 ,$$

we have

$$\begin{vmatrix} -\dfrac{\partial\Psi_1}{\partial r} & \dfrac{\partial\Psi_1}{\partial p_r^I} \\ -\dfrac{\partial\Psi_2}{\partial r} & \dfrac{\partial\Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial\Psi_1}{\partial r}\left\{ (1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$-\frac{\partial\Psi_1}{\partial p_r^I}\left\{ (1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial r} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r}\right\} \le 0$$

$$\begin{vmatrix} \dfrac{\partial\Psi_1}{\partial p_0^I} & -\dfrac{\partial\Psi_1}{\partial r} \\ \dfrac{\partial\Psi_2}{\partial p_0^I} & -\dfrac{\partial\Psi_2}{\partial r} \end{vmatrix} = \frac{\partial\Psi_1}{\partial p_0^I}\left\{ (1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial r} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r}\right\} \le 0 .$$

For iv), because

$$\frac{\partial\Psi_1}{\partial w_1} = -(1-s)f(\widehat{w_N^1})\frac{\partial\widehat{w_N^1}}{\partial w_1} - sf(\widehat{w_O^2})\frac{\partial\widehat{w_O^2}}{\partial w_1} < 0$$

$$\frac{\partial\Psi_2}{\partial w_1} = -(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial w_1} - sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial w_1} - \frac{\partial\Psi_1}{\partial w_1} \lesseqgtr 0,$$

we have

$$\begin{vmatrix} -\frac{\partial\Psi_1}{\partial w_1} & \frac{\partial\Psi_1}{\partial p_r^I} \\ -\frac{\partial\Psi_2}{\partial w_1} & \frac{\partial\Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial\Psi_1}{\partial w_1}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$-\frac{\partial\Psi_1}{\partial p_r^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial w_1} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial w_1}\right\} < 0$$

$$\begin{vmatrix} \frac{\partial\Psi_1}{\partial p_0^I} & -\frac{\partial\Psi_1}{\partial w_1} \\ \frac{\partial\Psi_2}{\partial p_0^I} & -\frac{\partial\Psi_2}{\partial w_1} \end{vmatrix} = \frac{\partial\Psi_1}{\partial p_0^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial w_1} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial w_1}\right\} < 0.$$

Proof of Proposition 5 This analysis is the same as in Proposition 4 except for the equilibrium conditions, which are slightly changed:

$$\Psi_1 = (1-s)[1-F(\widehat{w_N^1}(p_0^I, p_r^I))] - sF(\widehat{w_O^2}(p_0^I, p_r^I)) = 0$$
$$\Psi_2 = (1-s)[F(\widehat{w_N^1}(p_0^I, p_r^I)) - F(\widehat{w_N^2}(p_r^I))] - s[F(\widehat{w_O^1}(p_0^I, p_r^I)) - F(\widehat{w_O^2}(p_0^I, p_r^I))] = 0.$$

We abuse the notation and keep the same notation as before. Note that the same notation does not represent the same function. The law of supply and demand holds in each market and sales and leases of houses are substitutes. We additionally have $\frac{\partial\widehat{w_O^1}}{\partial p_0^I} < 0$ in addition to $\frac{\partial\widehat{w_O^2}}{\partial p_0^I}$, $\frac{\partial\widehat{w_N^1}}{\partial p_0^I}$, $\frac{\partial\widehat{w_O^1}}{\partial p_r^I}$, $\frac{\partial\widehat{w_N^2}}{\partial p_r^I} > 0$ and $\frac{\partial\widehat{w_O^2}}{\partial p_r^I}$, $\frac{\partial\widehat{w_N^1}}{\partial p_r^I} < 0$. The expressions $\frac{\partial\Psi_1}{\partial p_0^I}$, $\frac{\partial\Psi_1}{\partial p_r^I}$, and $\frac{\partial\Psi_2}{\partial p_r^I}$ are the same as in the proof of Proposition 4, and $\frac{\partial\Psi_2}{\partial p_0^I}$ changes to

$$\frac{\partial\Psi_2}{\partial p_0^I} = -\frac{\partial\Psi_1}{\partial p_0^I} - sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_0^I} > 0.$$

Equilibrium prices are not guaranteed to be stable because the derivative of excess demand is not necessarily negative definite. That is,

$$\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial p_0^I} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$+\frac{\partial \Psi_1}{\partial p_r^I} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_0^I} \lesseqgtr 0 ,$$

which can be negative.[16] If we restrict our attention to stable equilibrium prices, the comparative statics are determined by the sign of the numerator in Cramer's rule.

For i), because

$$\frac{\partial \Psi_1}{\partial s} = -\{[1-F(\widehat{w_N^1})] + F(\widehat{w_O^2})\} < 0$$
$$\frac{\partial \Psi_2}{\partial s} = -[F(\widehat{w_N^1}) - F(\widehat{w_N^2})] + [F(\widehat{w_O^1}) - F(\widehat{w_O^2})] < 0 ,$$

we have $\frac{\partial p_0^I}{\partial s}$, $\frac{\partial p_r^I}{\partial s} < 0$.

For v), let us assume another parameter ε such that $\frac{\partial F}{\partial \varepsilon} > 0$. The distribution with a higher ε is first-order stochastically dominated by the one with a lower ε. Because

$$\frac{\partial \Psi_1}{\partial \varepsilon} = -(1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^1}) - s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^2}) < 0$$
$$\frac{\partial \Psi_2}{\partial \varepsilon} = -\frac{\partial \Psi_1}{\partial \varepsilon} - (1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^2}) - s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^1}) ,$$

we have

$$\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \varepsilon} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \varepsilon} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial \varepsilon}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$-\frac{\partial \Psi_1}{\partial p_r^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^2}) + s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^1})\right\} < 0$$

[16] This is not likely because $-\frac{\partial \Psi_1}{\partial p_0^I} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I} \approx \frac{\partial \Psi_1}{\partial p_r^I} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_0^I}$. However, it cannot be excluded.

$$\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \varepsilon} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \varepsilon} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}(\widehat{w_N^2}) + s\frac{\partial F}{\partial \varepsilon}(\widehat{w_O^1})\right\} + \frac{\partial \Psi_1}{\partial \varepsilon} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_0^I} < 0 .$$

Thus $\frac{\partial p_0^I}{\partial \varepsilon}$, $\frac{\partial p_r^I}{\partial \varepsilon} < 0$.

For other comparative statics, the change in the threshold income levels, $\widehat{w_O^1}$, $\widehat{w_O^2}$, $\widehat{w_N^1}$, and $\widehat{w_N^2}$ should be considered. We have

$$\frac{\partial \widehat{w_O^1}}{\partial p_1} > 0, \quad \frac{\partial \widehat{w_O^2}}{\partial p_1} < 0, \quad \frac{\partial \widehat{w_N^1}}{\partial p_1} < 0, \quad \frac{\partial \widehat{w_N^2}}{\partial p_1} = 0$$
$$\frac{\partial \widehat{w_O^1}}{\partial w_1} > 0, \quad \frac{\partial \widehat{w_O^2}}{\partial w_1} > 0, \quad \frac{\partial \widehat{w_N^1}}{\partial w_1} > 0, \quad \frac{\partial \widehat{w_N^2}}{\partial w_1} > 0$$
$$\frac{\partial \widehat{w_O^1}}{\partial r} > 0, \quad \frac{\partial \widehat{w_O^2}}{\partial r} \ge 0, \quad \frac{\partial \widehat{w_N^1}}{\partial r} > 0, \quad \frac{\partial \widehat{w_N^2}}{\partial r} \ge 0$$
$$\frac{\partial \widehat{w_O^1}}{\partial \nu} < 0, \quad \frac{\partial \widehat{w_O^2}}{\partial \nu} = 0, \quad \frac{\partial \widehat{w_N^1}}{\partial \nu} < 0, \quad \frac{\partial \widehat{w_N^2}}{\partial \nu} = 0 .$$

The change in threshold income is similar to the results of the main analysis. An increase in ν will increase the utility of a house owner when keeping a house and that of a non-owner when buying a house.

For ii), because

$$\frac{\partial \Psi_1}{\partial p_1} = -(1-s)f(\widehat{w_N^1})\frac{\partial \widehat{w_N^1}}{\partial p_1} - sf(\widehat{w_O^2})\frac{\partial \widehat{w_O^2}}{\partial p_1} > 0$$
$$\frac{\partial \Psi_2}{\partial p_1} = -sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_1} - \frac{\partial \Psi_1}{\partial p_1} < 0 ,$$

we have

$$\begin{vmatrix} -\frac{\partial \Psi_1}{\partial p_1} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial p_1} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_1}\left\{sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I} + (1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I}\right\}$$

$$-\frac{\partial\Psi_1}{\partial p_r^I} sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_1} \lesseqgtr 0$$

$$\begin{vmatrix} \dfrac{\partial\Psi_1}{\partial p_0^I} & -\dfrac{\partial\Psi_1}{\partial p_1} \\ \dfrac{\partial\Psi_2}{\partial p_0^I} & -\dfrac{\partial\Psi_2}{\partial p_1} \end{vmatrix} = \frac{\partial\Psi_1}{\partial p_0^I} sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_1} - \frac{\partial\Psi_1}{\partial p_1} sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_0^I} \lesseqgtr 0 .$$

For iii), because

$$\frac{\partial\Psi_1}{\partial r} = -(1-s)f(\widehat{w_N^1})\frac{\partial\widehat{w_N^1}}{\partial r} - sf(\widehat{w_O^2})\frac{\partial\widehat{w_O^2}}{\partial r} < 0$$

$$\frac{\partial\Psi_2}{\partial r} = -(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial r} - sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r} - \frac{\partial\Psi_1}{\partial r} \lesseqgtr 0 ,$$

we have

$$\begin{vmatrix} -\dfrac{\partial\Psi_1}{\partial r} & \dfrac{\partial\Psi_1}{\partial p_r^I} \\ -\dfrac{\partial\Psi_2}{\partial r} & \dfrac{\partial\Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial\Psi_1}{\partial r}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$-\frac{\partial\Psi_1}{\partial p_r^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial r} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r}\right\} < 0$$

$$\begin{vmatrix} \dfrac{\partial\Psi_1}{\partial p_0^I} & -\dfrac{\partial\Psi_1}{\partial r} \\ \dfrac{\partial\Psi_2}{\partial p_0^I} & -\dfrac{\partial\Psi_2}{\partial r} \end{vmatrix} = \frac{\partial\Psi_1}{\partial p_0^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial r} + sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r}\right\}$$

$$-\frac{\partial\Psi_1}{\partial r} sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial r} < 0 .$$

For iv), because

$$\frac{\partial\Psi_1}{\partial w_1} = -(1-s)f(\widehat{w_N^1})\frac{\partial\widehat{w_N^1}}{\partial w_1} - sf(\widehat{w_O^2})\frac{\partial\widehat{w_O^2}}{\partial w_1} < 0$$

$$\frac{\partial\Psi_2}{\partial w_1} = -(1-s)f(\widehat{w_N^2})\frac{\partial\widehat{w_N^2}}{\partial w_1} - sf(\widehat{w_O^1})\frac{\partial\widehat{w_O^1}}{\partial w_1} - \frac{\partial\Psi_1}{\partial w_1} \lesseqgtr 0 ,$$

we have

$$\begin{vmatrix} -\frac{\partial \Psi_1}{\partial w_1} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial w_1} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial w_1}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$-\frac{\partial \Psi_1}{\partial p_r^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial w_1} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial w_1}\right\} < 0$$

$$\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial w_1} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial w_1} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial w_1} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial w_1}\right\}$$

$$-\frac{\partial \Psi_1}{\partial w_1} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_0^1} < 0\,.$$

For vi), because

$$\frac{\partial \Psi_1}{\partial v} = -(1-s)f(\widehat{w_N^1})\frac{\partial \widehat{w_N^1}}{\partial w_1} > 0$$

$$\frac{\partial \Psi_2}{\partial v} = -sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial v} - \frac{\partial \Psi_1}{\partial v} \lesseqgtr 0\,,$$

we have

$$\begin{vmatrix} -\frac{\partial \Psi_1}{\partial v} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial v} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} = \frac{\partial \Psi_1}{\partial v}\left\{(1-s)f(\widehat{w_N^2})\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}$$

$$-\frac{\partial \Psi_1}{\partial p_r^I} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial v} > 0$$

$$\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial v} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial v} \end{vmatrix} = \frac{\partial \Psi_1}{\partial p_0^I} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial v} - \frac{\partial \Psi_1}{\partial v} sf(\widehat{w_O^1})\frac{\partial \widehat{w_O^1}}{\partial p_0^I} < 0\,.$$

Proof of Proposition 6 The structure of the proof is almost the same as the proof of Proposition 4 except that the signs of $\frac{\partial \widehat{w_O^1}}{\partial r}$, $\frac{\partial \widehat{w_O^2}}{\partial r}$, $\frac{\partial \widehat{w_N^1}}{\partial r}$, and $\frac{\partial \widehat{w_N^2}}{\partial r}$ become ambiguous rather than positive. Thus, we only show the effect of the change in r while the other parts of the proof are omitted.

Consider $\frac{\partial \widehat{w_O^1}}{\partial r}$ for example. At $\widehat{w_O^1}$, we have either

$$u((1+\mu)\widehat{w_O^1}+H)+\beta u(w_1+p_1-(1+r)\mu\widehat{w_O^1})=u(c_0^{OL})+\beta u(c_1^{OL})$$

or

$$\begin{aligned}&u((1+\mu)\widehat{w_O^1}+H)+\beta u(w_1+p_1-(1+r)\mu\widehat{w_O^1})\\&=u((1+\mu)\widehat{w_O^1}+p_r)+\beta u(w_1+p_1-p_r-(1+r)\mu\widehat{w_O^1}).\end{aligned}$$

Note that house owners leasing their house are indebted in the second case and they can also be indebted in the first case. If they are indebted, the negative effect of an increase in interest rate can be even greater than when they keep the house. Therefore, $\frac{\partial \widehat{w_O^1}}{\partial r}$ can be negative. The same logic applies to other threshold incomes and p_0^I and p_r^I may increase.

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