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Effects of Mixed Oligopoly and Emission Taxes on the Market and Environment*

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The literature has mainly focused on the optimal emission tax rate to internalize the negative externalities of pollutants. Thus, we investigate firms' technology adoption behavior under an emission taxation system. We also examine the possibility of using a welfare-maximizing public firm with eco-friendly production technology to reduce environmental damage. Private firms can choose clean or normal technology for their production process. The analysis indicates several main results. First, under the emission taxation system, private firms adopt clean technologies even though they bear additional abatement costs in the production process if environmental damage caused by their production process is large. Second, mixed oligopoly is socially desirable when environmental damage is low. Thus, private firms take normal technology. Finally, when private firms adopt clean technology, mixed oligopoly is better than emission taxes if environmental damage is low and if the market is less competitive from a social welfare perspective.

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I. Introduction

Policy measures for environmental protection can be classified into two groups: command-and-control standards and market-based measures. Most economists endorse the latter, such as emission taxes, because they can provide dynamic incentives for the adoption and diffusion of cheap and superior contro1 technologies.[1] Emission taxes represent one market-based measure that can lessen environmental damage and make markets highly efficient.

Many studies consider emission taxes a form of environmental regulation that can control the behavior of firms that emit pollutants.2 Barret (1994) reveals that governments may have incentives to impose weak environmental standards on firms that compete in imperfectly competitive international markets. Conrad and Wang (1993) investigate the effects of emission taxes and abatement subsidies in a market with the endogenous entry of price-taking firms. Lahiri and Ono (2007) compare the effects of emission taxes and relative emission standard on welfare and pollution levels under oligopolistic market structures, and find that a relative emission standard is welfare-superior to an emission-equivalent tax when considering a fixed number of firms. Lee (2007) examines the effects of emission taxes levied on intermediate goods using the framework of vertically related markets with imperfect competition.

The abovementioned studies consider the imperfect competition market structure but do not account for the case of mixed oligopoly markets. However, we have witnessed mixed oligopolistic markets in a broad range of industries, such as oil, electricity, telecommunications, and power plants that emit pollutants during their respective production processes. Bárcena-Ruiz and Garzón (2006) reveal that emission taxes are lower in the mixed oligopoly than in the private oligopoly, even though environmental damage is greater when the government imposes an emission tax on firms to protect the environment. Naito and Ogawa (2009) combine a partial privatization model with a mixed oligopoly and compare environmental regulations in this market. Xu et al. (2016) compare a Cournot with a Bertrand duopoly in a differentiated mixed duopoly market and investigate the policy combination of optimal emission tax and the privatization of public firms.

Emission taxes are effective in some cases, but are not so easy to apply when solving every environmental problem. For example, when a government imposes taxes on emissions, which is a kind of environmental regulation, it must have full information on the demand and supply sides of the markets, particularly when the

[1] See Keohane *et al.* (1998).

[2] Many studies have analyzed the optimal emission tax under an oligopoly market, such as Ulph (1996), Yin (2003), Fujiwara (2009), Pal (2012), and Kim and Lee (2014). For example, Fujiwara (2009) insists that presence of free entry and product differentiation are significant for optimal environmental policies in the oligopoly market with product differentiation.

markets are under an imperfect competition. Effectively implementing environmental taxes necessitates some type of monitoring cost. Holland (2012) notes that emission taxes may not be the first-best measure for correcting environmental externalities in the presence of incomplete regulation. If the highest tax burden is already imposed on firms in the industry, the introduction of an environmental tax is undoubtedly met with strong tax resistance, which can cause the government to be reluctant in using environmental tax.

Many types of eco-friendly clean technology have been developed in recent years. Wind power and recycled energy are typical examples. These technologies are eco-friendly, but their adoption costs are greater than those for traditional technology. Private firms may not want to adopt eco-friendly technology in the production process if market circumstances have not changed.

The present study focuses on the possibility of using a public firm that utilizes an eco-friendly production technology and maximizes social welfare as an instrument in reducing pollution emissions in an oligopolistic market. We assume that the public firm produces good eco-friendly technology that emits fewer pollutants than normal technology. In our analysis, we call such eco-friendly technology "clean technology" and the public firm that employs it "ecological public firms." If the government can make such an ecological public firm enter the market, controlling private firms' pollution through market competition may be possible.

The purpose of this study is two-fold. First, unlike the previous literature that has mainly focused on the optimal emission tax rate to internalize negative externalities of pollutants, we investigate firms' technology adoption behavior under an emission taxation system, that is, if firms adopt clean technology when an emission tax is implemented. Second, we analyze the effects of mixed oligopoly with an ecological public firm and private firms on social welfare and environmental damage. We then assume that private firms can choose either clean or normal technology for production processes. The government cannot mandate that private firms choose clean technology. If the government hopes that firms convert, it must shift the market so that private firms choose clean technology voluntarily.

This study is organized as follows. Section 2 suggests the basic model that we use in the study. Section 3 considers an oligopoly market with private firms under an emission taxation system and investigates the possibility that a firm chooses clean technology. Section 4 considers the case when an ecological public firm enters the market and analyzes the behavior of the public firm and private firms. Section 5 compares social welfare under each case and investigates private firms' technology choices. Section 6 suggests policy implications based on results of our analysis. Finally, Section 7 presents the concluding remarks.

II. Model

The basic model starts with an oligopoly market by n private firms in which the output level of firm i is q_i. The inverse demand function is given by $P = a - Q$, where a is the demand scale in the economy, and Q is the total output level, $Q = \sum_{i=1}^{n} q_i$, $(n \geq 2)$. Each firm produces homogeneous goods and emits pollutants in the process of production. One unit of output emits one unit of pollutant. Firms have a choice in their production between producing goods using pollution abatement and those that do not. We identify the former as "clean technology" and the latter as "normal technology." Following Bárcena-Ruiz and Garzón (2006) and Pal (2012), we define cost function as

$$C(q_i, x_i) = \frac{c}{2} q_i^2 + \frac{\gamma}{2} x_i^2, \tag{1}$$

where x_i is the pollution abatement level of firm i when it adopts clean technology, and γ is the parameter of the abatement cost; in addition, pollution by firm emissions causes environmental damage.[3] Let the emission level of firm i be given by $e_i = \max\{q_i - x_i, 0\}$, we define environmental damage as

$$ED = \frac{k}{2}\left(\sum_{i=1}^{n} e_i\right)^2, \tag{2}$$

where k is the parameter for environmental damage caused by pollutant emissions. If firm i uses normal technology, then x_i becomes zero in Equations (1) and (2). Without regulations, private firms can only use clean technology when it is profitable. The government has difficulty forcing private firms to adopt clean technology. Thus, identifying a mechanism or market condition that would make private firms voluntarily choose clean technology is critical.

Emission taxation is a policy measure that internalizes negative externalities, such as pollution generated by firms' production activities. The previous literature on emission taxes has focused on the optimal tax rate for internalization. We are interested in exploring whether firms adopt clean technology when an optimal level of emission tax is levied. The timeline of the game is as follows. First, private firms choose collectively normal technology or clean technology for their production process. The primary interest in this study is not focused on the interaction among

[3] Although Bárcena-Ruiz and Garzón (2006) assume $k = 1$, we assume $c = \gamma = 1$ for simplicity because it plays a key role in our model. Qualitative results do not change if we consider $c = \gamma \neq 1$. Firms use only normal technology if γ is extremely higher than c; we ignore this case. The detailed calculations corresponding to this case are available from the author on request.

oligopolistic firms with respect to the adoption of environmental technology but on examining the effects of the market entry of an ecological public firm. Thus, we assume that private firms choose the same strategy with respect to technology adoption, which is the simplest formulation that captures the existence of environmental technology in the private firm. One interpretation for the collective activity of private firms in choosing technology is that private firms form joint ventures to develop and share environmental technology. Second, the government decides the level of emission tax that maximizes social welfare. Third, each firm chooses its output and abatement level simultaneously by taking its rivals' output and abatement levels as a given.[4]

Firms included in this study decide whether to adopt clean technology before the government's decision regarding tax rate. This may be perceived as abnormal in the sense that firms take action before the government.[5] However, the above setting can be justified by assuming that firms, with strong commitment power in adopting environmental technology, can signal the government that they are ready to reduce emissions by adopting clean technology.

III. Market Equilibrium under Private Oligopoly

Following the backward induction method, we first solve the firms' profit maximization problems and the government's policy on emission taxes under a private oligopoly that only consists of private firms. Private firms collectively determine in the first stage whether they want to adopt clean technology, and the two possible subgames are choosing clean technology or normal technology. From a profit maximization perspective, firms only adopt clean technology if it is more profitable than normal technology.

3.1. Normal Technology: Case N

If firm i chooses normal technology under emission tax t, $x_i = 0$ in Equation (1) and no pollution abatement costs are required in the production process. Therefore, the profit of firm i is given by

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[4] Many studies have compared the Cournot quantity competition with Bertrand price competition (e.g., Lee et al. (2017). However, we use quantity competition in this paper to focus on the technology choice of the firms.

[5] In the strategic trade policy literature, firms are assumed to choose a strategic variable first before the policy of the government (Brander and Spencer, 1987; Blonigen and Ohno, 1998; Konishi et al., 1999).

$$\pi_i[q_i, q_{-i} : t] = (a - Q)q_i - \frac{1}{2}q_i^2 - tq_i, \tag{3}$$

where Q demonstrates the total output, $Q = \sum_{i=1}^{n} q_i$. Solving the first-order condition of profit maximization gives

$$q_i = \frac{a-t}{2+n},\ \pi_i = \frac{3(a-t)^2}{2(2+n)^2},\ CS = \frac{n^2(a-t)^2}{2(2+n)^2}, T = \frac{tn(a-t)}{2+n},\ ED = \frac{kn^2(a-t)^2}{2(2+n)^2}, \tag{4}$$

where CS is the consumer surplus and T is the total tax revenue. Equilibrium output and profit are negatively related to the number of firms and tax rates. Social welfare (SW) is defined as the sum of consumer surplus, producer surplus, total tax revenue, and environmental damage, and is expressed as

$$SW = CS + \sum_{i=1}^{n} \pi_i + T - ED. \tag{5}$$

Solving the welfare maximization problem allows us to obtain optimal emission tax using the equation[6]

$$t^N = \frac{a(kn-1)}{1+n+kn}. \tag{6}$$

Emission tax is only meaningful when $t^N \geq 0$; thus, we assume $kn \geq 1$ to ensure $t^N \geq 0$ in Equation (6).[7] The optimal emission tax level is positively related to k. That is, the greater the extent of environmental damage is, the higher the optimal tax level becomes: $\partial t^N / \partial k > 0$. Substituting t^N into Equation (4) gives

$$\begin{aligned} &q_i^N[n,k] = \frac{a}{1+n+kn},\ \pi_i^N[n,k] = \frac{3a^2}{2(1+n+kn)^2}, \\ &CS^N[n,k] = \frac{a^2n^2}{2(1+n+kn)^2}, \\ &T^N[n,k] = \frac{a^2n(kn-1)}{(1+n+kn)^2},\ ED^N[n,k] = \frac{a^2kn^2}{2(1+n+kn)^2}, \quad \text{and} \end{aligned} \tag{7.1}$$

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[6] We use superscript N to denote normal technology and C, which we present in the next section, to denote clean technology.

[7] If $kn < 1$, t^N has a negative value, thus implying subsidization rather than taxation. We exclude this case.

$$SW^N[n,k] = \frac{a^2 n}{2(1+n+kn)} \quad . \tag{7.2}$$

Equations (7.1) and (7.2) allow us to obtain the following relationships through comparative statics:

$$\frac{\partial Q^N}{\partial k} < 0,\ \frac{\partial CS^N}{\partial k} < 0,\ \frac{\partial T^N}{\partial k} ?,\ \frac{\partial ED^N}{\partial k} ?,\ \frac{\partial \pi_i^N}{\partial k} < 0,\ \frac{\partial SW^N}{\partial k} < 0 . \tag{8}$$

The intuitions of the above comparative statics are as follows. The increase in emission tax resulting from the increased environmental damage parameter (k) increases the firm's production cost. This causes a reduction in each firm's output level and that of industry ($\partial Q^N / \partial k < 0$). Moreover, profits decrease as a result of an increase in k because industry output decreases $(\partial \pi^N / \partial k < 0)$, market price increases, and consumer surplus decreases ($\partial CS^N / \partial k < 0$). A decrease in consumer and producer surplus outweighs change in environmental damage and tax revenue. Social welfare decreases as k increases ($\partial SW^N / \partial k < 0$). However, the effects on the total tax revenue and environmental damage are ambiguous.

Differentiating T^N with respect to k gives

$$\frac{\partial T^N}{\partial k} = n[\underbrace{\frac{\partial t^N}{\partial k} q^N}_{\substack{tax\ rates\ effects \\ (+)}} + \underbrace{t^N \frac{\partial q^N}{\partial k}}_{\substack{output\ effects \\ (-)}}] = \frac{a^2 n^2 (3+n-kn)}{(1+n+kn)^3} . \tag{9}$$

An increase in environmental damage parameter generates two conflicting effects for the total tax revenue: a positive effect caused by an increase in tax rates (tax rates effects) and a negative effect caused by a decrease in output level (output effects). Thus, whether an increase in k raises or lowers the total tax revenue depends on the relative magnitude of the two opposing effects. From Equation (9), we have[8]

$$if\ k \le (>) \frac{3+n}{n} \Rightarrow \frac{\partial T^N}{\partial k} \ge (<) 0 . \tag{10}$$

Differentiating ED^N with respect to k gives us

[8] Here, $k \ge 1/n$ from the assumption.

$$\frac{\partial ED^N}{\partial k} = \underbrace{\frac{1}{2}(Q^N)^2}_{(+)} + \underbrace{knQ^N \frac{\partial q}{\partial t}\frac{\partial t^N}{\partial k}}_{(-)} = \frac{a^2 n^2 (1+n-kn)}{2(1+n+kn)^3}. \tag{11}$$

For the effects of tax revenue (Equation (9)), an increase in k generates two opposing effects on environmental damage: direct effect due to the rise in k with a positive sign, and indirect effect via a change in tax rate with a negative sign because $\frac{\partial q}{\partial t}\frac{\partial t^N}{\partial k} < 0$ in Equation (11). From Equation (11), we have

$$if\ k \le (>) \frac{1+n}{n} \Rightarrow \frac{\partial ED^N}{\partial k} \ge (<) 0\,. \tag{12}$$

From the market equilibria under case N, we can obtain

$$\frac{\partial Q^N}{\partial n} > 0,\ \frac{\partial CS^N}{\partial n} > 0,\ \frac{\partial T^N}{\partial n} > 0,\ \frac{\partial ED^N}{\partial n} > 0,\ \frac{\partial \pi_i^N}{\partial n} < 0,\ \frac{\partial SW^N}{\partial n} > 0\,. \tag{13}$$

If the number of firms (n) increases, the market becomes more competitive and the total output increases although the output of each firm decreases ($\partial Q^N / \partial n > 0$, $\partial q^N / \partial n < 0$). A firm's profit decreases because it is positively related to output level ($\partial \pi^N / \partial n < 0$). When total output increases, total tax revenue increases ($\partial T^N / \partial n > 0$) and market price decreases, thus causing consumer surplus to increase ($\partial CS^N / \partial n > 0$). Social welfare increases as n increases ($\partial SW^N / \partial n > 0$), implying that an increase in consumer surplus and tax revenue outweighs decreases in firm profits and increases in environmental damage.

3.2. Clean Technology: Case C

A firm that chooses clean technology under a given emission tax must pay pollution abatement costs while reducing emissions. Thus, the profit function of firm i is given by

$$\pi_i = \left(a - Q\right) q_i - \frac{1}{2} q_i^2 - \frac{1}{2} x_i^2 - te_i\,. \tag{14}$$

Each private firm chooses its output and abatement level simultaneously and independently. We can obtain the following solutions by solving the first-order conditions for maximizing profits of each firm.

$$q_i = \frac{a-t}{2+n},\ \ x_i = t,\ \ \pi_i = \frac{3a^2 - 6at + (7+4n+n^2)t^2}{2(2+n)^2}, \tag{15.1}$$

$$CS = \frac{n^2(a-t)^2}{2(2+n)^2},\ \ T = \frac{nt(a-(3+n)t)}{2+n},\ \ ED = \frac{kn^2(a-(3+n)t)^2}{2(2+n)^2}. \tag{15.2}$$

Optimal emission tax t can be obtained by solving the social welfare maximization problem, which is expressed as

$$t^C = \frac{a(kn(3+n)-1)}{5+n(5+n)+kn(3+n)^2}. \tag{16}$$

Substituting t^C of Equation (16) into Equations (15.1) and (15.2) allows us to obtain market equilibria under clean technology, which are expressed as

$$q_i^C[n,k] = \frac{a(3+n)(1+kn)}{Z},$$

$$\pi_i^C[n,k] = \frac{a^2[28+3n(6+n)+2kn(24+17n+3n^2)+4k^2n^2(9+6n+n^2)]}{2Z^2}, \tag{17.1}$$

$$CS^C[n,k] = \frac{a^2n^2(3+n)^2(1+kn)^2}{2Z^2},\ \ T^C[n,k] = \frac{a^2n(4+n)(kn(3+n)-1)}{Z^2}, \tag{17.2}$$

$$ED^C[n,k] = \frac{a^2kn^2(4+n)^2}{2Z^2},\ \ SW^C[n,k] = \frac{a^2n(4+n)(1+kn)}{Z^2}, \tag{17.3}$$

where $Z = 5+n(5+n)+kn(3+n)^2$. From these market equilibria given by Eqs. (17.1) to (17.3),

$$\frac{\partial Q^C}{\partial k} < 0,\ \frac{\partial CS^C}{\partial k} < 0,\ \frac{\partial x^C}{\partial k} > 0,\ \frac{\partial T^C}{\partial k}?,\ \frac{\partial ED^C}{\partial k} < 0,\ \frac{\partial \pi_i^C}{\partial k}?,\ \frac{\partial SW^C}{\partial k} < 0. \tag{18}$$

Most of the results are similar to the case for normal technology given in Equation (8). However, unlike the case for normal technology, the effects on environmental damage reveal an obvious positive sign relative to clean technology. Differentiating ED^C with respect to k gives

$$\frac{\partial ED^C}{\partial k} = \underbrace{\frac{1}{2}\left(\sum_{i=1}^{n} e_i\right)^2}_{(+)} + \underbrace{k\sum_{i=1}^{n} e_i\left(n\frac{\partial q}{\partial t} - \frac{\partial X}{\partial t}\right)\frac{\partial t^C}{\partial k}}_{(-)} < 0, \tag{19}$$

where $X=\sum_{i=1}^{n} x_i$. In Equation (11), the first term on the right-hand side of Equation (19) represents the direct effects of increasing environmental damage parameter with a positive sign, and the second term reveals the indirect effects via the change in the environmental tax with a negative sign. Unlike $\frac{\partial ED^N}{\partial k}$ in Equation (11), $\frac{\partial ED^C}{\partial k}$ is negative because firms engage in pollution reduction activities in the case of clean technology. Thus, $\frac{\partial X}{\partial t}(>0)$ appears in the clean technology case as an indirect effect, which is magnified compared with the case of normal technology. Consequently, if $k \geq \frac{1}{n}$, the second term dominates the first term and $\frac{\partial ED^C}{\partial k}<0$.[9]

The relationships between equilibria and the number of firms are given below.

$$\frac{\partial Q^C}{\partial n}>0, \frac{\partial CS^C}{\partial n}>0, \frac{\partial x^C}{\partial n}?, \frac{\partial T^C}{\partial n}?, \frac{\partial ED^C}{\partial n}<0, \frac{\partial \pi_i^C}{\partial n}<0, \frac{\partial SW^C}{\partial n}>0 \qquad (20)$$

Lemma 1: *Suppose that optimal emission taxes are levied. 1) If firms adopt normal technology, an increase in environmental damage parameter (k) increases (resp. reduces) environmental damage when k is higher (resp. lower) than $\frac{n+1}{n}$. Conversely, if firms adopt clean technology, an increase in k decreases environmental damage irrespective of the magnitude of k. 2) As the market becomes more competitive (as n increases), environmental damage under normal technology increases while that under clean technology decreases.*

3.3. Firms' Choice between two Cases

We investigate in this subsection the conditions under which firms collectively choose clean technology despite additional costs for pollution abatement activities. Firms choose clean technology if profits in doing so are greater than those under normal technology. In Equations (3) and (14), we look at emission tax rate gaps between normal and clean technologies because tax burdens on firms are key constituents in profit functions. Tax burden plays a key role in the firms' adoption of technology because firms prefer to adopt clean technology and reduce pollution emissions if tax rates under normal technology are excessively costly to endure. From Equations (6) and (16), we obtain

$$if \;\; k \geq (<) \tilde{k} \Rightarrow t^N - t^C = \frac{a(2+n)\{-(2+n)+kn(3+n)(kn-1)\}}{(1+n+k)(5+5n+9kn+n^2+6kn^2+kn^3)} \geq (<) 0 \,. \qquad (21)$$

where $\tilde{k}=\frac{3+n+\sqrt{33+26n+5n^2}}{2n(3+n)}$. In Eq. (21), if k is higher (resp. lower) than the

[9] See Appendix 1 for more details.

threshold level $\tilde{k}$, then emission tax level under normal technology is higher (resp. lower) than under clean technology. Threshold level $\tilde{k}$ is negatively related to number of firms n; hence, it decreases if n increases.[10] For example, if $n=2$, $\tilde{k}=0.762$ and if $n=10$, $\tilde{k}=0.158$. Therefore, t^N is higher than t^C if $k>0.762$ with $n=2$. Accordingly, if the value of k is sufficiently low, i.e., $k<\tilde{k}$, then t^N is lower than t^C and the tax rate gap between normal and clean technologies given in Eq. (21) is positively related to k. That is, $\frac{\partial(t^N-t^C)}{\partial k}>0$.

Conversely, t^N and t^C reveal different directions with respect to n. In case N, firms decide output level (q^N) under t^N, which is given. Since they do not abate their emissions, environmental damage (ED^N) becomes an increasing function with respect to total output (Q^N). Thus, t^N increases in line with n, and approaches $t^N \to ak/(k+1)$ when $n \to \infty$. However, in case C, firms decide their output level (q^C) and abatement level (x^C) under t^C, that is given. Since all the firms are identical and they choose $x=t$ from Eq. (15.1), environmental damage (ED^C) becomes the function of $\sum e_i (=Q^C-nt^C)$. We have from Eq. (15.1) that $Q^C-nt^C=\frac{n\{a-(3+n)t^C\}}{2+n}$. From this equation, we can obtain that if $n\to\infty$, $Q^C-nt^C\to-\infty$ when $t^C>0$. Since this contradicts the definition of $e_i=\max\{q_i-x_i,\ 0\}$, $t^C\to 0$ must be true. From the above equation and Eq. (17.1), we can confirm that if $n\to\infty$, $Q^C\to a$ and $X^C=nx\to a$. This means that if the number of firms is large and the market is close to perfect competition, each firm will fully abate its emissions under the emission taxation system with clean technology.[11]

We turn to firms' profit gap between the two production technologies. From Eqs. (7.1) and (17.1), we have

$$\pi^N[n,k]-\pi^C[n,k]=\frac{a^2\Gamma}{2(1+2+kn)^2\left\{5+n(5+n)+kn(9+6n+n^2)\right\}^2}, \tag{22}$$

where $\Gamma=47+2n(38+19n+3n^2)+2n(83+114n+50n^2+7n^3)k+n^2(83+46n-9n^2-8n^3-n^4)k^2-2n^3(3+n)(20+19n+4n^2)k^3-4n^4(3+n)^2k^4$. Calculation of profit gap is complicated, so we apply numerical analysis to identify solutions. Fig. 1 depicts profit gap on the $(k,\ \pi^N-\pi^C)$ space when $n=10$.[12] The curve $\pi^N-\pi^C$ has downward slope in k (i.e., $\frac{\partial(\pi^N-\pi^C)}{\partial k}<0$), and it holds that $[\pi^N-\pi^C]_{k<0.10343}>0$ and $[\pi^N-\pi^C]_{k>0.10343}<0$. Therefore, the function has the unique point $\bar{k}$, where $\pi^N-\pi^C=0$. Profit under clean technology (π^C) is higher than under normal technology (π^N) if $k>0.659$ with $n=2$ or $k>0.103$

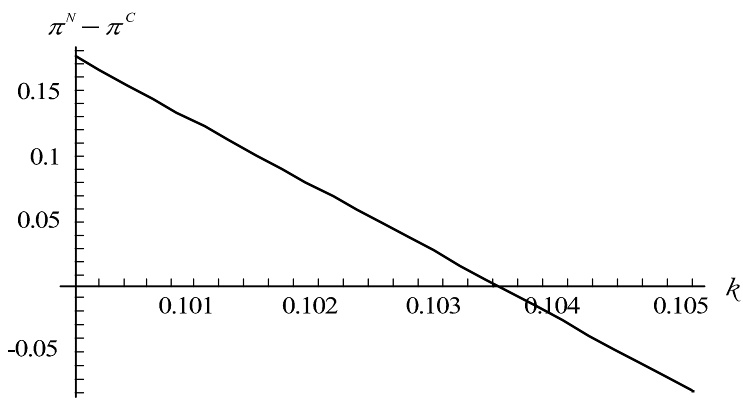
[10] That is, $\frac{\partial\tilde{k}}{\partial n}=-\frac{5n^2+3(11+\sqrt{33+26n+5n^2})+n(24+\sqrt{33+26n+5n^2})}{2n^2(3+n)\sqrt{33+26n+5n^2}}<0$.

[11] We would like to thank the anonymous referee for valuable comments in this regard.

[12] We set $a=50$.

with $n = 10$.

[Figure 1] Profit Gap of private Firm between Case N and Case C: $n = 10$



One interesting point is observed from the above analysis: As k increases, firms are more likely to adopt clean technology despite additional costs for pollution abatement activities, because it yields greater tax savings via lower environmental tax rate. Conversely, environmental damage under normal technology is always more extensive than under clean technology irrespective of the magnitude of k, because firms that use clean technology abate their emissions to match the tax rate ($x = t$).

$$ED^N[n,k] - ED^C[n,k] = \frac{a^2 kn^2(1+5kn+5kn^2+kn^3)(9+10n+13kn+2n^2+7kn^2+kn^3)}{2(1+n+kn)^2(5+5n+9kn+n^2+6kn^2+kn^3)^2} > 0. \quad (23)$$

Finally, we compare social welfare between both cases. From Eqs. (7.2) and (17.3), we have

$$SW^N[n,k] - SW^C[n,k] = \frac{a^2 n\{1+kn(1-4kn-kn^2)\}}{2(1+n+kn)^2(5+5n+9kn+n^2+6kn^2+kn^3)^2} < 0. \quad (24)$$

In the numerator of Eq. (24), because $kn > 1$ and $n \geq 2$ from the assumption, the sign of the parenthesis is obviously negative; hence, $SW^N < SW^C$ holds.[13] The following proposition can be obtained from the above analysis.

[13] In Eq. (24), if $k > \frac{1+\sqrt{17+4n}}{2n(4+n)}$, SW^C is larger than SW^N. Since $k > \frac{1}{n} > \frac{1+\sqrt{17+4n}}{2n(4+n)}$, we can say $SW^N < SW^C$.

Proposition 1: *Suppose that optimal emission taxes are levied. 1) If* k *is sufficiently high (resp. low), then emission tax under clean technology is greater (resp. less) than under normal technology and firms' profits with clean technology are greater (resp. less) than those with normal technology. 2) If* k *is sufficiently high, firms have incentive to choose clean technology even it incurs additional abatement costs. 3) Environmental damage under normal technology is more extensive than under clean technology irrespective of the magnitude of* k. *4) Social welfare under clean technology is always greater than under normal technology irrespective of the magnitude of* k.

IV. Market Equilibriums Under Mixed Oligopoly with Ecological Public Firm

We now examine the effects of mixed oligopoly in which a public firm uses clean technology in its production process. We assume in this case that government does not levy environmental taxes on emissions. One interpretation for this is that environmental taxes lead to overall increase in tax burden, which undoubtedly would be met with high tax resistance. Therefore, government must identify alternative measures rather than environmental taxes to reduce emissions. We can easily imagine that if the government uses entry deregulation instead of emission taxes, market structure will change from private to mixed oligopoly, and private firms' choices between normal and clean technology be different from under the emission taxation case. Our question is if this type of policy measure is more effective and socially desirable than the emission taxation system.

We consider a mixed oligopoly consisting of one public firm (firm 0) and n private firms. As in the basic model, inverse demand function is given by $P = a - Q$, where $Q = q_0 + \sum_{i=1}^{n} q_i$. Profit function of private firm i ($i = 1,2,\ldots,n$) is given by

$$\pi_i = (a - Q)q_i - \frac{1}{2}q_i^2 - \frac{1}{2}x_i^2 . \tag{25}$$

We assume that the public firm always adopts clean technology, and it determines its output level and pollution abatement level simultaneously. If private firms adopt clean technology, total pollution emission in the market is $\sum_{i=0}^{n} e_i$. In this case, environmental damage is given by $ED = \frac{k}{2}(\sum_{i=0}^{n} e_i)^2$. Social welfare is defined as the sum of consumer surplus (CS), producer surplus ($\sum_{i=0}^{n} \pi_i$) and environmental damage (ED) with negative effect on social welfare. Note that total tax revenue is eliminated here.

$$SW = CS + \sum_{i=0}^{n} \pi_i - ED. \tag{26}$$

Each private firm determines output level and pollution abatement level simultaneously, taking rivals' output and abatement levels as given. Because $\frac{\partial \pi_i}{\partial x_i} = -x_i < 0$ in Eq. (25), there is no incentive for private firms to adopt clean technology under mixed oligopoly system; that is, $x_i = 0$ for $i = 1, 2, \ldots, n$. Because all private firms are symmetric, output level of each private firm is the same in the equilibrium, i.e., $q_1 = q_2 = \cdots = q_n = q$. Thus, the best response function of a representative private firm is given by

$$q = \frac{a - q_0}{2 + n}. \tag{27}$$

Conversely, the public firm determines it's output and pollution abatement levels simultaneously to maximize social welfare. From the first-order condition of social welfare maximization, we can obtain the following best response functions with respect to output level and pollution abatement level:

$$q_0 = \frac{a - (1+k)nq + kx_0}{2 + k}, \tag{28.1}$$

$$x_0 = \frac{k(q_0 + nq)}{1 + k}. \tag{28.2}$$

From Eqs. (28.1) and (28.2), we can confirm that the public firm decreases output and increases abatement when a representative private firm increases output. This reflects the fact that q_0 and q are strategic substitutes in a Cournot competition and that the public firm is reducing environmental damage to maximize social welfare. From Eqs. (27), (28.1), and (28.2), equilibrium with respect to firms' output and abatement levels under mixed oligopoly are given as follows:[14]

$$q_0^M[k,n] = \frac{a(2 - k(n-2))}{4 + n + k(6+n)}, \tag{29.1}$$

$$q_i^M[k,n] = \frac{a(1+2k)}{4 + n + k(6+n)}, \tag{29.2}$$

[14] Superscript M denotes a mixed oligopoly.

$$x_0^M[k,n]\left(=q_0^M[k,n]\right)=\frac{a(2-k(n-2))}{4+n+k(6+n)}. \tag{29.3}$$

$$x_i^M=0. \tag{29.4}$$

Eq. (29.3) reveals that the public firm fully abates emissions and therefore generates no environmental damage.[15] In Eq. (29.1), the public firm does not produce output when k is relatively large ($q_0^M=0$).[16] Combining assumption $kn>1$ and condition of positive output of the public firm, $q_0^M>0$, we obtain

$$\begin{array}{ll} \text{If } \frac{1}{n}\le k<\frac{2}{n-2} & \Rightarrow \quad q_0^M>0 \\ \text{If } \frac{2}{n-2}<\text{k} & \Rightarrow \quad q_0^M=0. \end{array} \tag{30}$$

Note that if $\frac{2}{n-2}<k$, then market structure becomes a private oligopoly comprised of private firms and allowing entry of the ecological public firm into the market is meaningless. In this case, emission taxes rather than shifting the market to mixed oligopoly may be a valid policy measure to protect the environment.

Using Eqs. (28.1) to (28.3), we obtain market equilibriums such as total output, private and public firms' profits, consumer surplus, environmental damage, and social welfare under mixed oligopoly as follows:

$$Q^M[k,n]=\frac{a(1+k)(2+n)}{4+n+k(6+n)}, \tag{31.1}$$

$$\pi_0^M[k,n]=\frac{a^2k(2+n)(2-k(n-2))}{(4+n+k(6+n))^2}, \tag{31.2}$$

$$\pi_i^M[k,n]=\frac{3a^2(1+2k)^2}{2(4+n+k(6+n))^2}, \tag{31.3}$$

$$CS^M[k,n]=\frac{a^2(1+k)^2(2+n)^2}{2(4+n+k(6+n))^2}, \tag{31.4}$$

$$ED^M[k,n]=\frac{a^2n^2k(1+2k)^2}{2(4+n+k(6+n))^2}, \tag{31.5}$$

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[15] We obtain $x_0^M=\frac{ak(2+n)}{4+n+k(6+n)}$. However, this value is greater than equilibrium output level (i.e., $x_0^M=\frac{ak(2+n)}{4+n+k(6+n)}>q_0^M$). Therefore, since we define $e_i=\max\{q_i-x_i,\ 0\}$, it holds $x_0^M=q_0^M$ in equilibrium.

[16] Bárcena-Ruiz and Garzón (2006) insist that the public firm's output level is zero if $n\ge 2$ under mixed oligopoly; this result is obtained by assuming that $k=1$ and $x_i=0$. However, we suggest that the public firm has an incentive to choose a positive abatement level $(x_0>0)$ if its behavior is based on social welfare maximization. Eq. (29.3) supports this point.

$$SW^M[k,n]=\frac{a^2\{4+7n+n^2+(16+24n+n^2)k+(12+16n-5n^2)k^2-4n^2k^3\}}{2(4+n+k(6+n))^2}. \quad (31.6)$$

From market equilibriums above, it follows that

$$\frac{\partial q_0^M}{\partial k}=\frac{\partial x_0^M}{\partial k}<0,\ \frac{\partial q_i^M}{\partial k}>0,\ \frac{\partial Q^M}{\partial k}<0,\ \frac{\partial CS^M}{\partial k}<0,\ \frac{\partial ED^M}{\partial k}>0,$$
$$\frac{\partial SW^M}{\partial k}<0 \ for \ \frac{1}{n}\le k<\frac{2}{n-2}. \quad (32)$$

Intuitions for Eq. (32) are straightforward; as k increases, the public firm reduces its output while private firms increase outputs. Because the former overweighs the latter, total output decreases, resulting in decrease in consumer surplus. Environmental damage increases because outputs of private firms using normal technology in the production process increases. Consequently, social welfare decreases.

V. Comparison

In this section, we compare the efficiency of two different policy measures, that is, emission taxation (Case C and Case N) and allowing market entry of an ecological public firm (Case M). As confirmed in Fig. 1, private firms may adopt normal or clean technology even under the emission taxation system, depending on value of k. This implies that we should consider both cases— private firms' adoption of normal technology and clean technology— in comparing the two policy measures.

Under the emission taxation case, whether private firms choose normal (Case N) or clean (Case C) technology in their production processes depends on the magnitude of k. If k is relatively high, private firms adopt clean technology under the emission taxation case (Case C). Thus, we should compare Case C with Case M. However, if k is relatively low, because private firms adopt normal technology (Case N) under the emission taxation system, we should compare Case N with Case M. For example,

$$\begin{aligned}&\text{when } n=2, \begin{cases}0.659<k \Rightarrow \text{Case C and Case M}\\ 0.5<k\le 0.659 \Rightarrow \text{Case N and Case M}\end{cases}\\ &\text{when } n=10, \begin{cases}0.103<k<0.25 \Rightarrow \text{Case C and Case M}\\ 0.1<k\le 0.103 \Rightarrow \text{Case N and Case M}\end{cases}\end{aligned} \quad (33)$$

5.1. Case I: When k is Low

Because private firms have no motivation to adopt clean technology in this case, we compare Case N with Case M. Using Eqs. (7.1) and (7.2) and Eqs. (31.1)-(31.6), the relationships between the two cases are given by[17]

$$q_i^M - q_i^N = \frac{a(-3+16k+20k^2)}{2(7+8k)(11+10k)} < 0, \quad (34.1)$$

$$Q^M - Q^N = \frac{2a(-2+23k+30k^2)}{(7+8k)(11+10k)} > 0, \quad (34.2)$$

$$\pi_i^M - \pi_i^N = \frac{3a^2(-3+16k+20k^2)(25+48k+20k^2)}{9(7+8k)^2(11+10k)^2} < 0, \quad (34.3)$$

$$\left(\pi_0^M + \sum_{i=1}^{n} \pi_i^M\right) - \sum_{i=1}^{n} \pi_i^N = \frac{3a^2\left(-375+8k(281+491k+20k^2-150k^3)\right)}{4(7+8k)^2(11+10k)^2} < 0, \quad (34.4)$$

$$CS^M - CS^N = \frac{2a^2(-2+23k+30k^2)(68+103k+30k^2)}{(7+8k)^2(11+10k)^2} > 0, \quad (34.5)$$

$$ED^M - ED^N = \frac{25a^2k(-3+16k+20k^2)(25+48k+20k^2)}{2(7+8k)^2(11+10k)^2} < 0, \quad (34.6)$$

where $0.1 < k \leq 0.103$.

Above inequalities are straightforward. Two effects work for $q_i^N > q_i^M$. Compared to private oligopoly with normal technology (i.e., Case C), an increase in the number of firms due to the entry of the public firm in mixed oligopoly reduces each private firm's output level. The welfare maximizing public firm produces output to the level at which price equals marginal cost, reducing private firms' output level in the market because products are strategic substitutes in Cournot competition. Equilibrium profit of each private firm rises, if and only if, equilibrium output rises, implying $\pi_i^N > \pi_i^M$. Both because of increase in the number of firms in the industry and because of public firm's benevolent activities aiming at the maximization of social welfare, total output under mixed oligopoly is greater than that under private oligopoly with normal technology (i.e., $Q^N < Q^M$). We obtain $P^N > P^M$ from increased number of firms and market competition in mixed oligopoly, implying $CS^N < CS^M$. Due to the presence of the welfare maximizing public firm, producer surplus is lower in mixed oligopoly (Case M) than in pure private oligopoly comprised of private firms with normal technology (Case N), i.e., $\sum_{i=1}^{n} \pi_i^N > \pi_0^M + \sum_{i=1}^{n} \pi_i^M$. Since the public firm under Case M fully abates emissions

[17] We set $n = 10$. Qualitative results are not affected by value of n.

$(x_0^M = q_0^M)$, $q_i^N > q_i^M$ suggests that total environmental damage under private oligopoly with normal technology (Case N) is greater than that under mixed oligopoly (Case M), i.e., $ED^N > ED^M$ even though $Q^N < Q^M$. As for social welfare the difference between Case M and Case N, observes that

$$SW^M - SW^N = (\underbrace{CS^M - CS^N}_{(+)}) + (\underbrace{PS^M - PS^N}_{(-)} - (\underbrace{ED^M - ED^N}_{(-)}) \\ = \frac{a^2(-23 + 588k - 1304k^2 - 3840k^3 - 2000k^4)}{4(7+8k)^2(11+10k)} > 0 \tag{34.7}$$

where PS represents producer surplus before deducting taxation, i.e., $PS^N = \sum_{i=1}^n \pi_i^N + T^N$ and $PS^M = \pi_0^M + \sum_{i=1}^n \pi_i^M$. Social welfare under mixed oligopoly is greater than under the emission taxation system ($SW^N < SW^M$). To understand $SW^N < SW^M$, note that there are two conflicting effects that are general. Compared to private oligopoly with normal technology, reflecting presence of the welfare maximizing public firm using ecological technology in the production process and increased competition caused by entry of the public firm, mixed oligopoly enables an increase in consumer surplus and decrease in environmental damage with positive impact on domestic social welfare. Conversely, mixed oligopoly induces lower producer surplus before deducting taxation,[18] with negative impact on domestic social welfare, than private oligopoly because the public firm cares less about its profits than the private firm. The former effect dominates, resulting in $SW^N < SW^M$. We can summarize above results as the following proposition.

Proposition 2: *Suppose that firms in the private oligopoly market adopt normal technology since* k *is relatively low. 1) Environmental damage under private oligopoly with normal technology is more extensive than under mixed oligopoly with an ecological public firm (* $ED^N > ED^M$ *). 2) Mixed oligopoly with an ecological public firm (Case M) is more desirable than emission taxation under which private firms adopt normal technology (Case N) in their production processes (* $SW^N < SW^M$ *).*

In comparing Case M and Case C, the following point is worth noting. In Eqs. (34.1) -(34.7), there are two channels through which entry of the ecological public firm affects market variables in mixed oligopoly: one is competition effects due to increase in number of firms and the other is effects due to the public firm's benevolent (welfare maximizing) activities. Welfare gain (as well as environmental damage reduction) that comes from entry of the public firm can be disaggregated into two parts: (1) effects from increased number of firms by entry of the public firm,

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[18] From the definition of PS, it holds that $PS^N = \sum_{i=1}^n \pi_i^N + T^N$ and $PS^M = \pi_0^M + \sum_{i=1}^n \pi_i^M$. And since $\sum_{i=1}^n \pi_i^N > \pi_0^M + \sum_{i=1}^n \pi_i^M$ from Eq. (34.4), we get $\sum_{i=1}^n \pi_i^N + T^N > \pi_0^M + \sum_{i=1}^n \pi_i^M$.

and (2) effects of benevolent activities of the public firm (see Appendix 2).

According to conventional wisdom, increase in the number of firms in the industry raises economic activities in the production side, leading to more extensive environmental damage and, in some cases, less social welfare. In this study, however, we demonstrate that entry of the ecological public firm, which increases the number of firms and total output of the industry, can not only reduce emissions in an oligopoly market without other environmental policies such as emission tax but also increase domestic social welfare. Underlying logic for this that the benevolent public firm in mixed oligopoly increases output, which is fully pollution controlled, to the level at which price of the product equals marginal production costs while private firms in the market reduce output that emits pollution in their production process because quantities are strategic substitutes in Cournot competition. In this sense, Proposition 2 implies that mixed oligopoly with an ecological public firm may be a more useful measure than emission taxation under private oligopoly with normal technology.

5.2. Case II: When k is High

Because private firms adopt clean technology in this case, we compare Case C and Case M. From Eqs. (17.1) -(17.3) and Eqs. (31.1) -(31.6), the relationships between the two cases are given by

$$Q^C - Q^M = \frac{-2a(2+93k-26k^2)}{(7+8k)(31+338k)} < 0\,, \tag{35.1}$$

$$X^C - x_0^M = \frac{3a(-53+530k+1144k^2)}{(7+8k)(31+338k)} > 0\,, \tag{35.2}$$

$$ED^C - ED^M = \frac{a^2 k(41-1776k-3380k^2)(351+2224k+3380k^2)}{2(7+8k)^2(31+338k)^2} < 0\,, \tag{35.3}$$

where $0.103 < k \le 0.25$.

Case II is more complicate than Case I, because private firms choose clean technology under emission taxation but normal technology under mixed oligopoly $(x_i^C = t^C,\ x_i^M = 0)$. As in Eqs. (35.1) -(35.3), relative magnitude of variables between Case C and Case M are obvious with respect to total output, pollution abatement level, and the level of environmental damage. However, other variables such as social welfare and output and profits of each private firm are ambiguous.

Relative magnitude of variables is verified more clearly via numerical analysis. Table 1 specifies the range of k, where the corresponding relationship of variables given in the first line is satisfied for a given number of firms (n). For example, for

$\pi_i^N < \pi_i^C$ and $q_0^M > 0$, the k must be higher than 0.659, i.e., $0.659 < k$ when $n = 2$. The first column in Table 1 represents the condition in terms of k, under which Case C and Case M can be established.

[Table 1] Range of k to satisfy each term

n	$\pi^N < \pi^C$ and $q_0^M \geq 0$	$q_i^C < q_i^M$	$\pi_i^C < \pi_i^M$	$PS^C + T^C < PS^M$	$SW^C < SW^M$
2	$0.659 < k$	$1.1 < k$	$2.291 < k$	$1.1 < k$	$0.659 < k < 0.884$
3	$0.427 < k < 2$	$0.637 < k < 2$	$1.102 < k < 2$	$1.156 < k$	$0.427 < k < 0.441$
4	$0.311 < k < 1$	$0.444 < k < 1$	$0.742 < k < 1$	–	–
6	$0.195 < k < 0.5$	$0.275 < k < 0.5$	$0.471 < k < 0.5$	–	–
8	$0.137 < k < 0.333$	$0.199 < k < 0.333$	–	–	–
10	$0.103 < k < 0.25$	$0.155 < k < 0.25$	–	–	–

The second column shows the range of k for which output of a private firm under Case M is greater than under Case C, i.e., $q_i^C < q_i^M$. It is noteworthy that, depending on the magnitude of k, although output of a private firm under Case M may be greater or less than output under Case C, the total output under Case M is always greater than under Case C as we observe at Eq. (35.1), i.e., $Q^C < Q^M$. This reflects the output of the ecological public firm added in total output under Case M. Meanwhile, when we look at pollution abatement activities of firms for each case, although all private firms engage in abatement activities under Case C, only the public firm engages in those activities in Case M. Eq. (35.2) implies the abatement level accomplished by private firms under Case C is greater than the level accomplished by the public firm under Case M (i.e., $X^C > x_0^M$). Consequently, because $Q^C < Q^M$ and $X^C > x_0^M$, $ED^C < ED^M$ holds as in Eq. (35.3).

The third column specifies the condition for $\pi_i^C < \pi_i^M$. We can derive an interesting insight from numerical results for this column. Compared with Case I in Section 5.1, where $\pi_i^N > \pi_i^M$ holds for the domain of k, $\pi_i^C > \pi_i^M$ only holds if $k \in (0.659, 2.291]$ and $\pi_i^C < \pi_i^M$ if $k > 2.291$ when $n = 2$ in Case II. That is, if the value of k is sufficiently high, profits of a private firm under Case M are greater than under Case C. Intuitively, this can be explained as follows: Entry of a public firm into the market works to reduce equilibrium profits of private firms under Case M. However, if k is sufficiently high, emission tax levied on private firms becomes high, reducing their profits under Case C. Because the latter effects outweigh the former when k is large, we obtain $\pi_i^C < \pi_i^M$.

More importantly, the range of k that makes $\pi_i^C < \pi_i^M$ contracts as n increases. For example, $\pi_i^C < \pi_i^M$ requires that k lies between 0.659 and 2.291 (i.e., $k \in (0.659, 2.291]$) when $n = 2$, (0.427, 1.102) when $n = 3$, and (0.275, 0.471] when $n = 6$. Obviously, as n increases, the range of k that makes $\pi_i^C < \pi_i^M$ contracts, and eventually, when $n = 8$, the range of k for $\pi_i^C < \pi_i^M$ becomes extinct and $\pi_i^C > \pi_i^M$ holds for the domain of k. This means that if the market under Case C is relatively competitive, $\pi_i^C > \pi_i^M$ always holds. In Eq. (16), the emission tax under Case C approaches to zero if n approaches to infinity, i.e., if $n \to \infty$, then $t^C \to 0$. When the tax burden is not serious, and the market is less competitive than mixed oligopoly, the profit under Case C is larger than under Case M.

Comparing both cases requires that we should know the change of producer surplus. The fourth column reveals the range of k for which producer surplus under Case M is greater than the sum of producer surplus and tax revenue under Case C. As we observe in the third column, if the market is less competitive (n is small), $\pi_i^C < \pi_i^M$ under some conditions, so there exists some range of k that satisfies $PS^C + T^C < PS^M$. If n is large (for instance, $n \geq 4$ in Table 1), it always holds that $PS^C + T^C > PS^M$, irrespective of k.

Finally, we turn to the welfare comparison between Case C and Case M (the last column in Table 1). With respect to consumer surplus and environmental damage, we know that $CS^C < CS^M$ because $Q^C < Q^M$ in Eq. (35.1) and $ED^C < ED^M$ in Eq. (35.3). Therefore, adopting Case M instead of Case C means that the economy faces two opposing effects for social welfare: 1) increase in consumer surplus and 2) increase in environmental damage. Producer surplus differs depending on the magnitude of n. The final column in Table 1 indicates relative magnitude of social welfare between Case C and Case M depends on k and n. If the market is less competitive, for example, as $n \leq 3$, then social welfare under mixed oligopoly is greater than under Case C only if k is low. However, if the market is competitive (e.g., $n > 3$), then allowing the market entry of the ecological public firm (Case M) cannot be an effective policy measure irrespective of the value of k, and levying an emission tax on private firms that use clean technology in the production process (Case C) may be effective. The above discussion permits establishment of the following proposition.

Proposition 3: *Suppose that firms in the private oligopoly market take clean technology since k is relatively high. 1) Environmental damage under mixed oligopoly is more extensive than under private oligopoly with clean technology ($ED^C < ED^M$). 2) When the market is less competitive, i.e., n is small, $\pi_i^C > \pi_i^M$ (resp. $\pi_i^C < \pi_i^M$) holds if k is low (resp. high). In contrast, when the market is competitive, $\pi_i^C > \pi_i^M$ holds, irrespective of value of k. 3) From the perspective of social welfare, if the market is less competitive, mixed oligopoly with an ecological public firm becomes more desirable than emission tax; otherwise, taxing private firms' emissions is more desirable than the market entry of the ecological public firm.*

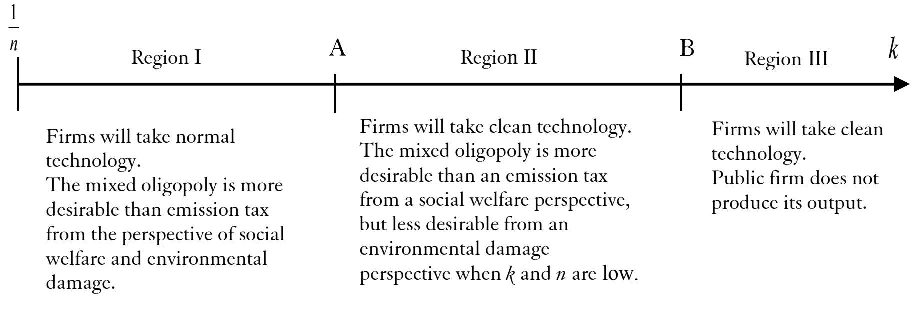
The intuition of Proposition 3 is as follows. The change of market structure from a private to mixed oligopoly affects private firms' behavior, so social welfare is changed. This change mainly depends on market competitiveness (n), and environmental damage parameter (k). Positive effects of the change in market structure is restrictive as we observe above. Even though the entry of a public firm makes the market more competitive and thus increases total output, the positive effect is only an increase in consumer surplus by increase in total output. Producer surplus only increase when the market is relatively less competitive, e.g., $n \le 3$ in Table 1. However, abatement level under mixed oligopoly is lower than under private oligopoly because only the public firm reduces its emission level. Accordingly, environmental damage increases under mixed oligopoly. Consequently, if the market is competitive, negative effects of mixed oligopoly such as decrease in producer surplus and increase in environmental damage outweigh positive effects such as increase in consumer surplus. These relations result in decreased social welfare.

VI. Policy Implications

Key results of our analysis in previous sections are summarized according to the level of k in Fig. 2. In the figure, the domain of $k(>\frac{1}{n})$ is divided into three regions by points A and B. Point A reveals the threshold level $\overline{k}$, where $\pi_i^N - \pi_i^C = 0$ is satisfied (Eq. (22)) and $\pi_i^N > \pi_i^C$ for $\frac{1}{n} < k < \overline{k}$ and $\pi_i^N < \pi_i^C$ for $k > \overline{k}$. Point B represents the level of $k\,(=\frac{2}{n-2})$, where $q_0^M = 0$ from Eq. (29.1). Additionally, if $k < \frac{2}{n-2}$, then $q_0^M > 0$, which implies mixed oligopolistic competition in the market, whereas if $k > \frac{2}{n-2}$, then $q_0^M = 0$, which implies private oligopolistic competition between private firms that use clean technology.

If k lies in region I, private firms will adopt normal technology and social welfare under mixed oligopoly is greater than under emission taxation $(SW^M > SW^N)$. This means that when k is low, allowing entry of the ecological public firm into the market (i.e., mixed oligopoly) may be a better environmental policy measure than emission taxation.

[Figure 2] The relationship between k and firms' behaviors



If k belongs to region II, then private firms will adopt clean technology under an emission taxation system. However, social welfare under mixed oligopoly is only greater than under emission taxes when k and n are low. This implies that when the value of k is intermediate, mixed oligopoly is desirable if k and n are relatively low, but emission taxes are desirable otherwise.[19] In region III, where $k > \frac{2}{n-2}$, mixed oligopolistic competition (i.e., Case M) does not appear because $q_0^M = 0$, and hence market structure becomes a private oligopoly comprised of private firms using clean technology. This implies that when the value of k is sufficiently high, allowing market entry of the public firm may not be as useful as environmental policy measure; rather, emission tax is more desirable in such a case. In this regard, if the public firm exists in the market, then privatizing that firm and introducing an emission tax is recommended as an environmental policy from a social welfare perspective.

VII. Conclusion

In this study, we have examined the effects of mixed oligopoly with an ecological public firm on the market and environment. Research motivation is based on the question of if there may exist an alternative policy measure that can replace emission taxes. The reason we consider the alternative measure is that emission taxes do not necessarily contribute to improving social welfare or encouraging private firms' efforts to reduce pollution. Addressing the above points requires focusing on environmental damage parameter k, more specifically, the magnitude of the negative externalities associated with pollution. The main results are as follow.

First, under the emission taxation system, private firms adopt clean technology (resp. normal technology) in their production processes if the k is sufficiently high (resp. low). In this case, social welfare under clean technology is always greater than under normal technology irrespective of the magnitude of k. Second, if k is less than the threshold level (i.e., $\frac{1}{n} < k < \overline{k}$), mixed oligopoly is socially more desirable than the emission taxation system, which leads private firms to engage in normal technology. In contrast, if k is sufficiently high (i.e., $k > \frac{2}{n-2}$), mixed oligopoly may not be beneficial for the economy. Rather, emission taxation on private firms is more desirable from the social welfare perspective because they motivate private firms adopt clean technology. If k is intermediate level (i.e., $\overline{k} < k < \frac{2}{n-2}$), then market structure matters; that is, if the market becomes less competitive, then the mixed oligopoly is more socially desirable, and vice versa. Finally, comparing environmental damage between mixed oligopoly and emission

[19] Mixed oligopoly is more desirable than an emission tax from a social welfare perspective, but less desirable from an environmental damage perspective.

taxation, we found that if k is sufficiently low (resp. high), and hence private firms adopt normal technology (resp. clean technology) in their production processes, then environmental damage under emission tax is greater (resp. less) than under mixed oligopoly.

Above results allow us to conclude that although it holds within a restrictive condition, using mixed oligopoly as an alternative environmental policy measure is valid from the perspective of social welfare. In particular, when private firms engage in normal technology, mixed oligopoly is always better than emission taxation. Using an emission taxation system allows governments to induce private firms to voluntarily engage in clean technology so that they reduce pollution. However, according to our analysis, the emission taxation system has limited effectiveness, particularly when k is low, because private firms will not adopt clean technology in such a case due to additional costs associated with pollution abatement initiatives.

The following two points deserve attention in interpreting results of this study. The first point is that two effects are integrated in welfare gain as well as environmental damage reduction by using an eco-friendly public firm: one is the effect of intensified competition induced by increased number of firms in the market and the other is that comes from entry of a benevolent public firm in an oligopolistic market keeping total number of firms constant. Therefore, it is desirable to disaggregate welfare gain and environmental damage reduction into above mentioned two parts (see Appendix 2).

The second point is that the government does not levy environmental tax on private firms in mixed oligopoly. Levying environmental tax on private firms in mixed oligopoly would increase social welfare, if environmental damage parameter is sufficiently high. However, our primary interest in this study is to examine the possibility of using a public firm directed at maximization of social welfare as an instrument to directly regulate pollution. To investigate this issue, we have postulated two different cases of market competition (i.e., one is private oligopoly with environmental tax and the other is mixed oligopoly without environmental tax) and have examined if entry of a public firm with eco-friendly technology into a market instead of imposing an emission tax could increase social welfare by comparing above two cases. In this context, the setting that the government does not levy environmental tax on private firms in mixed oligopoly is more suitable for our analysis. Therefore, we would like to leave the issue on the interplay between environmental externalities and public firm's behavior with environmental taxation as an interesting challenge for further study.

Appendix 1: Proof of $\frac{\partial ED^C}{\partial k} < 0$ and $\frac{\partial ED^C}{\partial n} < 0$

From Eq. (17.3), we obtain

$$\frac{\partial ED^C}{\partial k} = \frac{a^2 n^2 (4+n)^2 \{5+5n+n^2 - kn(9+6n+n^2)\}}{2Z^3}. \tag{A.1}$$

Eq. (A.1) indicates that

$$if\ k \geq (<) \frac{5+5n+n^2}{n(9+6n+n^2)} \left(\equiv Z_1\right) \Rightarrow \frac{\partial ED^C}{\partial k} \leq (>) 0 .$$

It follows from the assumption that $k \geq \frac{1}{n}$. And we can easily confirm that $\frac{1}{n} - Z_1 = \frac{4+n}{n(9+6n+n^2)} > 0$, implying that $\frac{\partial ED^C}{\partial k} < 0$ holds for all values $k \geq \frac{1}{n}$. Using the same way as $\frac{\partial ED^C}{\partial k} < 0$, we can verify that $\frac{\partial ED^C}{\partial n} < 0$. From Eq. (17.1), we obtain

$$\frac{\partial ED^C}{\partial n} = \frac{a^2 kn(4+n)\{20+10n+n^2 - kn^2(15+8n+n^2)\}}{Z^3}. \tag{A.2}$$

Eq. (A.2) indicates that

$$if\ k \geq (<) \frac{20+10n+n^2}{n^2(15+8n+n^2)} \left(\equiv Z_2\right) \Rightarrow \frac{\partial ED^C}{\partial n} \leq (>) 0 . \tag{A.3}$$

We can easily find that $\frac{1}{n} - Z_2 = \frac{5n+7n^2+n^3-20}{n^2(15+8n+n^2)} > 0$ for $n \geq 2$. This implies that $\frac{\partial ED^C}{\partial n} < 0$ holds for all values $k \geq \frac{1}{n}$. ■

Appendix 2: Disaggregation of Social Welfare and Environmental Damage

Welfare gain (as well as environmental damage reduction) which comes from entry of a public firm can be disaggregated into two parts: (1) the effects from the increased number of firms by entry of the public firm, and (2) the effects of benevolent initiatives of the public firm. Social welfare difference between Case M and Case C can be rewritten as

$$SW^M[n,k]-SW^N[n,k]=\underbrace{(SW^N[n+1,k]-SW^N[n,k])}_{\substack{\text{the effects of increased}\\ \text{number of firms}}}+\underbrace{(SW^M[n,k]-SW^N[n+1,k])}_{\substack{\text{the effects of benevolent initiatives}\\ \text{of the public firm}}}, \quad \text{(A.4)}$$

where n denotes the number of private firms in each market structure. In above equation, the first term of right hand side reveals effects of increased number of firms while the second term captures effect from entry of a benevolent public firm in an oligopolistic market keeping total number of firms constant. From Eqs. (7.2) and (31.6), we have

$$SW^N[n+1,k]-SW^N[n,k]=\frac{a^2}{2(1+n+kn)(2+n+k(1+n))}>0, \quad \text{(A.5)}$$

$$SW^M[n,k]-SW^N[n+1,k]=\frac{a^2\Omega}{2(2+n+k(1+n))(4+n+k(6+n))^2}, \quad \text{(A.6)}$$

where $\Omega=-8-6n-4k^4n^2(1+n)+k(-12+7n+12n^2)+k^2(4+36n+18n^2-5n^3)+k^3(12+28n+3n^2-9n^3)$.

Applying numerical analysis, we obtain $SW^N[n+1,k]-SW^N[n,k]=\frac{a^2}{2(11+10k)(12+11k)}>0$ and $SW^M[n,k]-SW^N[n+1,k]=\frac{a^2(-34+629k-1418k^2-4204k^3-2200k^4)}{4(7+8k)^2(12+11k)}>0$ within domain of $k(\in(0.1,0.103])$ when $n=10$. This implies that welfare gain from mixed oligopoly comes from increased competition and public firm's benevolent initiatives.

In a similar way as in social welfare, ED^M-ED^N can be disaggregated into two parts, as follows:

$$\begin{aligned}ED^M[n,k]-ED^N[n,k]=&\underbrace{(ED^N[n+1,k]-ED^N[n,k])}_{\substack{\text{the effects of increased}\\ \text{number of firms}}}\\ &+\underbrace{(ED^M[n,k]-ED^N[n+1,k])}_{\substack{\text{the effects of benevolent initiatives}\\ \text{of the public firm}}},\end{aligned} \quad \text{(A.7)}$$

where

$$ED^N[n+1,k]-ED^N[n,k]=\frac{a^2k(1+4n+2kn+2n^2+2kn^2)}{2(1+n+kn)^2(2+n+k(1+n))^2}>0, \quad \text{(A.8)}$$

$$ED^M[n,k]-ED^N[n+1,k]=\frac{-a^2k\begin{pmatrix}4+6k+3n+2kn\\ -2k^2n-2kn^2-2k^2n^2\end{pmatrix}\begin{pmatrix}4+6k+7n+12kn+2k^2n\\ +2n^2+4kn^2+2k^2n^2\end{pmatrix}}{2(2+n+k(1+n))^2(4+n+k(6+n))^2}, \quad \text{(A.9)}$$

Applying numerical analysis, we obtain $ED^N[n+1,k]-ED^N[n,k]=\frac{a^2 k(241+220k)}{2(11+10k)^2(12+11k)^2}>0$ and $ED^M[n,k]-ED^N[n+1,k]=\frac{a^2 k(-17+87k+110k^2)(137-263k+110k^2)}{2(7+8k)^2(11+12k)^2}<0$ within domain of k when $n=10$ As a result, $ED^M[n,k]-ED^N[n,k]=\frac{25a^2 k(-3+16k+20k^2)(25+48k+20k^2)}{2(7+8k)^2(12+10k)^2}<0$, implying that environmental damage reduction caused by the public firm's activity dominates increase in environmental damage from increased number of pollution emitting private firms. ■

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