

## **External Debt and Optimum Exchange Rate for Developing Economies: An Asset Approach**

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In any exchange regime which is not perfectly and permanently flexible, or likewise fixed for that matter, the government plays a potentially important role in the determination of foreign exchange rates. This covers a wide range of real-world exchange regimes, from adjustable peg to managed floating, and any number of their variations. The impact of such government intervention is well recognized in the literature. However, little is known about what motivates government to intervene in foreign exchange markets in the first place.

The normative policy question implied here is relevant in any economy where there is a conscious government exchange operation. However, from both the normative and predictive perspective, the government action is of much greater, in fact, of paramount importance for exchange rates in less developed countries (LDCs). The rule is that, more often than not, the exchange rates are dictated by the government in these countries, while government intervention in developed countries, in contrast, not only is smaller but usually takes the form of maneuvering indirectly through the market.

Exchange-rate policy, without question, is recently being used more actively in LDCs. The attitudes of various governments toward exchange policy, however, are more varied than uniform. Some pursue a "hard" currency policy, others permit devaluation more easily. Relegating this entirely to anecdotes and case studies would be unsatisfactory.

This raises the question as to how the government should, and does, optimally define the external value of its currency. A market equilibrium rate, such as the rate given by purchasing power parity, may come to mind as a possibility. Its applicability to a typical LDC, however, is limited by the notorious market distortions existing not only in foreign exchange but in other sectors of the economy. More importantly, the market equilibrium approach does not purport to describe a micro decision-making process of a government planner.

The literature in this area mostly has focused on such macro issues as the choice of exchange rate regimes and the effectiveness of government stabilization policies. Stanley Black [1] surveys various problems in

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extending the standard internal and external balance analysis to LDCs. Various authors [3, 5, 10, 15] examine the use of the currency basket, which has become popular in small LDCs.

This paper develops a model of optimal exchange rates for the government of a typical LDC. It takes as given such important related issues as what degree of government intervention, if any, is desirable, and how the government chooses its mix of policy instruments. Absent also is the role of market expectation. These issues have been examined intensively in the literature. The present paper focuses on the determination of target exchange rates in an optimization framework in an environment of an open, developing economy.<sup>1)</sup>

Having set the objective this way, some arbitrary specification of the government's or societal preference is unavoidable. Many observers of exchange policy in LDCs would agree that two major policy goals stand out: the financing of real growth and concern over price stability. Exchange rate changes affect both. Further, there are added "developmental" problems of having to perform in and improve on the country's "dualistic" production and trade structures.

Government policy in LDCs is further constrained by external finance. Debt cannot be marketed internally because of "fragmented capital market's" *a la* McKinnon [16]. Hence, external financing is necessary to meet investments required for growth objectives. Since external debt is denominated in foreign currency, devaluation of the domestic currency increases the debt-service burden of domestic borrowers. In an imperfectly-indexed economy, the interest of this group therefore represents a powerful constraint against devaluation.

The present model incorporates all these dimensions of exchange policy-making in developing countries. The government objective is to promote growth and price stability subject to national income and balance of payment constraints in the economy. The analytic framework is basically asset-theoretic, because the optimal exchange rate in the model depends on the gains and the cost of external financing. The structural effects of exchange rate changes as well as the government's preference affect the optimal exchange rates. In the dynamic context, the amount of existing debt and debt services also matter. The basic

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1) However, there is a connection between environment and government intervention. An argument for government exchange intervention in LDCs is found in Lipschitz [14]. See Frenkel and Aizenman [11] for the optimal degree of government intervention in foreign exchange markets.

model is then extended to incorporate implications of industry structures and endogenous changes in borrowing costs.

## I. The Basic Model

A simple dynamic constrained maximization is hypothesized for government. The government maximizes the future stream of expected utility subject to the national income identity and the balance-of-payments constraint of the economy. The expected utility function summarizes the government objective of promoting real growth and price stability.

Formally, the decision model is

$$\max \int_0^{\infty} U(Y, \pi(\dot{X}, r)) e^{-it} dt \quad (1)$$

subject to

$$-B(x) + \dot{D} = m \quad (2)$$

$$Y = Q(sD, A) - xrD \quad (3)$$

and  $D(0) = D_0$

Notations are:

$U$  = expected utility of the government

$Y$  = real income

$Q$  = the value of real domestic output

$A$  = domestic factors of production

$D$  = net external debt outstanding

$\dot{D}$  = changes in net external debt per unit of time

$B$  = the current account balance

$m$  = changes in reserve account

$\pi$  = domestic price inflation

$x$  = real exchange rate, expressed by the value of foreign currency in domestic money unit

$\dot{x}$  = time rate of change of  $x$ .

$r$  = other variables than  $x$  affecting domestic inflation

$r$  = real interest rate on external borrowing

$i$  = subjective rate of time preference

$s$  = an adjustment factor to equate a given amount of foreign-currency loan to a physical unit of capital.

Inclusion of inflation rates in the objective function reflects both the policy tradeoff and the welfare cost of inflation in an imperfectly-indexed economy. Inflation rates are dependent upon exchange rates, among others. Real income is defined as real output minus external debt service

payments. The balance-of-payments equation summarizes the external constraint of the economy. It is presupposed that the government derives its utility, as least partly, from improvement in economic conditions of the country. The model, therefore, abstracts from the well-known problems in using a social utility function such as the impact of income distribution within the country.

The utility function is strictly concave, separable and time-additive. With reversed sign,  $\dot{D}$  describes the net lending as well as borrowing of a country. The exchange rate,  $x$ , is defined as a subset of an admissible, nonempty and positive compact set. All nonparametric variables are continuously differentiable.

The role of foreign capital in economic development is represented in the real income equation. This specification makes most sense if foreign funds,  $D$ , are used to purchase foreign capital equipment in foreign currency for investment; the coefficient "s" accounts for consumption leakages and other residuals. This implies that domestic residents are subject to exchange risk in that, with unforeseen devaluation, a given amount of foreign-currency loan is repayable with greater domestic resources.<sup>2)</sup> Following the two-gap model, it is assumed that domestic financing is an imperfect substitute for foreign financing.

Basically, the present model takes a portfolio balance approach or an asset view to exchange rate determination [2, 8, 12, 13]. In this approach, at a given point in time, stock demand for and supply of net foreign asset (or debt) determine the exchange rate, but, over time, the current account affects the value of the net foreign assets outstanding. Put differently, per unit of time, an equilibrium exchange rate must equilibrate net flow demand in the current account with a time rate of change in demand for net foreign asset.<sup>3)</sup>

The present model builds on a number of simplifications, the most important one perhaps being the lack of the monetary sector. The monetary sector can be accommodated by relating changes in official reserve to the money market equilibrium. While such an extension will certainly broaden the analysis, it is not essential for the present purpose. In many situations concerning LDC lendings, it often does not really matter who the borrower is inside the country, as much as what the country borrows and owes as a whole.<sup>4)</sup> The present analysis concerns the

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2) The present model, however, does not explicitly consider the effect of exchange rate variability.

3) A more refined asset model of exchange rates endogenizes the reserve balance. In the present model, the reserve balance is exogenous.

4) Another supporting point is that international lending to a private borrower is not greatly different from that to a government, because many of the "private" loans in LDCs are effectively guaranteed by the government.

country's net aggregate borrowings.

The above is a problem in optimal control. The Hamiltonian is:

$$H(x, D, h, t) = e^{-it} \{ U[Q(sD, A) - xrD, \pi(\dot{x}, r)] + h(m + B(x) - \dot{D}) \}, \quad (4)$$

where  $H$  is the Lagrangian multiplier.

The first-order conditions are:

$$\frac{\partial H}{\partial x} - \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{x}} \right) = -rDU_1 + U_2 \pi_{,i} + hB' = 0 \quad (5)$$

$$\frac{\partial H}{\partial D} - \frac{d}{dt} \left( \frac{\partial H}{\partial \dot{D}} \right) = e^{-it} [U_1 (sQ_D - xr) + (h - i h)] = 0 \quad (6)$$

where prime or subscripted variables indicate partial derivatives. The transversality condition is:

$$\lim_{t \rightarrow \infty} e^{-it} U_1 D = 0.$$

This implies that  $e^{-it} \rightarrow 0$  faster than  $U_1 D \rightarrow \infty$ , which requires a positive real discount rate. The transversality condition, coupled with twice-differentiable, strictly concave utility function, assures that the necessary conditions are also sufficient.

Equation [5] entails:

$$h = (rDU_1 - U_2 \pi_{,i}) / B' \quad (7)$$

and

$$\dot{h} = U_1 (\dot{D}r + r\dot{D}) / B'. \quad (8)$$

In taking time derivatives, all parameters concerning the structure of the economy and government's preference were taken as given. The optimal exchange rate is obtained by substituting [7] and [8] in [6]:

$$x = \frac{1}{r} [sQ_D - i^2bk + \frac{rD(f - i + r/r)}{B'}] \quad (9)$$

where  $b \equiv -U_2/U_1 > 0$  is the government's marginal rate of substitution of growth for price stability,  $k \equiv \pi_1/B'$  summarizes the structural impact of exchange rate changes on inflation and the current account, and  $f \equiv \dot{D}/D$ .

The optimum exchange rate target is thus determined as the present value of three perpetual flows: the marginal productivity of foreign capital, government valuation of the structural impacts of exchange rate changes, and the burden of loan repayment.

Before examining the equation in detail, some general observations are

in order. First, consider the special cases. If  $f = i - \dot{r}/r$ , the debt service term drops out in (9). This case amounts to an indefinite rollover of existing debt. If  $i = r$  in addition, depicting the case of "perfect" international loan markets, [9] reduces to

$$x = \frac{sQ_D}{r} - r b k. \quad (10)$$

Hence, the optimum exchange rate is the present value of the marginal productivity of foreign capital minus government's valuation, in interest-cost terms, of the likely structural impacts of exchange rate changes. This underscores the fact that basically the present model reflects the asset views that the exchange rate represents net contribution of foreign capital relative to returns from domestic assets.

Second, the last point implies that the determination of an optimum exchange rate is related to that of an optimum borrowing. To understand this more clearly, rewrite [9] as:

$$Q_D = \frac{r}{s} \left[ x + \frac{i_2}{r} b k - \frac{D(f - i + \dot{r}/r)}{B'} \right] \quad (11)$$

This shows that an optimum foreign borrowing occurs at a point where the marginal productivity of foreign capital equals the interest rate on borrowing, adjusted for exchange rates and others.

Third, there is an economic constraint that the exchange rate obtained in (9) be positive and bounded. The constraint is met if:

$$sQ_D > i_2 b k - \frac{rD(f - i + \dot{r}/r)}{B'} \quad \text{and} \quad (12a)$$

$$B' \neq 0. \quad (12b)$$

Condition (12a) stipulates that the productivity of foreign capital should be greater than the time-adjusted and government-preference-adjusted structural impact of exchange rate changes, minus a disequilibrium term which depends on whether the net borrowing (in excess of what is warranted by time preference and dynamic changes in interest costs) can be offset by an improvement in the current account. Continued reliance on external financing when this condition is violated is unstable. Condition (12b) is similarly required for stability: the case of  $B' = 0$  is explosive in this model since, if so, there are no means of ever repaying the debt.

To examine the dynamic connection between external borrowing and exchange rates, observe from (6) that the shadow price of foreign capital,  $h$ , depends on the exchange rate and the marginal productivity of foreign capital. Consider the case of  $h = 0$ .

Then,

$$\left. \frac{dh}{dx} \right| \begin{array}{l} \dot{h} = 0 \\ Q_D = Q_D^* \end{array} = -U_1 r / i < 0.$$

$$\left. \frac{dh}{dQ_D} \right| \begin{array}{l} \dot{h} = 0 \\ x = x^* \end{array} = U_1 s / i > 0.$$

The dynamic path of the shadow price of foreign capital is shown in Figure 1. In the right quadrant,  $h$  is a decreasing function of  $x$ , and, in the left quadrant, an increasing function of  $Q_D$ . In both quadrants, the  $\dot{h} = 0$  loci are obtained with the third variable set at their respective steady levels. The area above the locus  $\dot{h} = 0$  is defined  $\dot{h} > 0$ , the area below  $\dot{h} < 0$ .

The saddle paths to steady state points  $(Q_D^*, x^*)$  are shown with dashed lines. In the right quadrant, at point A, for instance, a falling shadow price of foreign capital implies that net foreign borrowing should decrease to keep the marginal productivity of foreign capital unchanged, say, at its steady state level  $Q_D^*$ . This means that, for dynamic stability, the current account should increase with devaluation. Similarly, the path through Z is a saddle path since an upvaluation of the domestic currency is associated with an increased borrowing and a decreased current account. As expected, the saddle path requires a usual static stability condition in foreign exchange market.

In the second quadrant, the saddle path requires that when  $\dot{h} < 0$  and  $Q_D > Q_D^*$ ,  $Q_D$  should decline toward  $Q_D^*$ , and when  $\dot{h} < 0$  and  $Q_D < Q_D^*$ ,  $Q_D$  should rise toward  $Q_D^*$ , until the shadow price,  $h$ , reached the level  $h^*$  consistent with the steady state exchange rate.

Suppose  $Q_D$  rises above  $Q_D^*$  due to a change in any of the exogenous variables in (11). This will shift the  $\dot{h} = 0$  locus to the left in the left quadrant (Figure 2). The steady state  $x^*$  will move from  $x_0^*$  to  $x_1^*$  as a result. That is, a *ceteris paribus* increase in marginal productivity of foreign capital causes an increase in the domestic-currency price of foreign money.

Now assume that the currency in question follows the optimum path prescribed by equation (9). The following propositions on normative exchange policy then emerge from comparative dynamics. Proofs are in the Appendix.

**Proposition 1:** Assuming that  $f \leq i - \dot{r}/r$ , the case for devaluation increases with the strength of expected current-account improvement arising from it.

**Proposition 2:** Devaluation is more acceptable, (a) the less is the inflation effect of devaluation (assuming  $B' > 0$ ) and (b) the greater is the government's preference of growth to price stability on the margin.

These two propositions recast and formalize the familiar effects of devaluation in terms of exchange rate targeting. An interesting and empirically-verifiable result is the effect of government preference implied in Proposition 2: an ambitious and impatient government, growth-wise, tends to opt for a devaluation much more easily than those with a more balanced and long-term outlook.

Equation (9) also suggests several interesting points on the role of external financing.

**Proposition 3:** The more productive foreign capital is, the more desirable (or acceptable) devaluation becomes.

**Proposition 4:**  $B' > 0$ , devaluation is less desirable than/equally desirable to/more desirable than an increase in debt, according as  $f \lesseqgtr i - r/r$ .

**Proposition 5:** Assuming  $B' > 0$ , a rise in interest rates on external borrowing affects the optimality of devaluation ambiguously.

Proposition 3 stems from the fact that the value of a foreign currency *ceteris paribus* rises with the productivity of a foreign-currency loan: a devaluation of the domestic currency, hence, is in order when the foreign capital becomes more productive. In Proposition 4, as long as the debt service burden is binding ( $f \leq i - r/r$ ), devaluation is not desirable because it raises the burden in real terms. An interesting point here is that target exchange rates result from the comparison of alternative means of adjustment: the cost of structural adjustment in the real sector and the cost of financing the current-account deficit by external borrowing. If the cost of financing is greater, the real adjustment with devaluation (which is possible with stable foreign exchange markets) is superior; if the financing cost is lower, then it may not be too bad an idea to postpone the structural adjustment in the real economy.

Proposition 5 combines two conflicting effects. A rise in interest rates, over time, raises the debt service burden, with implied reduction in the optimality of devaluation. At a point in time, however, the increased cost of financing induces a shift from the financial to the real sector. The real-sector adjustment incurs a cost of increased debt service burden. The desirability of the devaluation therefore becomes ambiguous with a

rise in interest rates in this case<sup>5)</sup>

## II. Further Implications and Extensions

This section extends the basic model to examine implications of industry structures and endogenous changes in borrowing rates. The latter is to account for market imperfections in external financing.

### 1. Industry Structure

The basic model treated the inflation impact of devaluation in aggregate form:  $\pi = \pi(\dot{x}, y)$ . Price changes caused by devaluation, however, are hardly homogeneous sectorally. The devaluation immediately raises the domestic price of tradeables, but the spillover into nontradeables is slow and indirect. The spillover depends on the industry structure as well as the indirect transmission through income and monetary channels (Choi [4]).

To formalize, write changes in aggregate price index as

$$\pi = w\pi^t + (1-w)\pi^n = \pi^n + w(\pi^t - \pi^n), \quad 0 < w < 1$$

where  $\pi^t$  and  $\pi^n$  denote the inflation rates in tradeable and nontradeable goods sectors respectively, and  $w$  is the proportion of the tradeable sector in the economy. Since prices of tradeables are directly affected by exchange rates, i.e.,

$$\pi^t = g(\dot{x}, \pi^*)$$

where  $\pi^* =$  world inflation of tradeable goods, it is immediately clear that the overall inflation is affected by industry structure:

$$\frac{\partial \pi_1}{\partial w} \equiv \frac{\partial(\partial \pi / \partial \dot{x})}{\partial w} = g_1 > 0. \quad (13)$$

Interpreting this result within the basic model suggests that an increase in the openness of the country—as measured  $w$ —is associated with the increased inflationary impact of a given devaluation, and hence with a lower acceptability of devaluation as an optimal policy choice.

**Proposition 6:** Devaluation is less desirable *ceteris paribus*, the more open the country is in its trade structure.

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5) With  $B' < 0$ , devaluation becomes unambiguously less desirable with a rise in interest rates.

## 2. Endogenous Changes in Borrowing Cost

The basic model treated the interest rates as independent of funds to be borrowed, implying a perfect-market assumption for international financing. The financing for LDCs, however, is largely imperfect for such a variety of reasons as government controls, political risk, poorly-functioning domestic financial markets, concessionary financings, etc. A simple way of incorporating these market imperfections is to hypothesize a positive correlation between interest rates and the amount of existing debt:

$$r = r(D), \quad r' > 0.$$

The growth of debt is now constrained by the effect of interest rates as well as its marginal productivity.

Re-solving the model, equation (9) now becomes

$$x = \frac{1}{r(1 + e_{rD})} [sQ_D - i^2 b_k + \frac{rD}{B} (f - i + \dot{r}/r)], \quad (14)$$

where  $e_{rD} > 0$  is the absolute elasticity of interest rates with respect to debt.

The new optimum rate equation is different from the one in the basic model only by the presence of the elasticity term. Qualitatively, earlier results hence still go through, but there is a difference in magnitude. For instance,  $\partial x / \partial D$  is now smaller in absolute value than the same in (9). This result can be related to default or credit risk of the borrowing country. However, that will take another model and hence is beyond the scope of this paper.<sup>6)</sup>

## III. Summary and Conclusions

Despite frequent references to "overvaluation" or "undervaluation" of a currency, it is unclear just how the baseline and supposedly optimum exchange rates are determined. The market parity can be accepted as the target in an equilibrium sense. However, in an environment of imperfect markets, there still remains a microeconomic question of how the government, as a market participant, could view the exchange rates normatively.

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6) For the analysis of correlation between debt and the probability of default, see Eaton and Gersovitz [9].

The present paper derived the optimum exchange rate from the standpoint of a government. The motivation was rather "practical". For both predictive and normative purposes, it hoped to model the exchange rate targeting behavior of a typical LDC government constrained by external debt. Basically, two points were raised on the optimality of exchange rates. First, it is possible for the government to view the "optimal" exchange rates in terms of its objectives and preferences, coupled with expected structural impacts of exchange rate changes. Second, the determination of the optimal exchangerate target also requires an evaluation of the relative cost of adjustment through real and financial sectors. To the extent that external financing is more (or less) costly, then the structural adjustment in the real sector through devaluation is more (or less) acceptable and therefore more (or less) optimal. Countering this, however, is the dynamic effect of increasing debt-service burden with higher interest rates, which reduces the optimality of devaluation.

The basic model has been extended to incorporate the effects of industry structures, and endogenous determination of interest rates on external borrowing. With these extensions, the exchange-rate targeting should also take into account the openness of trade structure, and the extent to which borrowing costs depend on existing debt.

## Appendix

Five propositions in the basic model can be proved by taking partial derivatives of equations (9) in the text. Assuming  $B' = 0$ ,

$$(A1) \quad \frac{\partial x}{\partial B'} = \frac{1}{B'^2} [i^2 b \pi_1 / r - D(f - i + \dot{r}/r)] > 0 \text{ if } f \leq i - \dot{r}/r.$$

$$(A2) \quad \frac{\partial x}{\partial \pi_1} = \frac{-i^2}{r B'} b > 0 \text{ if } B' < 0.$$

$$\frac{\partial x}{\partial b} = \frac{-i^2 k}{r} > 0 \text{ if } B' < 0.$$

$$(A3) \quad \frac{\partial x}{\partial Q_D} = \frac{s}{r} > 0.$$

$$(A4) \quad \frac{\partial x}{\partial D} = \frac{(f - i + \dot{r}/r)}{B'} < 0 \text{ if } f > i - \dot{r}/r \text{ and } B' > 0.$$

(A5)     $\frac{\partial x}{\partial r} = \frac{-1}{r^2} (sQ_D - i^2bk) \begin{cases} \leq 0 & \text{if } B' > 0, \\ > 0 & \text{if } B' < 0. \end{cases}$

and  $< 0$  if  $B' < 0$ .

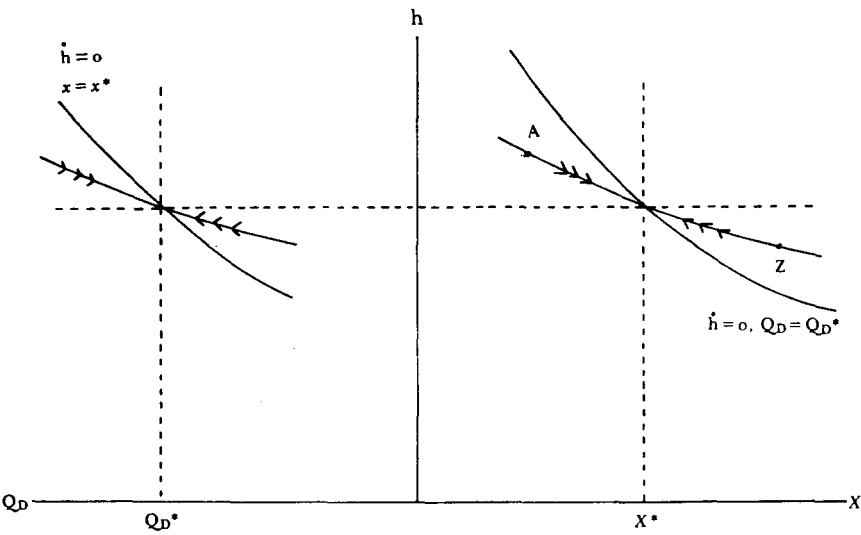


Figure 1

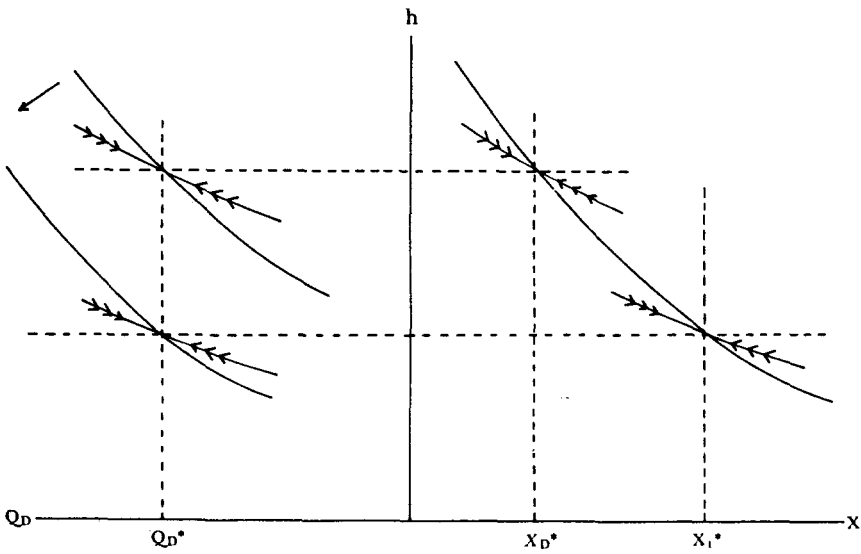


Figure 2

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