The Different Distribution of Randomness Across Countries and Heckscher-Ohlin Model

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Batra (1975) and Helpman and Razin (1978) reestablished the factor price equalization theorem in the Heckscher-Ohlin model under uncertainty assuming the identical distribution of randomness across countries. This paper investigate if there are any systematic movements of factor prices in two countries when this condition fails to hold. To single out the effect of the different distribution of randomness across countreis, we take an extreme case in which randomness is present in one of two countreis and is the only difference between these two countries. We obtain factor price divergence in a free trade regime and commodity price divergence with factor mobility then we extend our discussion to the general case with different factor endowment across countries.

I. Introduction

One of the major achievements in the field of international trade theory under uncertainty is the reestablishment of the basic theorems of international trade theory (Rybczynski, Stolper-Samuelson, Factor price equalization, and Heckscher-Ohlin theorems) in a stochastic neoclassical setting.

Among various models employed to get these results, the most widely used formulation is characterized by the following: (1) ex-ante resource allocation and ex-post trade, i.e., the production decisions are made under uncertainty and the trade is realized after the uncertainty resolves; (2) representative consumer (or representative entrepreneur) bears all the risk, i.e., fixed stock of risk bearer (an exception is the study of Mayer (1976); (3) small country; (4) single source of randomness (multiplicative production randomness or price randomness).

In addition, as Pomery (1979) clearly points out, the reestablishment of the basic theorems heavily relied on the assumption that the random variables are distributed identically across the countries and on the mechanism introduced to obtain the constancy of the risk premium in terms of numeraire commodity.

The objective of this study is to highlight the role of these assumptions

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and to emphasize the fact that commodity trade is itself one form of the risk sharing scheme. To emphasize the different distribution of random variables across countries, we take an extreme case in which randomness is present in one of two countries and is the only difference between these two countreis. Then commodity trade occurs as the result of introduction of randomness. The main results we obtain are:

a) factor price divergence[1] in a free trade regime;
b) commodity price divergence when factors are completely mobile and no commodity trade;
c) complete specialization in free commodity trade and perfect factor mobility.

In the final section we have extended our discussion to the general case with different factor endowment across countries.

II. Model

Our model is a familiar $2\times 2\times 2$ neoclassical. Each country produces two commodities, denoted by X_1 and X_2, with the aid of two primary factors of production, capital (K) and labor (L), and constant returns to scale. In addition to the usual assumption of identical technology we assume identical endowment so that there is no trade under certainty. we confine our discussion to a particular home country.

The production function of the home country's first industry is subject to random multiplicative (and therefore factor neutral) disturbances.

$$X_1 = \theta F_1(K_1, L_1)$$
$$X_2 = F_2(K_2, L_2)$$
$$K_1 + K_2 = K$$
$$L_1 + L_2 = L$$

where θ is a random variable such that $\bar{\theta} > \theta > \underline{\theta} > 0$ and $E = 1$, Furthermore, F_1 is assumed to be more capital intensive than F_2 for all possible factor price so that there is no factor intensity reversal. Let the transformation fuction derived from F_1 and F_2 be F; then,

$$X_1 = \theta F(X_2). \tag{1}$$

The function F is supposed to be decreasing and strictly concave. Let P_i denote commodity prices and let c_i and x_i denote the consumption and production of X_i, respectively. Furthermore, it is assumed that the utility function is strictly concave so that consumers are risk averse.

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1) The price divergence, in this paper, has a specific meaning which will become clear soon.

Production

Before θ is realized, the home country must choose a production point (x_1, x_2) from the production possibility frontier defined by (1). After θ is realized, the home country chooses its optimal consumption and, in turn, trades in the usual fashion by maximizing its utility function $u(c_1, c_2)$ subject to the budget constraint $c_1+pc_2=x_1+px_2=g$ where $p=p_2/p_1$ is relative price of X_2 in terms of X_1. This yields demand functions $c_i=c_i(p,y)$ that satisfy the budget constaint. Given these demand functions we can then define the indirect utility function:

$$V(p,y) \equiv u(c_1(p,y), c_2(p,y)).$$

Since $y=x_1+px_2$, the indirect utility function gives the utility level that results from any production decision and θ realization. The expected utility of any production decision is then simply

$$EV(p, \theta F(x_2)+px_2) \tag{2}$$

and the problem to be solved is

$$\underset{x_2}{MAX}\ EV(p, \theta F(x_2) + px_2).$$

Solving this problem, we get the quilibrium condition

$$EV_Y(\theta F'(x_2) + p) = 0 \tag{3}$$

where $V_y \equiv \partial V/\partial Y$

Autarky Equilibrium

In equilibrium, factor prices w and r are determined such that all domestic markets clear. Using the notation employed in Jones (12), the factor market clearing conditions for labor and capital can be written

$$\begin{aligned} a_{L1}\cdot Ex_1 + a_{L2}\cdot x_2 &= L \\ a_{K1}\cdot Ex_1 + a_{K2}\cdot x_2 &= K \end{aligned} \tag{4}$$

where x_1 is substituted by Ex_1 so that $a_{L1}=L_1/Ex_1$ and $a_{K1}=K_1/Ex_1$.

The competitive zero profit condition is

$$\begin{aligned} (w\cdot a_{L1} + r\cdot a_{K1})(1+z_\theta) &= 1 \\ w\cdot a_{L2} + r\cdot a_{K2} &= Ep-z_P \end{aligned} \tag{5}$$

where $q_i=(w\cdot a_{Li}+r\cdot a_{Ki})$ $(i=1, 2)$ is the unit cost attributable to non-random factor payments to labor and capital in industry i, and z's are the risk premium paid to reduce the randomness.

Let $q = q_2/q_1$ be ratio of unti costs; then

$$q(1+z_\theta)^{-1} + z_p = Ep \tag{6}$$

Since the marginal rate of transformation in expected output space is precisely the ratio of unit costs, i.e., $q = -F'(x_2)$, (3) and (6) give

$$q = -F'(x_2) = EpV_y/E\theta V_y = (Ep - z_p)(1+z_\theta) \tag{7}$$

Hence, $(1+Z\theta) = EV_y/E\theta V_y$ or $Z_\theta = -\mathrm{cov}(\theta, V_y)/E\theta V_y$

and $z_p = -\mathrm{cov}(p, V_y)/EV_y$.

Finally, the commodity market clearing condition is written by

$$\theta F(x_2) = \alpha(p)\ x_2. \tag{8}$$

where α (p) denotes denotes the known slope of the Engel curve derived from our homothetic preference and supposed to be $\alpha(p) > 0$ and $\alpha'(p) > 0$.

III. General Equilibrium

For any given expectation of θ, (i.e., a given probability distribution of θ) and for given $p(\theta)$, the home country's production decision is given by equation (3). The foreign country's production decision is made according to the similar marginal condition of expected utility maximization

$$EV_Y(F'(x_2^*) + p) = 0 \tag{9}$$

where the starred variables indicate foreign.

Since the world commodity price ratio p is determined by the actual supply and demand, even though production decisions are made prior to knowledge of the random variable θ, the actual price ratio is determined only after the actual value of θ becomes known. Thus, we have as many commodity market clearing equations as there are different state of nature. The equilibrium price p is the solution of the system of simultaneous equations consisting of (3), (9), and

$$\theta F(x_2) + F(x_2^*) = \alpha(p)\ (x_2 + x_2^*). \tag{10}$$

IV. Properties of Heckscher-Ohlin Model

In this section we will investigate how the introduction of uncertainty affects the factor price equalization theorem. Since the peculiarity of our model consists in the different distribution of randomness across countries, the single country propositions such as the Stolper-Samuelson and Rybczynski theorems do not differ from the results of existing uncertainty literature.

Factor Price Equalization Theorem

It is well known that the strict version of this theorem does not hold under uncertainty without additional assumptions. Batra (2, Ch. 4) shows that if randomness is distributed identifically across countries there is a tendency toward equalization. (Helpman and Razin need both the identical distribution of random variables across countries and the security market to get the strict version of the theorem.)

Since the randomness is different in each country factor prices do not equalize. In fact, we show that free commodity trade makes the factor prices of the two countries diverge.[2] The underlying logic of factor price equalization, in a uncertainty free standard model, is a univalent relation between commodity prices and factor prices which is independent of factor endowment in the absence of factor intensity reversal. In our model, we have price distribution instead of a unique price and the relation between commodity prices and factor prices depends on the factor endowments.

But the unit cost defined in (5) retains the same relation with factor prices that the commodity prices in certainty model have with the factor prices. Furthermore, the marginal rate of transformation in expected output space is precisely the ratio of unit cost, the factor price change can be studied by observing the optimal production plan.

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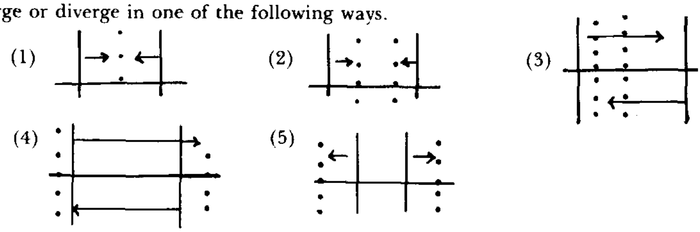
2) One would think that the failure of factor price equalization implies factor price divergence. It is not so. Since the factor prices in two countries move to the opposite direction, the factor prices either converge or diverge in one of the following ways.



The failure of factor price equalization might mean that

i) We have the price movement of the type (4) or (5),
ii) We have the equalization in one occasion and divergence in another, or
iii) Simply we do not know what happens with prices. The factor prices divergence we are going to show refers the price movement of the type (5).

If the two countries have the same resource allocations in autarky equilibrium, in turn, have same factor prices, and different production plans under free trade, then we have the factor price divergence. We present this special case of factor price divergence formally.

Proposition1: In a closed economy, if the elasticity of substitution in consumption is unitary, then the introduction of multiplicative technological uncertainty in one sector does not affect the resource allocation. This holds regardless of the attitude towards risk.

Proof: $\sigma = 1$ implies the linearity of price function, i.e.,

$$p(\theta) = \alpha^{-1}(\theta F(x_2)/x_2) = \theta F(x_2)/kx_2.$$

Substituting into the first order condition

$$-F'(x_2) = [E(\theta F(x_2)/kx_2)V_Y]/E\theta V_Y = F(x_2)/kx_2 = p(1) \qquad (11)$$

This proposition asserts that if the elasticity of substitution is unitary then the effect of induced uncertainty in price compensates exactly the effect of technological uncertainty.

Proposition 2: Under a free trade regime, the two countries of our model have distinct equilibrium resource allocation.

Proof: Suppose that they have the same resource allocation, $x_2 = x_2^*$.

From the market clearing equation we have:

$$P(\theta) = \alpha^{-1}[(\theta+1)F(x_2)/2x_2]$$

Substituting into the first order conditions (3) and (9) yields:

$$EV_Y(x_2;\theta)\{\theta F'(x_2) + \alpha^{-1}[(\theta+1)F(x_2)/2x_2]\} = 0$$

$$EV_Y(x_2;\theta)\{F'(x_2) + \alpha^{-1}[(\theta+1)F(x_2)/2x_2]\} = 0$$

Combining, we get

$$F'(x_2)[EV_Y\theta - EV_Y] = 0$$

which, in turn, implies $\text{cov}(\theta, V_y) = 0$.

This cannot happen because

$$\begin{aligned}\partial V_Y/\partial\theta &= (\partial V_Y/\partial p)(\partial p/\partial\theta) + (\partial V_Y/\partial Y)(\partial Y/\partial\theta) \\ &= [(x_2-c_2)V_{YY} - V_Y(\partial c_2/\partial y)](\partial p/\partial\theta) + V_{YY}F(x_2) \\ &= V_{YY}F(x_2) - V_y(\partial c_2/\partial Y)(\partial p/\partial\theta) < 0 \qquad (12)\end{aligned}$$

The above two propositions establish the factor price divergence for the special case of Cobb-Douglas utility. Now we remove this additional assumption and prove the factor price divergence in general.

Proposition 3: If the autarky equilibrium resource allocations in two countries ($\bar{x}_2$, $\bar{x}_2^*$) satisfy

$$\bar{x}_2 < \bar{x}_2^* - \varepsilon \text{ (or } \bar{x}_2 > \bar{x}_2^* + \varepsilon)$$

$$\text{and } \underline{\theta} F(\bar{x}_2^* - \varepsilon) / (\bar{x}_2^* - \varepsilon) = F(\bar{x}_2^*) / \bar{x}_2^*$$

$$(\text{or } \bar{\theta} F(\bar{x}_2^* + \varepsilon) / (\bar{x}_2^* + \varepsilon) = F(\bar{x}_2^*) / \bar{x}_2^*),$$

for some $\varepsilon > 0$, then the free trade equilibrium ($\hat{x}_2$, $\hat{x}_2^*$) is such that

$$\hat{x}_2 < \bar{x}_2 < \bar{x}_2^* < \hat{x}_2^* \text{ (or } \hat{x}_2 > \bar{x}_2 > \bar{x}_2^* > \hat{x}_2^*)$$

Proof: Let W and W* be the marginal expected utility.

$$W(x_2, x_2^* ; \theta) = E(\theta F' + p) V_y$$

$$W^*(x_2, x_2^* ; \theta) = E(F' + p) V_Y$$

Given the competitive market assumption, the maximization of expected utility implies the adjustment mechanism of the economy is given by the differential equation $(dx_2/dt) = g(W)$ where g is a sign preserving function. Thus, to prove this proposition, it suffices to show that $W(\bar{x}_2, \bar{x}_2^*) < (>) 0$ and $W^*(\bar{x}_2, \bar{x}_2^*) > (<) 0$ as $\bar{x}_2 < \bar{x}_2^*$ ($\bar{x}_2 > \bar{x}_2^*$). By assumption, $QF(x_2)/x_2 > F(x_2^*)/x_2^*$ for all Q. This, in turn, implies that the home country's autarky price is higher than that of the foreign country and that the world market clearing price with production plan ($\bar{x}_2$, $\bar{x}_2^*$) is bounded by these two prices, i.e., $\bar{p} > p > \bar{p}^*$

$$\text{where } \bar{p} = \alpha^{-1} (\theta F(\bar{x}_2) / x_2)$$

$$\bar{p} = \alpha^{-1} (F(\bar{x}_2^*) / \bar{x}_2^*)$$

$$p = \alpha^{-1} [(\theta F(\bar{x}_2) + F(\bar{x}^*)) / (\bar{x}_2 + \bar{x}_2^*)],$$

To evaluate W at $\bar{x}_2$ for given price $P(\theta)$, we write W explicitly as

$$W = \sum_k \Pi_k [\theta^k F'(x_2) + p(\theta^k)] V_y(p(\theta^k), \theta^k F(x_2) + p(\theta^k) x_2)$$

Differentiating with respect to $p(\theta^k)$,

$$\left.\frac{\partial W}{\partial p(\theta^k)}\right|_{\substack{p=\bar{p} \\ x_2=\bar{x}_2}} = \Pi_k \{V_{yp}(\theta^k) [\theta^k F'(\bar{x}_2) + p(\theta^k)] + V_y(\theta^k)\} = \Pi_k V_y(\theta^k) \left[\frac{(1 - {}^{PX}{}_2)}{y} - \frac{\theta^k c_2 F'(x_2)}{y}\right] > 0$$

$$W(\bar{x}_2\,;p)-W(\bar{x}_2\,;\bar{p})=\sum_k(\partial W/\partial p(\theta^k))(p_k-\bar{p}_k)$$

Since $W(\bar{x}_2\,;\bar{p})=0$, $W(\bar{x}_2,p)<0$ and $(dx_2/dt)<0$.

Therefore, the free trade equilibrium production plan must be smaller than $\bar{x}_2$. In similar fashion, we can show that $\hat{x}_2^*>\bar{x}_2^*$.

Hence we have the factor price divergence. This is a very strong one. Consequently, one is led to ask what role uncertainty plays in obtaining this result and whether this result depends on any additional hypotheses of our model.

Since the risk premium z's in the competitive zero profit condition (6) can be interpreted as a payment for risk-bearing service, it is possible to consider our model as one with three factors of production [e.g., Pomery (1978)]. Once given this interpretation one can easily see that the above result hinges entirely on the presence of the third factor of production, namely, the risk-bearing services, and the peculiar assumption of identical physical factor endowment across countries. The presence of a third factor in the home country departs from the standard uncertainty free model not only by varying from the two-by-two structure but also by assuming identical technology across countries. This, therefore, justifies the failure of the factor price equalization, but does not explain the factor price divergence. The second cause mentioned, the identical factor endowment across countries, is responsible for the factor price divergence, because this assumption permits us to obtain the kind of price relations of two countries we have, i.e., $\bar{p}>p>\bar{p}^*$. (We will discuss this point in detail in a later section.)

V. International Factor Mobility

It is well known that in Heckscher-Ohlin trade model commodity movements and factor movements are substitutes. However, this is not the case, as we have shown in previous section, if there exists uncertainty. Instead we established factor price divergence under free trade. A natural question arises: is there commodity price divergence when factors of production are mobile across countries?[3)]

3) The randomness of the home country's equilibrium price does not constitute a problem in answering this question because the price change caused by capital mobility is uni-directional for all states of nature. Thus it suffices to observe one particular state of nature or the expected price.

Let us consider the autarky equilibria of both countries. If these countries have identical resource allocation, capital mobility will not disturb the equilibria because there is no incentive for capital movements. Without loss of generality, we assume that the home country's autarky production of X_2 is greater than that of foreign country. Now assume all impediments to capital mobility are removed while trade of commodities is not allowed. Since the marginal productivity of capital in the foreign country is higher than that in the home country, the capital begins to flow from home to abroad and the production possibility frontier expands in the foreign country and contracts at home. To make clear the implication of capital flow, we will first see what happens in each country.

1. Foreign Country

The inflow of capital to the foreign country shifts the transformation curve outward. If prices remain unchanged, the production will adjust along the Rybczynski line while consumption remains unchanged. Consequently, there will be excess supply of X_1, and excess demand for X_2, if the prices remain at the autarky equilibrium level. The relative price of X_2, therefore, must increase until the new equilibrium is established.

2. Home Country

Since the relationship between commodity and factor prices is not independent of factor endowment under uncertainty, no simple assertion can be made without additional assumption when capital outflow occurs from the home country.

Consider the equilibrium conditions

$$E[\theta F'(x_2;K)+p]V_Y(p,\theta F(x_2;K)+px_2)=0$$

$$\theta F(x_2;K)-\alpha(p)x_2=0$$

Differentiating these equations we obtain

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} dp/dk \\ dx_2/dK \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

where

$$M_{11} = EV_Y[1-(\partial c_2/\partial_y)(\theta F'+p)]$$

$$M_{12} = E[V_{YY}(\theta F'+p)^2+V_Y\theta F'']$$

$$M_{21} = \alpha'$$

$$M_{22} = \alpha - \theta F'$$
$$N_1 = -E[V_{YY}\theta(\theta F' + p)(\partial F/\partial K) + V_Y\theta(\partial(F')/\partial K)]$$
$$N_2 = \theta(\partial F/\partial K)$$

This system can be solved with the help of Cramer's Rule to obtain

$$\frac{dp(\theta)}{dK} = \frac{N_1 M_{22} - N_2 M_{12}}{M_{11} M_{22} - M_{12} M_{21}}$$

Since the denominator is positive, the sign of (dp/dK) is determined by the sign of numerator. It can be shown that the numerator is positive, if any of the following condition is met:

a) the Arrow-Pratt relative risk aversion is non-decreasing and the elasticity of substitution in consumption is smaller than one;
b) the Arrow-Pratt relative risk aversion is non-increasing and the elasticity of substitution in consumption is greater than one. (The proof is relegated to Appendix.)

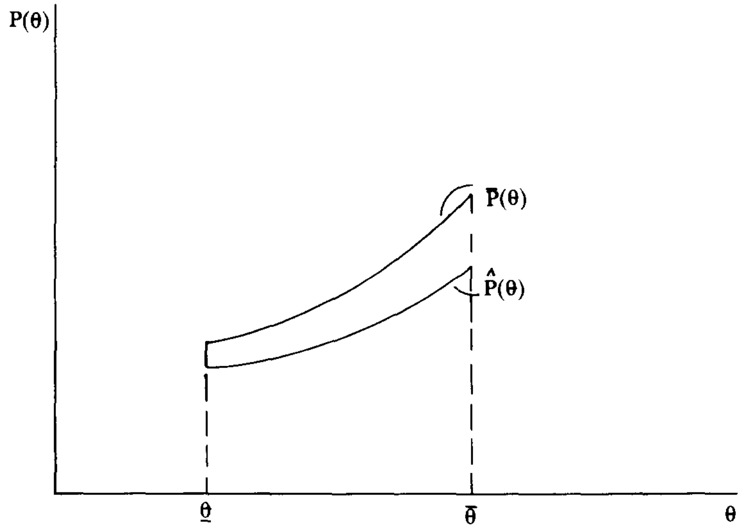
These conditions are sufficient conditions for $(dp/dK) > 0$. When these conditions are not met, the sign of (dp/dK) is indeterminate.

3. General Equilibrium

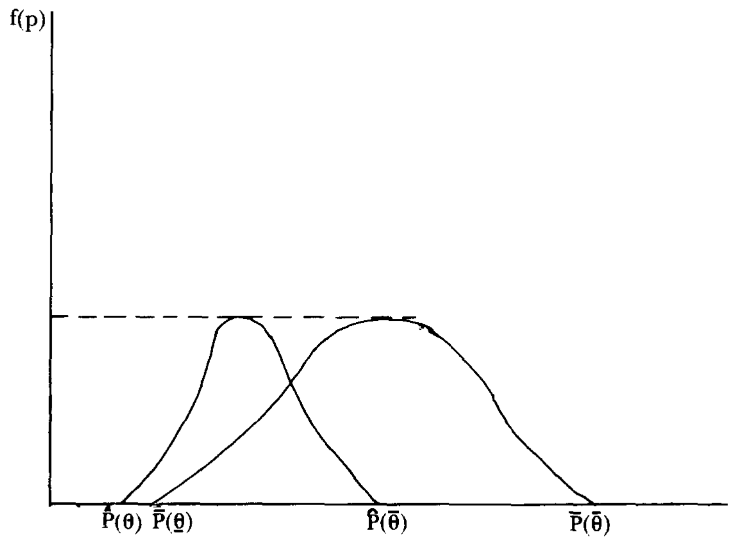
As all impediments to capital mobility are removed, the capital moves from home to abroad and this capital flow will continue until the factor prices in two countries are equalized. In the above discussion we have shown that the relative price of X_2 increases in the foreign country as capital inflow continues while the opposite occurs at home. Since, initially, $\bar{x}_2 > \bar{x}_2^*$, (by the assumption of Proposition 2)

$$\bar{p}(\theta) = \alpha^{-1}(\theta F(\bar{x}_2)/\bar{x}_2) < \bar{p}^* = \alpha^{-1}(\theta(\bar{x}^*)/\bar{x}^*) \text{ for all } \theta.$$

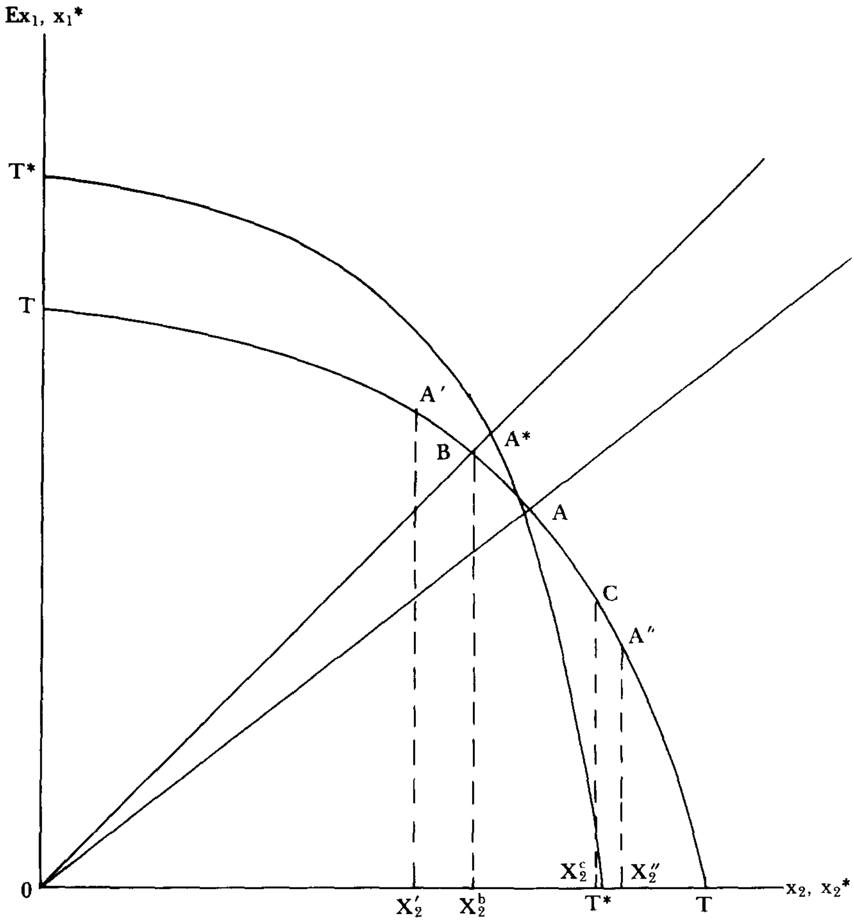
Therefore, we have $\hat{p}^* > p^* > p(\theta) > \hat{p}(\theta)$ for all θ, whenever the relative risk aversion coefficient and the elasticity of substitution in consumption are combined such a way that satisfy the condition established in previous subsection.



[Figure 1]



[Figure 2]



[Figure 3]

VI. Generalization

Throughout this paper, we have confined our attention to a model in which the physical factor endowments are identical across countries. A generalization to the case of different factor endowments in two countries is in order.

6.1 Suppose that the home country is labor abundant. Since X_2 is assumed to be labor intensive, the transformation curves of two countries will appear as drawn in Figure 3. The autarky equilibria in the absence of uncertainty are represented by A and A*. Let C be the point where the home country's marginal rate of transformation is equal that of A*. Unless, the elasticity of substitution in consumption is unitary, the home country's autarky equilibrium will change with the introduction of uncertainty. Denoting the home country's equilibrium by $\bar{x}_2$, either

$$\alpha)\ \bar{x}_2 \in (x_2^b - \varepsilon, x_2^c)$$

or $$\beta)\ \bar{x}_2 \notin (x_2^b - \varepsilon, x_2^c),\ \text{i.e.},\ \bar{x}_2 \in (0, x_2^b - \varepsilon] \text{ or } \bar{x}_2 \in [x_2^c, T].$$

Actually the prevalence of one case over another depends on the relative intensity between the degree of difference of physical factor endowments and the degree of risk aversion. As the factor endowments of two countries become similar, the interval (x_2^b, x_2^c) will contract. in the limiting case of identical physical factor endowments, this interval degenerate to a point. This is the case for our model of section 2.

If the case ß) prevails, we can apply the argument used in the proof of Proposition 3 to establish the factor price divergence. But if α) is the case, it becomes ambiguous. Since that case corresponds the autarky equilibrium price relation in two countries such that $\bar{p}^* > p$ and $(w^*/r^*) > (w/r)$. Under free trade, $\bar{p}^*$ and (w^*/r^*) must fall while the opposite occurs at home. Thus, one of the following will happen:

i) $(w^*/r^*) > (\widehat{w^*/r^*}) > (\widehat{w/r}) > (\widehat{w/r})$

ii) $(w^*/r^*) > (\widehat{w^*/r^*}) = (\widehat{w/r}) > (\widehat{w/r})$

iii) $(w^*/r^*) > (\widehat{w/r}) > (\widehat{w^*/r^*}) > (\widehat{w/r})$

iv) $(\widehat{w/r}) > (w^*/r^*) > (w/r) > (\widehat{w^*/r^*})$

i)-iii) correspond to the case of factor price equalization and iv) is a case of factor price divergence.

6.2 In this section we have considered a general Heckscher-Ohlin model of uncertainty in which not only the endowment of physical factors but also the endowment of randomness across countries is different. Identical randomness across countries of Batra and Helpmand and Razin is one extreme case and the identical physical factor endowments is the other.

In the identical randomness model, the factor price equalization theorem holds and in the identical physical factor model factor price divergence occurs. And what happens in the middle depends on the closeness to these extremes. If the physical factor endowments in two countries are markedly different and the risk aversion is not very strong, then there will be a tendency toward factor price equalization under free commodity trade and, conversely, if the difference in physical factor endowment is insignificant and the risk aversion is strong, there will be factor price divergence.

VII. Conclusion

A general equilibrium model with technological uncertainty has been analysed to see the effect of the difference in randomness across countries. Our discussion has established that a) factor prices diverge in a free trade regime and b) commodity prices diverge under complete factor mobility.

When our model is combined with the standard Heckscher-Ohlin model, it is clear that there is conflict between the effects of different endowments of randomness and the effects of different endowments of physical factors, so that the relative force of these opposing effects will determine the net effect.

Appendix

Proof that $dp(\theta)/dk > 0$ if any of the conditions a) and b) of section 5.2 holds.

$$\text{numerator} = (\theta F' - \alpha)\ (\partial F/\partial K)\ EV_{YY}\theta\,(\theta F' + p) + (\theta F' - \alpha)\ \frac{\partial(F')}{\partial K}\ EV_Y\theta$$
$$-\theta\,(\partial F/\partial K) EV_{YY}\,(\theta F + p)^2 - \theta\,(\partial F/\partial K)\, F'' EV_Y\theta$$

since the sign of first term is ambiguous while the others are all positive, we will show that the first and the third term together must be positive under the specified conditions.

$$\begin{aligned} 1st+3rd &= (\partial F/\partial K)\ [(\theta F' - \alpha) EV_{YY}\theta\,(\theta F' + p) - \theta EV_{YY}(\theta F' + p)^2\,] \\ &= (\partial F/\partial K)\ [-\alpha EV_{YY}\theta\,(\theta F' + p) - \theta EV_{YY}p\,(\theta F' + p)\,] \\ &= -(\partial F/\partial K)\,(\theta/x_2)\ [F(x_2) EV_{YY}\theta\,(\theta F' + p) + x_2 EV_{YY}p(\theta F' + p)\,] \\ &= -(\partial F/\partial K)\,(\theta/x_2) EV_{YY}Y(\theta F' + p) \end{aligned}$$

Now we show that $EV_{YYY}(0F' + p)$ is negative if the relative risk aversion is non-decreasing and the elasticity of substitution in consumption is smaller than one. The proof is an adaptation of the technique elaborated by Sandmo (1971, p. 68).

Let $\hat{\theta}$ be the value of θ such that $-F(x_2) = p(\theta)/\theta$.

Then, $\theta F' + p(\theta) > (<) \theta$ as $\theta > (<) \theta$, because the inelastic substitution in consumption implies greater than proportionate change of p relative to that of θ. Let $\hat{Y}$ be the corresponding θ. Then, $Y \gtrless \hat{Y}$ as $\theta \lesseqgtr \hat{\theta}$. By non-decreasing relative risk aversion

$$R(Y) \geq R(\hat{Y}) \quad \text{for } \theta > \hat{\theta}.$$

Multiplying both side by $(\theta F' + p)$, we get

$$R(Y)\ (\theta F' + p) \geq R(Y)\ (\theta F' + p) \quad \text{for } \theta > \hat{\theta}.$$

However, this holds for all θ, because for $\theta < \hat{\theta}$, $Y < \hat{Y}$, so that $R(Y) \leq R(\hat{Y})$. Then multiplying both sides with negative expression $(\theta F' + p)$ will invert the inequality. Thus

$$R(Y)(\theta F' + p) \geq R(Y)(\theta F' + p) \quad \text{for all } \theta.$$

Rewriting R explicitly,

$$YV_{YY}(\theta F' + p) \leq -R(Y)\, V_Y(\theta F' + p) \quad \text{for all } \theta.$$

Applying the expectation operator to both sides and noing that R(Y) is given number, we obtain

$$EV_{YY}(\theta F' + p) \leq -R(Y)\, EV_Y(\theta F' + p) = 0$$

Hence, $EYV_{YY}(\theta F' + p) \leq 0$.

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