

BOND PORTFOLIO IMMUNIZATION WITH IMPERFECT CORRELATION OF FORWARD RATES IN STABLE PARETIAN MARKETS

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I. INTRODUCTION

The interest-rate risk immunization is a complete interest-rate risk hedging strategy to keep constant the net worth of a bond portfolio. An(1989) develops risk-minimization model for immunization, treating all forward rates as the state variables and allowing the imperfect correlation of forward rates across maturities. Furthermore, he shows that the risk minimization model is better than the other duration models based on the duration concepts in terms of the management of interest rate risk for bond portfolio.¹ In that paper he studies the immunization in the Gaussian markets which all forward rates are assumed to follow a multivariate normal distribution.

It is well known that when the joint probability distribution of security returns follow a multivariate stable distribution with characteristic exponent α , $0 < \alpha \leq 2$, portfolio returns follow a univariate stable distribution with the same characteristic exponent α . In addition, the distribution of portfolio returns belong to a two parameter(location, scale) family of distributions for symmetric nonnormal multivariate stable distributions as defined by Press(1982) and a linear dependence structure with nonnormal multivariate stable distributions as defined by Fama (1963) or Samuelson(1967). For these distributions, mean is the appropriate return

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¹Macaulay (1938) defines the duration as a weighted average of dates of the coupon and final payment of the bond, where the weight is the array of actual discount function. Fisher and Weil (1971) use the duration for the bond portfolio immunization. Khang (1979) and Cox, Ingersoll, and Ross (1979) develop one-factor model based on their own duration measure. Ingersoll (1983), Brennan and Schwartz (1983), and Nelson and Schaeffer (1983) approach the immunization in terms of two-factor duration model.

measure while scale parameter is the appropriate risk measure. Thus for an arbitrary value of α , $1 < \alpha \leq 2$, a portfolio can be obtained using mathematical programming techniques that are very similar, though not identical to ones for the case of Gaussian distributions. The scale parameter of the portfolio replaces the standard deviation used in the case of Gaussian distribution and the underlying optimizing problem being solved is a parametric convex programming problem.

Since Mandelbrot's study (1963), many financial economist have applied the stable distributions for asset price behavior to many financial economics areas (Fama (1963, 1965), McCulloch (1985)).² However, the Paretian stable distribution has been neglected in the immunization literature. If the underlying distribution in the interest-rate market is not a Gaussian but stable, it would be desirable that the immunization is studied in Paretian stable markets.

The purpose of this paper is to extend the risk minimization model for immunization to the Paretian stable markets and to examine whether it is desirable that the immunization strategy is studied in the stable markets. The remainder of the paper is organized as follows. In section I, the nomenclature is established. In section II, risk-minimization model is developed in stable markets. In section III, the correlation coefficients for forward rates are estimated with an Adaptive Conditional Heteroskedasticity (ACH) in an effort to estimate the scale-coscale matrix of the bond returns. In section IV, the performance of stable model is compared to that of Gaussian model. Finally, in section V, conclusions are drawn.

II. NOMENCLATURE

Let $\delta(t, m)$ denote the price at time t of a pure discount bond maturing at time $s = t + m$ with unit value:³

$$(1) \delta(s, s) = 1.$$

Let $\eta(t, m)$ denote the continuously compounded yield to maturity at time t on a zero-coupon bond with maturity m . The price of a pure discount bond with maturity date $s = t + m$ at time t is:

$$(2) \delta(t, m) = \exp [-m \cdot \eta(t, m)]$$

Let $\phi(t, s)$ denote the instantaneous forward rate on hypothetical point-payment forward loans to be contracted at time t , to begin at time $s = t + m$, and to be repaid

²Differently, Clark (1973) uses a subordinate stochastic process as a model for speculative price changes. He finds that a member of finite-variance distributions subordinate to the normal distribution, lognormal-normal, fits cotton prices better than stable distributions, using Kolmogorov-Smirnov tests.

³A pure discount bond means a default-free and option-free discount bond.

instantaneously. The yield to maturity can be regarded as the average rate of the forward rates spanning the given period, so that $\eta(t, m)$ can be formulated, as follows:

$$(3) \eta(t, m) = \frac{1}{m} \int_t^{t+m} \phi(t, s) ds.$$

This generates the term structure of interest rates at time t as a function of m by letting m range from 0 to ∞ . Note that shifts in the term structure of the interest rates are governed by movements in the forward rates at different maturities.

III. RISK MINIMIZATION MODEL

1. Scale and Coscale Matrix

In a discrete framework, the one-month holding-period rate of return from t to $t + \Delta t$, where $\Delta t = 1$ month, on a default-free pure discount bond of maturity m at time t is given by dividing the log-price relative by the length of the holding period because it can be sold for $\delta(t + \Delta t, m - \Delta t)$ after one month:

$$(4) R(t, m, \Delta t) = \frac{1}{\Delta t} \log \frac{\delta(t + \Delta t, m - \Delta t)}{\delta(t, m)}.$$

Let Z_i denote the holding-period rate of return over Δt on the zero-coupon bond with maturity m_i . Z_i is composed of a short-term interest rate and changes in forward rates:

$$(5) Z_i = \eta(t, \Delta t) - \frac{1}{\Delta t} \int_{t+\Delta t}^{t+m_i} [\delta(t + \Delta t, s) - \phi(t, s)] ds$$

If we evaluate the integral in (10) by summing trapezoids of width Δt , the rate of return on a zero coupon bond becomes:⁴

$$(6) Z_i \approx \eta(t, \Delta t) - \frac{1}{\Delta t} \sum_{k=1}^J \Delta \delta(k) \\ = \eta(t, \Delta t) - \frac{1}{\Delta t} V_j' \Delta \Phi'$$

⁴This holds for short but discrete time intervals, or for stable distributions. In the Ito process, Z has the drift term which depends on the variance-covariance structure of the forward rates. See Brennan and Schwartz (1983) and Nelson and Schaeffer (1983) for the model of the rate of return in the Ito process.

where $V'_j = [\frac{1}{2}, 1, \dots, 1, \frac{1}{2}, 0, \dots, 0]$ and $\Delta\Phi' = [\Delta\phi(1), \dots, \Delta\phi(j)]$ is a vector of changes in forward rates. V'_j is a row vector, which consists of $\frac{1}{2}$ in the first and j th elements, unity between the second element and the $j-1$ th element, and zero from the $j+1$ to the m th elements.

Let A_j denote the rate of return on the j th asset which is regarded as a coupon bond. Let a_{ij} represent a portion of the present value of asset j that comes from its coupon payments on maturity i :

$$(7) \quad a_{ij} = \frac{c_{ij} \delta(t, i)}{\sum_{i=1}^m c_{ij} \delta(t, i)}$$

where c_{ij} is the coupon payment on asset j at maturity m_i . Since a coupon bond is simply a portfolio of zero coupon bonds, the rate of return on the asset A_j is expressed by a linear combination of rates of return on zero coupon bonds, as follows:⁵

$$(8) \quad A_j = \sum_{i=1}^m a_{ij} Z_i = a_j \cdot Z,$$

where $Z' = [Z_1, \dots, Z_m]$ is a vector of rates of return on zero coupon bonds and $a_j = [a_{1j}, a_{2j}, \dots, a_{mj}]$ is a vector of the portion of the present value of asset j that derives from its payments. The return on assets depends upon the changes in the forward rates at different maturities.

Scale-coscale matrix is defined as an array of the square scales and coscales, as defined by Bawa, Elton and Gruber (1979). We may let Σ^s denote the scale-coscale matrix of forward rates.

$$(9) \quad \Sigma^s = (C_{ij}),$$

where $C_{ij} = C_i^2$ is the square scale of the i th forward rate, and $C_{ij} = p_{ij} C_i C_j$ is the coscale between the i th and j th forward rates.

Using (6), we get the scale-coscale matrix of zero-coupon bond, as follows:

$$(10) \quad \begin{aligned} \Omega^s &= (\omega_{ij}^s)/(\Delta t)^2 \\ &= V' \Sigma^s V/(\Delta t)^2, \end{aligned}$$

where Ω^s is scale-coscale matrix of the zero-coupon returns, ω_{ij}^2 is $C^\alpha(Z_i)$ $= (V'_i \Sigma^s V_i)^{\alpha/2}$, and $\omega_{ij}^s = C^\alpha(Z_i, Z_j) = (V'_j \Sigma^s V_j)^{\alpha/2}$.

⁵This also holds for short but discrete time intervals, or for a stable distribution.

Also, the scale-coscale matrix of asset returns are constructed, as follows:

$$(11a) \quad S^s = (S_{ij}^s) \\ = A' [V' \Sigma^s V] A \\ = B' \Sigma^s B.$$

where S^s is the scale-coscale matrix of asset returns,

$$(11b) \quad S_{jj}^s = C^\alpha (A_j) = (a_j' \Omega^s a_j)^{\alpha/2}, \text{ and}$$

$$(11c) \quad S_{ij}^s = C^\alpha (A_i A_j) = (a_i' \Omega^s a_j)^{\alpha/2}.$$

(11b) measures the interest rate volatility of the assets in the stable market.

2. Specification

In establishing risk-minimization model for immunization we focus on a small self-financing project. We do not consider the outside capital, and do not allow short selling. We define an investor, perhaps a commercial bank, with a single-payment liability at a future date. We assume that the bank wishes to purchase n different assets (coupon bonds) with a value equal to the liability so as to immunize the net present value of its position against interest-rate risk. The composite portfolio consists of one zero-coupon bond representing the liability, and n different coupon bonds on the asset side.⁶ Let the liability be the 0th asset and A^* denote the $n+1$ vector (A_0, A_1, \dots, A_n) of individual random returns. We let X_i denote the proportion of the i th asset. Since A_0 represents the liability, $X_0 = -1$. The portfolio proportions of assets on the asset side are restricted to

nonnegative, because short selling is not allowed, so that $\sum_{i=1}^n X_i = 1$ and $X_i \geq 0$,

$i = 1, 2, \dots, n$. The mixed portfolio then satisfies: $\sum_{i=1}^n X_i = 0$, $X_0 = -1$, $X_i \geq 0$, $i = 1,$

$2, \dots, n$. The return on the mixed portfolio becomes:

$$(12) \quad R = X^{*'} A^* = X^{*'} E_R + v \\ E(v | E_R) = 0, \text{ and} \\ X^{*'} U = 0,$$

where $X^{*'} = (-1, X_1, \dots, X_n)$; $X' = (X_1, X_2, \dots, X_n)$, $A^* = (A_0, A_1, \dots, A_n)$, E_R is a vector of expected returns, v is the unexpected return, and U is a unit vector.

The scale of the portfolio is given by (11), as follows:

⁶This can be extended to the case of a coupon bond in the liability.

$$(13) C_R^\alpha = (X^{*'} S^s X^*)^{\alpha/2}$$

Our risk minimization problem becomes:

$$(14) \min_X C_R = (X^{*'} S^s X^*)^{1/2}$$

$$\text{s.t. } X^{*'} U = 0, X > 0.$$

Necessary and sufficient conditions for the minimum-risk portfolio are provided by the Kuhn-Tucker conditions since C_R^α is a convex function of X .⁷ In the stable market, C_R is minimized when $(X^{*'} S^s X^*)$ in (14) is minimized.⁸

IV. ESTIMATING CORRELATION COEFFICIENTS FOR FORWARD RATES

1. Specification of Correlation Coefficients for Forward Rates

In our model, the scale-coscale matrix of the return on assets, as shown in (11), can be derived from the correlations of forward rates of different maturities. To estimate the coscale structure, we must first estimate the correlation coefficients for forward rates.

The correlation coefficients for forward rates may be estimated by the linear projection of the residuals of the i th-maturity forward rate on those of the change in the j th-maturity forward rate.

The changes in the forward rate are:

$$(15) \Delta \phi_{it} = d_i + \varepsilon_{it}, i = 1, 2, \dots, m.$$

where E_{it} is the forward-rate shock and d_i is the mean of the changes in forward rates. A nonzero d_i is generated by the term premium. Under uncertainty, each forward rate incorporates a term premium, $\phi(t, s) = E_t \phi(s, s) + \pi(t, m)$, $m = s - t$, where E_t denotes the expected value as of time t and $\pi(t, m)$ is the term premium.⁹ A linear regression of the residuals in (15) on one another is:

$$(16) \hat{\varepsilon}_{it} = b_{ij} \hat{\varepsilon}_{jt} + \varepsilon_{ijt}, i \neq j = 1, 2, \dots, m,$$

⁷See Ziemba (1974) for the proof of it.

⁸See Samuelson (1967) for the solution of the similar problem for a Paretian stable distribution. His problem is the risk minimization of a portfolio without allowing short-selling.

⁹McCulloch (1975a) finds the constant positive term premium using the U.S. Treasury security data ranging from June 1946 to August 1970. However, Kane (1983) rejects the constant term premium using survey data. Fama (1984) finds that the term premium is time-varying by regressing the excess return on the forward-spot rate differential. There are other numerous papers on this subject.

where $\hat{\varepsilon}_{it}$ are the residuals of the i th rate, b_{ij} is the regression coefficient, and ε_{ijt} is the disturbance term. The forward-rate shock ε_{it} is assumed to have a stable distribution with zero mean and a constant scale. Because the regression coefficient $b_{ij} = \rho_{ij}C_i/C_j$, ρ_{ij} is the correlation coefficient between the i th and the j th maturity forward rates, C_i is the standard scale of the i th maturity forward rate, and C_j is the standard scale of the j th maturity forward rate, we may derive estimates of correlation coefficients as:

$$(17) \rho_{ij} = b_{ij} \frac{C_j}{C_i}.$$

The correlation coefficient in a stable market is calculated as the product of the regression coefficient and the ratio of the standard scales of two forward rates. The coscale of forward rates is constructed as the product of the correlation coefficient and the geometric means of the respective forward-rate scales.

$$(18) C_{ij} = \rho_{ij} C_i C_j.$$

where C_{ij} is the coscale between the i th and the j th maturity forward rates.

2. The Data

To estimate the correlation coefficients for forward rates, term structure data on United States Treasury Securities are employed. The data, ranging from December 1946 to July 1983, are derived by applying a Cubic Spline term-structure fitting technique to average monthly bid and asked prices for the U.S. Treasury Securities.¹⁰

The Federal Reserve System intentionally stabilized interest rates on United States Treasury securities before the Accord of March 1951 so that the period is not representative of interest-rate volatility. We, therefore, discard the pre-Accord data, and use only the post-Accord period data beginning from March 1951 to July 1983, totalling 389 observations. Each calculated term structure contains a maximum of 59 forward rates and a minimum of 32 rates at pre-determined round-figure maturities (i.e., 0 month, 1 month, 2 month, ..., 18 month, 21 month, 24 month, 30 month, 3 year, 4 year, etc.). Unobserved maturity rates at each month are dealt with as missing observations. We analyze 1-month through 25-year rates since observations are not frequently made for more than 25 years.

Note that if the term premium incorporated in the forward rate is time-varying, then the forward-rate shock ε_{it} may be affected by a pure forecasting error, a

¹⁰See McCulloch (1975b) for more information about his term structure data.

time-varying term premium, or errors in measuring the forward rates.¹¹ Because of the time-varying term premium and the measurement errors, the shocks in forward rates may be serially correlated although the pure forecasting errors should be serially uncorrelated. $\Delta\pi(t, i)$ and $\Delta\pi(t + \Delta t, i)$, where $\Delta\pi(t, i)$ is the change in the term premium incorporated in the i th forward rate at time t , are both dependent on the term structure for time $t + \Delta t$. Also, the measurement errors of change in forward rates in the adjacent term structure will not be independent. Accordingly, the observed changes in forward rates may be serially correlated. This serial correlation gives us a negative bias on the variance (σ_i) and biases the regression coefficients (b_{ij}). Consequently, it may not only distort the correlation coefficients (ρ_{ij}), but also give us an erroneously small confidence interval. To avoid the serial correlation problem, we use alternate observations instead of using all observations. As a result, we have at most 194 observations available.¹²

3. Estimates of Correlation Coefficients

To estimate (15), we need changes in forward rates, $\Delta\phi(t)$. For the rates with maturities of more than 18 months, we use the interpolated value in calculating the changes. because each maturity becomes $m-1$ month after 1 month, but the data do not provide the rate for such maturities at the following month.

The system (15) is estimated equation by equation, based on maximum-likelihood estimation. We use symmetric-stable maximum-likelihood linear regression (SMSTRG) program for this purpose.¹³ If the forecasting errors are not homoskedastic, the scale of the forward rates are biased and the residuals are not efficient. In consequence, the calculated correlation coefficients based on equation (17) may be biased and may not be efficient estimates. Therefore, we should investigate whether or not the forward rates shocks are homoskedastic, and make corrections if heteroskedasticity is detected. An (1989) rejects homoskedasticity in favor of heteroskedasticity for the forecasting errors, and finds that Adaptive Conditional Heteroskedasticity (ACH) is the preferable specification for interest rates over the proportional heteroskedasticity (PH). Following his results, we adopt the ACH system for heteroskedasticity model in this study. To conduct the estima-

¹¹Since the term-structure data are derived from monthly bid and asked prices for the U.S. Treasury securities, they cannot be measured exactly and any estimator of a value derived from them contains measurement error.

¹²To identify the serial correlation problem, An (1989) applies an ARMA model to both the full observations and the alternate ones (odd observations). In the full-observation case, the shocks exhibit the AR(1) process. The coefficients of the autoregressive process are statistically significant at the 5% level except for the 6-month and 3-year forward rate shocks. In the odd-observation case, estimates of the AR(1) coefficients indicate serial correlation in only the 1-month forward rate. This is attributed to a large measurement error in the one-month forward rate difference, which employs the following month's O -maturity rate.

¹³See McCulloch (1979) for the discussion about SMSTRG program.

[Table 1] Correlation Coefficients ($\hat{\rho}$) for Forward Rates across Maturities in Stable Model with ACH($\alpha = 1.553$)

$m_i \backslash m_j = m_i + \Delta m$	Δm														
	1yr	2yr	3yr	4yr	5yr	6yr	7yr	8yr	9yr	10yr	11yr	12yr	13yr	14yr	15yr
1 yr	0.61	0.60	0.61	0.45	0.32	0.25	0.22	0.18	0.16	0.17	0.14	0.11	0.11	0.11	0.14
2 yr	0.93	0.41	0.30	0.23	0.19	0.17	0.17	0.20	0.22	0.19	0.18	0.17	0.16	0.13	0.10
3 yr	0.47	0.34	0.27	0.22	0.20	0.19	0.21	0.21	0.20	0.19	0.18	0.18	0.17	0.14	0.12
4 yr	0.86	0.62	0.51	0.41	0.27	0.21	0.16	0.10	0.05	0.01	0.01	0.02	0.01	-0.01	0.01
5 yr	0.92	0.80	0.56	0.39	0.31	0.22	0.15	0.08	0.04	-0.01	-0.06	-0.06	-0.11	-0.11	-0.04
6 yr	0.94	0.81	0.64	0.51	0.37	0.23	0.13	0.06	-0.03	-0.06	-0.09	-0.17	-0.19	-0.12	
7 yr	0.94	0.83	0.58	0.49	0.30	0.20	0.11	0.00	-0.04	-0.09	-0.18	-0.23	-0.17		
8 yr	0.95	0.83	0.61	0.41	0.32	0.23	0.09	0.03	-0.03	-0.14	-0.18	-0.18			
9 yr	0.95	0.84	0.65	0.60	0.48	0.27	0.18	0.08	-0.05	-0.11	-0.18				
10yr	0.96	0.86	0.82	0.68	0.46	0.33	0.21	0.08	-0.02	-0.15					
11yr	0.95	0.91	0.76	0.58	0.44	0.30	0.18	0.06	-0.08						
12yr	0.97	0.88	0.76	0.59	0.43	0.32	0.19	0.00							
13yr	0.96	0.90	0.75	0.64	0.50	0.33	0.12								
14yr	0.98	0.93	0.80	0.68	0.48	0.30									
15yr	0.97	0.93	0.82	0.59	0.41										

Note: When $\Delta m = 0, \rho_{ij} = 1$

tion, we should have a common α value across the equations. The Common characteristic exponent α , is restricted to be 1.553¹⁴.

The estimates of correlation coefficients for forward rates are reported in Table 1. The Table is reported in terms of m_i and $m_j = m_i + \Delta m$, $\Delta m = 1$ year, ..., 20 year.¹⁵ When $\Delta m = 0, \hat{\rho}_{ij} = 1$.

The results exhibit that the forward-rate movements are by no means perfectly correlated. The correlation coefficients smoothly decrease with maturity differences. Close-by forward rates are nearly perfectly correlated. The forward rates prove to be strongly correlated for near maturities and weakly correlated for distant maturities. Some forward rates are even negatively correlated.

¹⁴The value of $\alpha = 1.553$ stems from the following estimation procedure. First, we estimate the charateristic exponent, α , of each equation and then calculate 95-percent confidence intervals for α . Next, we find the intersection of these confidence intervals and take the mean of the lower bound and upper bound of the intersected confidence interval across equations as the common charateristic exponent.

¹⁵We put more than 21 year rates out of the analysis because we could not have the common characteristic exponent, α , on those rates. Moreover, the reported format of Table 1 is not a usual one. The reason why we use this format is that we intend to inverstigate whether or not there exists the consensus correlation coefficient in terms of the maturity difference. Unfortunately, Table 1 does not show the consensus correlation coefficients.

[Table 2] Test for Normality of Forward Rates using the Studentized Range

m	obs.	studentized	log likelihood		2Δlog L
		range	normal	stable(α = 1.553)	
1 mo	194	8.50	-191.561	-169.036	45.050
2 mo	194	7.86	-137.142	-121.935	30.414
3 mo	194	10.04	-154.586	-130.492	48.188
4 mo	194	10.25	-156.996	-126.864	60.264
5 mo	194	8.26	-149.246	-133.053	32.386
6 mo	194	7.66	-160.711	-135.950	49.522
7 mo	194	7.43	-169.246	-140.114	58.264
8 mo	194	6.78	-168.248	-144.211	48.074
9 mo	194	7.05	-160.722	-143.955	33.534
10mo	194	6.59	-150.094	-138.779	22.630
11mo	194	6.23	-138.270	-131.614	13.312
1 yr	194	7.00	-131.440	-124.093	14.694
2 yr	194	9.34	-130.755	-87.963	85.584
3 yr	194	9.32	-114.892	-71.019	87.746
4 yr	194	7.91	-47.945	-44.066	7.758
5 yr	194	11.82	-50.065	-17.370	65.390
6 yr	194	9.48	-30.625	-11.070	39.110
7 yr	194	9.96	-20.574	-8.731	23.686
8 yr	194	9.85	-29.692	-5.711	47.962
9 yr	194	9.58	-26.008	-0.004	52.008
10yr	194	9.66	-14.666	4.714	38.760
11yr	194	8.88	-7.767	7.406	30.346
12yr	194	8.42	-10.283	-0.159	20.248
13yr	194	8.36	-22.646	-16.147	12.998
14yr	188	8.06	-25.411	-12.435	25.952
15yr	178	8.97	-36.759	-13.455	46.608
16yr	177	10.32	-35.494	-17.532	35.924
17yr	177	9.92	-28.466	-18.540	19.852
18yr	177	8.55	-17.572	-3.475	28.194
19yr	177	7.58	-27.198	-21.739	10.918
20yr	172	9.91	-41.849	-30.679	22.340

* These are forecasting errors with ACH under the normality assumption.

Confidence Interval for Normality, 194 observations**

(Two-tailed Test):

.90 4.76 to 6.13

.95 4.65 to 6.57

.99 4.48 to 7.01

** Interpolated from H. A. David, H.O. Hartley, and E.S. Pearson(1954).

4. Specification Test for Distribution

Since this study is motivated by the postulation that the interest rates would follow stable distribution, not Gaussian, we might check first whether the for-

ward rates follow the normal distribution. Even if the normal distribution is rejected by a certain test procedure, we could not replace at once the Gaussian distribution with the Paretian stable distribution. It should be decided after testing whether which is more preferred distribution between normal and the Paretian stable distribution.

To test whether the interest rates have the Gaussian distribution, we use the "studentized range" statistic. The studentized range is defined as the ratio of the sample range to the standard deviation.¹⁶ Table 2 shows the 'studentized range' statistic for forward-rate shocks with ACH. Normality is rejected for 10-month and 1-year forward-rates at five percent significance level and for 11-month forward rate at ten percent level. Except for these rates, normality is rejected for all maturities at the one percent level. This result suggests that the normality assumption is a misspecification.

To establish a preferred distribution for the forward rates, we perform a likelihood-ratio specification test between Gaussian and stable distributions. We set the null hypotheses that the forward rates have Gaussian distribution and the alternative that the forward rates have the Paretian stable distribution. The null hypothesis is rejected at the 1-percent significance level for all maturities. The results are in favor of a stable distribution in the specification for the forward rates, and thus in the specification for immunization model. However, the significance of the difference for the immunization models cannot be assessed by such statistics. It must be evaluated in the context of specific portfolio performances. To evaluate the models we compare their immunizing performances in the next section.

V. COMPARISON OF IMMUNIZATION STRATEGIES

To test the immunization model we adopt the approach, employed by Brennan and Schwartz (1983) and Nelson and Schaeffer (1983), which is to investigate the one-month (or six month) holding period return on the portfolio. It is designed to avoid the serial correlation problem.¹⁷ Recognizing that estimated data is inevitably smoother than actual bond prices so that the smoothly fitted data may bias the true results, we use actual bond price data in the CRSP tape.¹⁸ We test

¹⁶See David, Hartley, and Pearson (1954).

¹⁷There is another approach which is to replicate the single payment of a certain maturity (5-, 10-, or, 20-year maturity)(see Fisher and Weil (1971) and Ingersoll (1983)). At date zero, a portfolio immunizing a target bond is constructed, and the portfolio is rebalanced on the certain maturity cycle. The difference between the returns on the immunizing portfolio and the target bond is observed at the end of the particular period. This approach has the drawback that the returns of the portfolio are serially correlated due to those use of overlapping sample. These serial correlations bias the standard deviations (or RMSE). The resulting statistics may draw erroneous conclusions in tests of model performance.

¹⁸Ingersoll (1983) uses estimated (index) data to test immunization strategies. He shows that the two-

the immunization models both for estimation period (March 1951-July 1983) and for outside-estimation period (August 1983-June 1986).

1. Empirical Test in Estimation Period

We assume that the matrix of the correlation coefficients of forward rates estimated in section III is stationary over time. First, we construct the scale-coscale matrix of the bond returns, using the estimated correlation coefficients for forward rates.¹⁹ Next, we select an immunized portfolio on the basis of the risk-

[Table 3a] The Results of the Alternative Immunization Strategies for 5-year Target Bond Using CRSP Data in the Estimation Period (1951/3-1983/7)

	Duration Strategies			Risk Minimization	
	Simple	One-Factor	Two-Factor	Gaussian	Stable
Portfolio A (3-year and 8-year Bonds: two-factor model adds 10-year bond)					
AR(1)	-0.170 (0.127)	-0.182 (0.126)	-1.816 (1.372)	-1.145 (0.128)	-0.149 (0.128)
Mean	-0.917 (0.799)	-0.976 (0.816)	-1.560 (0.838)	-0.776 (0.766)	-0.797 (0.771)
STD	6.396	6.524	6.706	6.126	6.165
RMSE	6.461	6.597	6.885	6.175	6.216
Portfolio B(3-year and 10-year Bonds: two-factor model adds 15-year bond)					
AR(1)	-0.312 (0.132)	-0.305 (0.133)	-1.091 (0.138)	-0.290 (0.127)	-0.287 (0.126)
Mean	-0.742 (0.821)	-0.818 (0.834)	-2.172 (0.809)	-0.092 (0.806)	-0.103 (0.805)
STD	6.657	6.674	6.471	6.446	6.440
RMSE	6.609	6.724	6.826	6.446	6.441
Portfolio C(3-year and 15-year Bonds: two-factor model adds 10-year bond)					
AR(1)	-0.273 (0.127)	-0.253 (0.127)	-0.091 (0.138)	-0.290 (0.123)	-0.287 (0.123)
Mean	-0.836 (0.686)	-0.968 (0.685)	-2.172 (0.809)	-0.019 (0.424)	0.062 (0.818)
STD	6.485	6.481	6.471	6.471	6.547
RMSE	6.548	6.566	6.826	6.471	6.547

'Mean' is the average of the difference of the one-month holding period rates of return between the asset and the liability. 'STD' is the standard deviation of the return difference. 'RMSE' is the root mean square error. The values in parentheses are the asymptotic standard errors.

factor model is substantially better than the duration model with estimated data. On the other hand, Brennan and Schwartz (1983) and Nelson and Schaeffer (1983) achieve inconclusive results on the two-factor model with actual bond price data.

¹⁹Following the same procedure with the stable case, we construct the variance-covariance matrix of asset returns in the Gaussian case. See An(1989) for the detailed estimation procedure of the Gaussian case.

[Table 3b] The Results of the Alternative Immunization Strategies for 10-year Target Bond Using CRSP Data in the Estimation Period (1951/3-1983/7)

	Duration Strategies			Risk Minimization	
	Simple	One-Factor	Two-Factor	Gaussian	Stable
Portfolio D(8-year and 15-year Bonds: two-factor model has no solution)					
AR(1)	-0.252 (0.125)	-0.261 (0.125)		-0.172 (0.129)	-0.194 (0.129)
Mean	-0.479 (2.089)	-0.486 (2.152)		-0.478 (1.842)	-0.383 (1.865)
STD	16.712	17.213		14.734	14.924
RMSE	16.719	17.220		14.742	14.929
Portfolio E(5-year and 15-year Bonds: two-factor model has no solution)					
AR(1)	-0.019 (0.131)	-0.018 (0.131)		-0.046 (0.134)	-0.054 (0.134)
Mean	-0.075 (1.805)	-0.174 (1.818)		0.426 (1.759)	0.665 (1.745)
STD	14.440	14.544		14.074	13.961
RMSE	14.440	14.545		14.080	13.977
Portfolio F(3-year and 15-year Bonds: two-factor model has no solution)					
AR(1)	-0.002 (0.130)	-0.003 (0.130)		-0.040 (0.132)	-0.043 (0.132)
Mean	-0.319 (1.839)	-0.443 (1.857)		-0.835 (1.773)	1.028 (1.776)
STD	14.714	14.859		14.182	14.209
RMSE	14.718	14.866		14.207	14.246

'Mean' is the average of the difference of the one-month holding period rates of return between the asset and the liability. 'STD' is the standard deviation of the return difference. 'RMSE' is the root mean square error. The values in parentheses are the asymptotic standard errors.

minimization model demonstrated in section II. Furthermore, we assume that 3-year, 5-year, 8-year 10-year, and 15-year bonds are available to be traded.

We consider two portfolio cases. One is a 5-year target case for which the liability consists of a 5-year coupon bond. The other is a 10-year target case for which the liability consists of a 10-year coupon bond. For each case we consider every possible immunizing portfolio among the available assets. For 5-year target, an immunizing portfolio, called Portfolio A, contains component bonds with maturities of 3 and 8 years. Another immunizing portfolio, called 'Portfolio B', contains 3-year and 10-year bonds, and the other portfolio, 'Portfolio C', contains 3-year and 15-year bonds. For 10-year target, the immunizing portfolio which is labelled as 'Portfolio D', contains component bonds with maturities of 8 and 15 years. Another immunizing portfolio, called 'Portfolio E', contains 5-year and 15-year bonds, and lastly, 'Portfolio F' contains 3-year and 15-year bonds. The test procedure is very similar to the Nelson and Schaeffer (1983) approach. We use the prices and cash flow data from the CRSP tape to estimate the term struc-

ture, and read directly the returns on the target bond and the coupon bonds from the tape. For each strategy, the one-month holding period rate of return on the liability is compared to that on the asset side.

The main results are contained in Tables 3a and 3b, which report the summary statistics of the models, including simple duration model, one-factor duration model, and two-factor duration model. These statistics are estimated by MLE. Results are given for the 30-year period of 1951-1983. Each table provides the mean of the difference between the one-month holding period returns on the asset and liability sides, standard deviations, and the RMSE of the return differences. One-month holding period returns presumably contain serial correlations because of the measurement errors and the time varying term premium if it exists. To investigate the serial correlation, we apply the ARMA model to the errors of the returns. However, the coefficients of an Ar(1) process are not significant at the 1-percent level. Therefore, the standard deviations and RMSE reported here are unbiased statistics so that they may be reliable.

Since the purpose of this paper is to evaluate the stable model over the Gaussian model, we will leave out the detailed comparison between risk minimization model and the other duration models (simple duration model, one-factor model, and two-factor model), and briefly introduce the main results of the earlier paper. The results of the earlier paper are as follows: 1) The risk-minimization model outperforms the other models in reducing interest-rate risk over both estimation period and outside estimation period. 2) Especially, when the term-structure shifts in a non-preserving-shape fashion, the risk-minimization model performs much better. 3) The simple duration model is the second best performer; the one-factor model is the third; the two-factor model is the fourth. 4) The simple duration model is still useful for bond portfolio immunization when the term structure shifts in a shape-preserving fashion.

Taking a broad perspective, there are no large differences between Gaussian and stable models, but the Gaussian model is slightly superior to the stable model. Only in 'portfolio B' and 'portfolio E', the stable model is marginally better than the Gaussian model. In the 5-year target case, the RMSEs in the Gaussian markets are smaller than those in the stable markets by 4.1 and 7.6 basis points in 'portfolio A' and 'portfolio C', respectively. The difference of RMSEs in 'portfolio B' between the stable and Gaussian models is only 0.5 basis points. In the 10-year target case, the RMSEs of the Gaussian model are smaller than those of the stable model by 18.7 and 3.9 basis points in 'portfolio D' and 'portfolio F', respectively. The difference of RMSEs in 'portfolio E' is only 10.3 basis points.

2. Empirical Tests Outside Estimation Period

To evaluate more clearly which one is better, it is necessary that the performance should be tested outside estimation period. The period of August 1983-June

1986 is considered. The test procedure is exactly same as is in the estimation period.

The results are presented in Tables 4a and 4b. As is true of the results of estimation period, the Gaussian model is slightly superior to the stable model in out-of-estimation period. For 5-year target, the RMSEs of the Gaussian model are lower than those of the stable model by 1.50 and 1.70 basis points in 'portfolio A-out', and 'portfolio B-out', respectively. In 'portfolio C-out', the stable model has the smaller RMSE by 4.8 basis points than the Gaussian model. For 10-year target case, the RMSEs of the Gaussian model are lower than those of the stable model by 11.9 and 2.2 basis points in 'portfolio D-out' and 'portfolio F-out', respectively. The stable model has the smaller RMSE by 5.3 basis points than the Gaussian model.

[Table 4a] The Results of the Alternative Immunization Strategies for 5-year Target Bond Using CRSP Data outside of the Estimation Period (1983/8-1986/6)

	Duration Strategies			Risk Minimization	
	Simple	One-Factor	Two-Factor	Gaussian	Stable
Portfolio A-out (3- and 8-year Bonds: two-factor model adds 10-year bond)					
AR(1)	-0.380 (0.165)	-0.372 (0.166)	-1.027 (0.178)	-0.364 (0.166)	-0.370 (0.166)
Mean	-0.487 (0.564)	-0.546 (0.578)	1.504 (0.986)	0.238 (0.536)	0.285 (0.538)
STD	3.241	3.321	5.662	3.079	3.090
RMSE	3.228	3.366	5.858	3.088	3.103
Portfolio B-out(3- and 10-year Bonds: two-factor adds 15-year bond)					
AR(1)	-0.186 (0.174)	-0.176 (0.174)	-0.203 (0.173)	-0.459 (0.157)	-0.456 (0.156)
Mean	-0.343 (0.963)	-0.345 (0.927)	-1.613 (1.021)	-0.156 (0.743)	-0.159 (0.746)
STD	5.533	5.609	5.864	4.266	4.283
RMSE	5.544	5.620	6.082	4.269	4.286
Portfolio C-out(3- and 15-year Bonds: two-factor adds 8-year bond)					
AR(1)	-0.078 (0.185)	-0.058 (0.186)	-0.203 (0.173)	-0.127 (0.179)	-0.132 (0.179)
Mean	-0.343 (0.885)	-0.345 (0.943)	-1.613 (1.021)	-0.983 (0.721)	-0.953 (0.719)
STD	5.083	5.416	5.864	4.174	4.132
RMSE	5.125	5.469	6.082	4.288	4.240

'Mean' is the average of the difference of the one-month holding period rates of return between the asset and the liability. 'STD' is the standard deviation of the return difference. 'RMSE' is the root mean square error. The values in parentheses are the asymptotic standard errors.

[Table 4b] The Results of the Alternative Immunization Strategies for 10-year Target Bond Using CRSP Data outside of the Estimation Period (1983/8-1986/6)

	Duration Strategies			Risk Minimization	
	Simple	One-Factor	Two-Factor	Gaussian	Stable
Portfolio D-out(8- and 15-year Bonds: two-factor model has no solution)					
AR(1)	-0.133 (0.173)	-0.126 (0.174)		-0.145 (0.176)	-0.140 (0.175)
Mean	-0.437 (0.899)	-0.400 (0.933)		0.824 (0.902)	0.831 (0.903)
STD	5.316	5.357		5.180	5.300
RMSE	5.334	5.372		5.245	5.364
Portfolio E-out (5- and 15-year Bonds: two-factor model has no solution)					
AR(1)	0.209 (0.173)	0.203 (0.173)		0.230 (0.172)	0.260 (0.171)
Mean	1.131 (1.790)	0.1112 (1.790)		0.214 (1.755)	0.279 (1.745)
STD	10.285	10.285		10.079	10.025
RMSE	10.286	10.286		10.082	10.029
Portfolio F-out(3- and 15-year Bonds: two-factor has no solution)					
AR(1)	0.141 (0.175)	0.139 (0.175)		0.134 (0.175)	0.147 (0.175)
Mean	0.264 (1.867)	0.248 (1.865)		0.279 (1.840)	0.349 (1.857)
STD	10.722	10.713		10.645	10.665
RMSE	10.725	10.716		10.649	10.671

'Mean' is the average of the difference of the one-month holding period rates of return between the asset and the liability. 'STD' is the standard deviation of the return difference. "RMSE" is the root mean square errors

VI. CONCLUSIONS

This paper investigates whether the stable model is more suitable to the study of the bond portfolio immunization than the Gaussian model. To conduct this, we first derived the scale-coscale matrix of forward rates, using the term structure of the interest rates, and estimated it with McCulloch's monthly term structure data of 1951:3-1983:7. Next, we compare the performance of the stable model to that of the Gaussian model for bond portfolio immunization. However, we did not find a solid evidence that the stable model is better than the Gaussian model for the immunization strategy, even though the specification test shows that the stable distribution is more desirable distribution for the forward rates movements. Rather, the Gaussian model is shown to be slightly superior to the stable model in terms of the implications for interest-rate risk management. Therefore, the stable model is not necessarily a better specification for the immunization.

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