Theoretical Presentation of Interregional Competition Model of the Beef Industry

by

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I. Introduction

Space is a significant factor in economic life and economists have given it increased attention in recent years.[1] Traditionally, the time dimension has been emphasized, and the spatial dimension has been minimized in economic analysis. Space has a different economic meaning for each commodity considered, due to regional differences in resource endowments and differing transportation costs. The purpose of this study is to embody the locational factors into an interregional competition model of the beef economy. By its nature, however, this economic model can only be a crude guide. The usefulness of the analytical model lies in future prediction in response to changing economic conditions. Reliable predictions will contribute to policy decisions.

The basic methodology of the interregional competition model requires construction of linear demand and linear supply functions for each of the regions. Trades are effected from regions with low prices to regions with high prices, resulting in shifts in the supply functions of each region entering trade. Trade continues until no profitable trades remain. The final result requires that price differences exactly equal cost of transportation between regions that enter the trade pattern. This iterative process has been discussed in some detail by previous writers on spatial price equilibrium.[2]

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1) Bestil Ohlin, Interregional and International Trade, Harvard University Press, Cambridge, 1935, p. 9.

2) Stephen Enke, "Equilibrium Among Spatially Separated Markets; Solution by Electric Analogue," Economica (Jan., 1951), pp. 40~47. Paul A. Samuelson, "Spatial Price Equilibrium and Linear Programming," American Economic Review (June, 1952), pp. 283~303. Kaul A. Fox, "A Spatial Equilibrium Model of the Livestock- Freed Economy in the U.S., Econometrica (Oct, 1953), pp. 547~566.

II. Functional Formulation of the Model

One of the first attempts to show that competitive market price level is determined by supply and demand functions was by A.A. Cournot in his analysis of the communication of markets.[3] He concentrated on price relations with two spatially separated markets.

A single-market partial equilibrium analysis begins with the intersection of a negatively sloping demand functions and a positively sloping supply function. The present model involves largely the derivation of linear supply and demand functions of beef, and the solution of the equilibrium model is found by comparing price differences based on these supply and demand curves with transportation costs. The mathematical model of each function is as follows:

DEMAND FUNCTION:

$$Q_{di}=f(Y_i,\ U_i,\ N_i,\ C_i,\ P_{bi}),\ (i=1,\ \dots,\ n) \cdots\cdots (1)$$

where Q_{di}=quantity of beef demanded in ith region,

Y_i=per capita income in ith region,

U_i=urbanization factor in ith region,

N_i=population in ith region,

C_i=per capita consumption of beef in pounds in ith region,

and P_{bi}=price of beef on carcass weight basis in ith region.

As a final formula for the single demand model, (1) is expressed

$$P=A_i-B_i\ Q,\ (i=1,\ \dots,\ n) \cdots\cdots (2)$$

where A_i=intercept of the demand curve in ith region, and

B_i=slope of the demand curve in ith region.

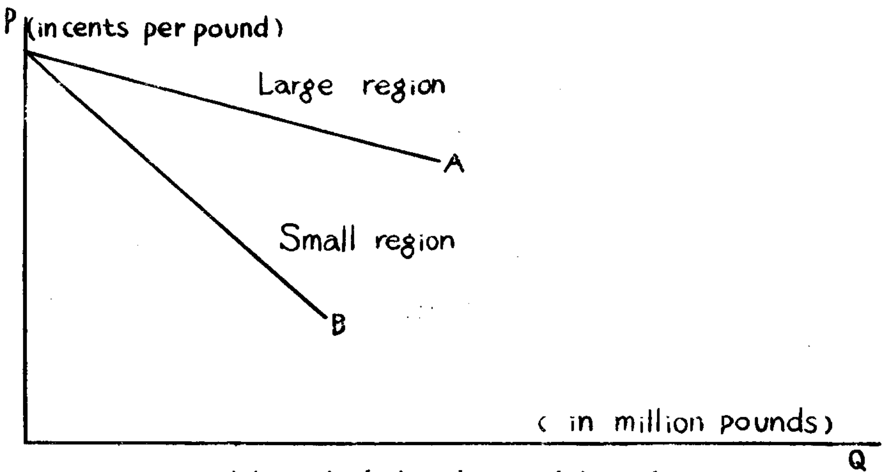
The intercept of the demand function depends on per capita income, per capita consumption of beef, and an urbanization factor in the data of population. From the derived per capita national demand function, the slope of the demand curve for each region depends on the population of the region. Thus, the general shape of the demand curve is shown in Figure 1.

SUPPLY FUNCTION:

$$Q_{si}=f(E_{1i},\ E_{2i},\ E_{3i},\ E_{4i},\ P_{bi}),\ (i=1,\ \dots,\ n) \cdots\cdots (3)$$

3) A.A. Cournot, Researches into the Mathematical Principles of the Theory of Wealth (The MacMillan Co., N.Y., 1929), pp. 117~126.

FIG. 1. HYPOTHETICAL DEMAND CURVES



A is a region having a large population, and
B is a region having a small population

where Q_{si}=quantity of beef supplied in *i*th region,

E_{1i}=total available energy therms for beef from concentrates in *i*th region,

E_{2i}=total available energy therms for beef from roughages in *i*th region,

E_{3i}=total available energy therms for beef from grazing on pasture in *i*th region, and

E_{4i}=total available energy therms for competitive sectors of livestock industries from all the natural resources in *i*th region.

The final formula of (3) as a single product model is as follows:

$$P = a_i + b_i\, Q, \quad (i=1, \ldots\ldots, n) \qquad \cdots\cdots (4)$$

where a_i=intercept of the supply curve in *i*th region, and

b_i=slope of the supply curve in *i*th region.

For the final supply model with foreign imports to be presented in the open model, function (4) is altered to the following formula with the explicit consideration of I_i;

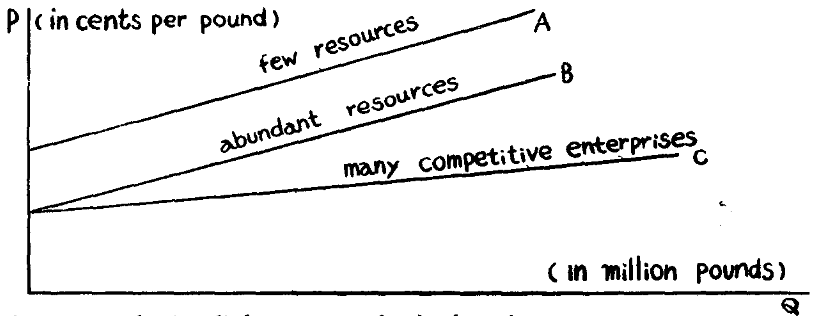
$$P = a_i + b_i(Q - I_i), \quad (i=1, \ldots, n) \qquad \cdots\cdots (5)$$

where I_i=total imports of beef from abroad in *i*th region.

Resource endowments determine the intercept(a_i) of the supply function. Therefore, given the same value of the supply slope, the region with greater resource endowments has a lower value of a_i, since the latter's supply curve is on the right of that of the other region.

The slope of the supply function(b_i) is determined by the level of livestock industries competitive to beef, such as dairy cattle, hogs, sheep and lambs, and other feed consuming enterprises. Suppose the same value of the intercept is given, the region having many alternatives to the beef industry has a flatter supply curve, which is shown in Figure 2.

FIG 2. HYPOTHETICAL SUPPLY CURVES



A is a region having little resources for beef production,

B is a region having much resources for beef production, and

C is a region having many alternative chocies to the beef industry.

SPATIAL EQUILIBRIUM SOLUTION:

The simultaneous solution of (2) and (4) produces the following result:

$$Q=\frac{A_i-a_i}{B_i+b_i} \quad \cdots\cdots (6)$$

$$P=\frac{A_ib_i+a_iB_i}{B_i+b_i}=\frac{a_iB_i+A_ib_i}{B_i+b_i} \quad \cdots\cdots (7)$$

where $i=1, \ldots, n$.

Let $Q=Q_i$ and $P=P_i$ from (6) and (7),

where Q_i=quantity of deef at the equilibrium before trade, and

P_i=price of beef at beef at the equilibium before trade. Both are in ith region.

The method of finding a spatial equilibrium in this study is based on the gradient method, developed by A. B. Larson at the University of Hawaii.[4] In the detailed explanation of this method, Larson stated as follows:

> ......Trade is the analogue of heat transfer, price is the analogue of temperature, and demand and supply, or perhaps more accurately, consumption and production, are the analogues of cooling and heating, respectively.
>
> Transportation cost is the analogue of the combined effects of the constant of thermal conductivity, the cross-sectional area, and the length of the conducting material. Trade occurs across a price gradient in much the

4) Arnold B. Larson. The Computer Program for the Gradient Method (FORTRAN IV for the IBM 360/Model 65), unpublished program, University of Hawaii. 1968.

same way as heat transfer occurs across a temperature gradient......[5]

The formula in terms of price and transportation is, therefore, expressed as:

Gradient: $G(i,j)=\frac{P_j-P_i}{T_{ij}}$, $(P_j - P_i)$ ....(8)

where P_j=price level of beef in jth region before trade,

P_i=price level of beef in ith region before trade, and

T_{ij}=transportation cost from ith region to jth region.

(i=1, ..., n, and j=1, ..., m)

Even if the demand function of the individual consumer in each region tends to be similar because of the development among the regions of communication and common ways of life, the supply functions are different by regions due to differences in the natural resource endowments of the beef industry and possibly due to differences in technical coefficients of production. Therefore, the values of P_i and P_j are different for each of the regions, and regional trade is held as long as the price differences between the trade patterns are greater than the transportation cost from the ith region to the jth region. At the stage of spatial equilibrium among the spatially separated regions, the gradient (G) of (8) is 1, for each set of partners between which trade actually occurs.

Ⅲ. Redundant Trade Routes

At intermediate stages in the iterative process of convergence to the final optimum competitive equilibrium pattern of trade, many false steps are taken. For example, two regions may both ship to two other regions. More complicated patterns also arise in which total transportation cost is greater than the minimum possible. The redundant trade routes are eliminated by recourse to the simplex algorithm of linear programming, to minimize cost of transportation, every twentieth iteration.

Let Q_i=amount of beef to be shipped from ith region,

Q_j=amount of beef to be received by jth region,

T_{ij}=transportation cost per pound of beef from ith region $j$to th region, and

Q_{ij}=amount of beef which is shipped from ith region to jth region.

Then, the formulation of linear programming gives the following relations:[6]

5) Arnold B. Larson, A Gradient Method of Determing Competitive Equilibrium Patterns of Interregional Trade and Price (Ditto Report, University of Hawaii. 1968).

6) Robert Dorfaman. Paul A. Samuelson and Robert M. Solow. Linear Programming and Economic Analysis, The RAND Series, (McGraw-Hill Book Company, Inc., New York, 1958) pp. 106~129.

$$\text{minimize} \quad C=\sum_{i=1}^{n} \sum_{j=1}^{m} Q_{ij} \cdot T_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{m} Q_{ij}=Q_i, \quad (i=1, \ldots, n),$$

$$\sum_{i=1}^{n} Q_{ij}=Q_j, \quad (j=1, \ldots, m),$$

$$\sum_{j=1}^{i} Q_i=\sum_{j=1}^{m} Q_j, \text{ and } Q_{ij} \geq O, \text{ for all } i \text{ and } j.$$

The final optimum solution, thus, is found by the simplex method with the solution of $n+m-1$ number of restraining equations. That is, $n+m-1$ of shipments are held at the positive levels for a minimum-cost set of routes.

Linear programming is used to eliminate redundant trade routes. It should perhaps be emphasized that the trade pattern that emerges is the simplest pattern which connects the existing patterns of production and consumption, and in reality there may be considerable cross-hauling, especially of brand name products such as frankfurters, bologna, and other special beef products. Also, trade patterns such as shipments of feeder cattle from region to region 2 where they are fattened, slaughtered, and shipped in carcass from to region 3 would not be accurately reflected in the results of the computer algorithm. Instead, the results would show shipments from reg. 1 direct to region 3, reflecting the contribution of the feeder cattle in region 1 to the beef consumed in region 3, and shipments from region 2 to region 3, reflecting the contribution of fattening process to the total beef consumed in region 3. A multiple product analysis would be required to show the entire pattern of shipments accurately.

The price level at the destination of the trade must not be greater than the price level at the origion of the trade plus transportation cost from the origin to the destination of the trade. That is, the beef price level at the jth region should be equal to or less than the beef price at the ith region plus the transportation cost from the ith to the ith region for the final spatial equilibrium.

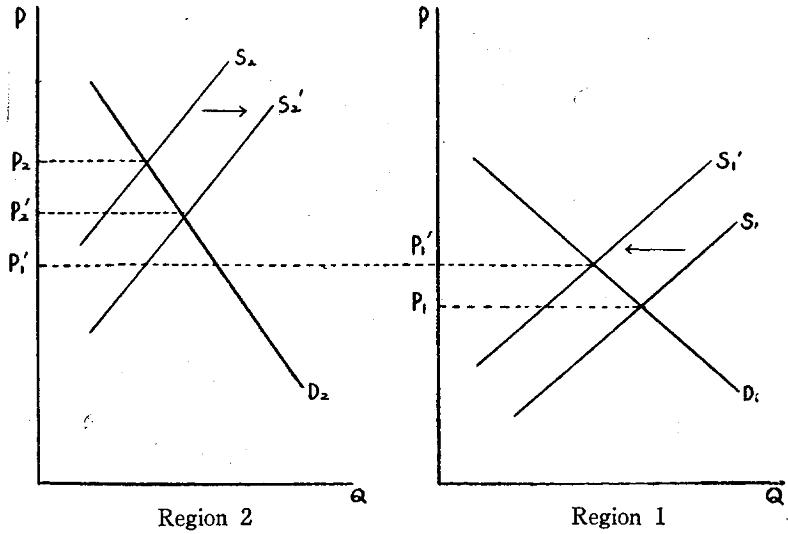
IV. Graphical Analysis

The two partners and the three partners cases, expressed in the following graphical analysis, can be generalized to other cases.

TWO PARTNERS CASE:

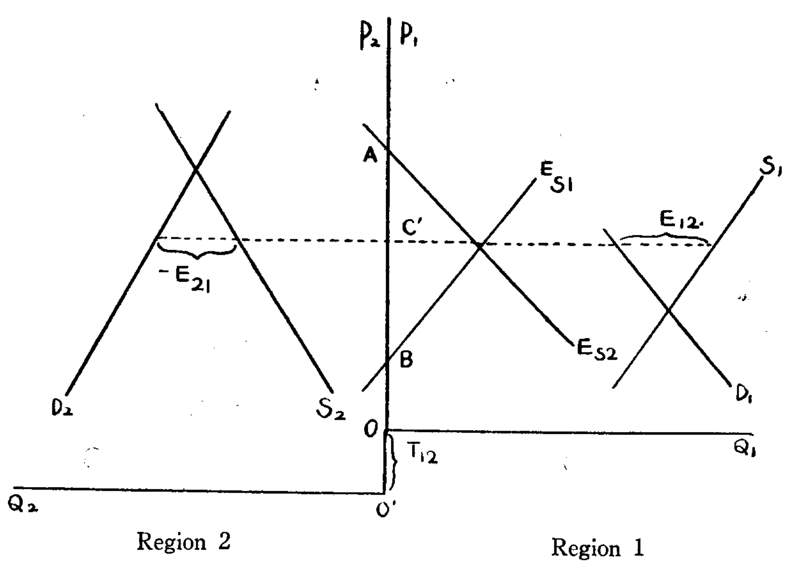
Suppose region 2 has a higher price level at the equilibrium before trade compared to region 1. Region 1, then, exports to region 2 as long as the price differences cover the transportation cost from region 1 to region 2.

FIG. 3. HYPOTHETICAL DEMAND AND SUPPLY CURVES



Because of the trade from region 1 to region 2, S_2 shifts to the right and S_1 shifts to the left, and the amounts of the price decrease and increase depend on the

FIG. 4. HYPOTHETICAL DEMAND AND SUPPLY CURVES
AND EXCESS SUPPLY CURVES



slopes of the respective supply and demand curves. The new equilibrium level of the price in region 2 and in region 1 is P_2' and P_1', where $P_2'-P_1'=T_{12}$.

The excess supply function of the region is presented in Figure 5.

The price level at the equilibrium before trade is $P_1=OB$ in region 1, and $P_2=O'A$ in region 2. Assuming that $Ab>00'$, where $00'=T_{12}$, trade occurs from region 1 to region 2 until $P_2-P_1=T_{12}$ and $E_{12}=-E_{21}$, which is the level of trade under the spatial equilibrium.

That is,

if $P_2=P_1+T_{12}$, $E_{12}>0$,

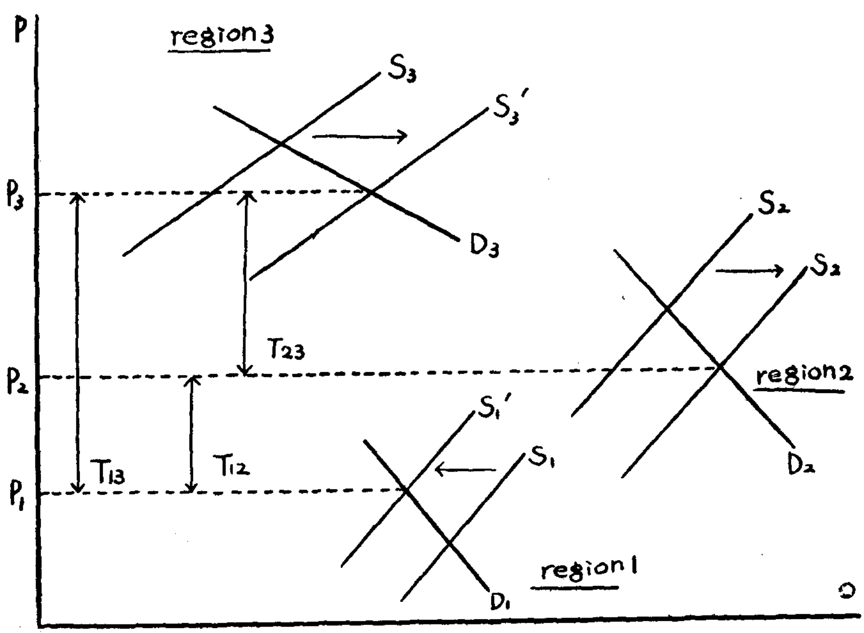
if $P_2<P_1+T_{12}$, $E_{12}=0$, and

if $P_1<P_2+T_{12}$, $E_{21}=0$.

THREE PARTNERS CASE:

Having three regions, only region 1 is assumed to export to region 2 and 3 at the same time. The supply curves in region 2 and 3 then shift to the right and that of region 1 shifts to the left.

FIG. 5. HYPOTHETICAL TRADE AMONG THE REGIONS



In the closed model, all the exports are equal to the import amount.

Let E_1=export amount of region 1,

I_{12}=import amount of region 2 from region 1, and

I_{13}=import amount of region 3 from region 1.

Then, $E_1=I_{12}+I_{13}$.

Also $P_3-P_1=T_{13}$, $P_2-P_1=T_{12}$, and $P_3-P_2=T_{23}$.

In general,

$$\sum_{i=1}^{n} E^i=\sum_{j=1}^{m} I_j,$$

$P_j-P_i=T_{ij}$ in case of trade,

where $i=1$, ……, n, and $j=1$, ……, m.

Ⅴ. Conclusion and Summary

The purpose of this paper was to present a logical procedure and functional structure to construct a spatial equilibrium model of the beef industry, largely centering on the derivation of supply and demand functions of beef for each region. The spatial equilibrium model of one product, beef, was presented. Perfect competition was assumed to prevail in the trade of the beef industry. The base year of this model is determined by the cross-sectional data availability for each region.

The method of finding a spatial equilibrium is based on the gradient method developed by A.B. Larson at the University of Hawaii, and the solution is found by comparing price differences based on the supply and demand functions of each region with transportation costs. As one of the principal determinants of the pattern of production and trade of beef in this presentation, transportation is derived through computer progamming and on the grid map coordinates of the regions.

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