A Dynamic Dual-Sector Model of Migration, Unemployment and Development with Sector-Specific Human Capital

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I. Introduction

After Harris and Todaro (1970. HT hereafter) and Todaro(1969) raised unemployment issues and discussed unemployment and development policy issues with some success in the developing countries, various directions of extensions or refinements have been pursued in the migration and unemployment literature for the developing countries. One profitable direction of them is to release the existence of the industrial minimum wage and to endogenize the high and stricky industrial wage rate such as the labor turnover model by Stiglitz (1974) and the efficiency wage hypothesis by Mirrlees (1975) and Stiglitz (1976). Another direction is to recognize the existence of an informal sector and to combine it as a third sector in the agricultural and industrial dichotomy suggested by Stiglitz(1982), Basu (1984) and others.

However, further research will be necessary for those remaining issues challenging the Ht model and/or its existence. Some challenging findings revealed by recent empirical studies are as follows : (i) The unemployment level predicted by the HT model, based on the simple wage difference between sectors, is higher than the actual level. (See Harris and Sabot 1982, for example). (ii) Most unemployed workers are found to be among the young and educated. (See Todaro 1976, Berry 1975, Blomovist (1978). (iii) There exist unemplovment as well as informal sector. Which has much different characteristics and implications from the description by Todaro (1969) in its nature, earnings, and mobility to the formal sector. (See ILD

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1972, Stiglitz 1982, Banerjee 1983, House 1984).

On the theoretical basis, the following improvements are desired: (i) A more realistic model is desirable to give better policy implications. As Meier (1984, 144-50) points out, if one is to identify the structural relationships involved in the development process, one must understand the dual structure of the modern and traditional. Ont structure or sector is significantly different from the other in the aspects of technology, distribution system and labor markets like the criteria adopted by ILO(1972). (ii) A more explicit general equilibrium approach is demanded, since the main topics in the migration, unemployment and development are to examine the relationship between sectors and to deal with some welfare issues which cannot be properly handled in the partial equilibrium approach. (iii) A more dynamic framework is desirable to deal with migration and unemployment following the HT type literature, and development policy issues of the dualistic economy following the long tradition of the economic development literature. Those phenomena like migration, job creation and economic development are obviously dynamic. The static framework to analyze these dyanmic phenomena may be misleading. Probably one desirable direction of research is to incorporate the migration and unemployment issues raised by HT (1970) and Todaro (1969) within the dual sector economic development framework initiated by Lewis (1954).

II. Basic Model

1. The Economic Decision of Migration

A. A Dynamic Dual Economy

The present paper is modelling a dynamic dual economy, a typical recently developing economy. Where some modern elements are contained among the traditional elements, while incorporation the recent empirical findings discussed above. We clarify the dual structure and classify it into two sectors, the modern sector and the traditional sector. We rather accept the criteria to classify the formal and informal sectors adopted by ILO (1972) in order to classify the modern sector and the traditional sector.

This definition on the modern sector probably is narrower than common uses in the migration and unemployment literature, but is rather similar to those in the traditional economic development literature. Basically, we are going to incorporate the migration and unemployment issues first raised by HT (1970) and Todaro (1969) within the dual sector economic development framework initiated by Lewis (1954).

The modern sector here produces a homogeneous nonstorable output, which is used for consumption and investment purposes, with modern neoclasical technology using labor (l) trained in the modern education institutions and homogeneous capital (k). The modern sector here operates in the profit motives and exists in the urban areas. Skilled workers in the modern sector earn the binding minimum wage when they are employed and earn higher wages as time passes according to the seniority rule based on age. Employed workers remain at the same firm, which is competitive and homogeneous to others, once they are employed and as time passes new workers are employed thanks to the growth of the modern sector, especially due to the capital accumulation.

The traditional sector here produces a nonstorable consumption good with traditional technology using one variable input, labor. Land uses, capital accumulation, and technical progress can be introduced but they may be fixed or be determined exogeneously. The traditional sector uses skilled workers, trained in a traditional way such as apprenticeship or unskilled workers. The traditonal sector is owned by the whole family or by the whole community which is homogeneous and individual property right is not introduced, and the output is equally distributed among production participants. If a worker leaves the traditional sector, he looses the chance to share the output.

The traditional sector here is composed of two subsectors; the rural traditional sector (agriculture) and the urban traditional sector (informal sector). Only for the purpose of simple presentation, we suppose that these two subsectors in the traditional sector produce the same output or perfect substitutes and use the same technology as a simplifying assumption. And we suppose that people are born only in the rural area or new-born people have pretended to move to the rural area once they are born in the urban area, since there are no mobility costs between areas, which is what we assume. By these assumptions we can even pretend that the modern sector only exists in the urban areas and the traditional sector exists in the rural areas. Obviously releasing these simplifying assumptions will not change the main results of the present paper, only complicating the structure of the model and its presentation.

In this setup workers are born homogeneously and can choose either type of skills by receiving training in the modern institution or by obtaining skill in the traditional way. However, there is some differences in obtaining the skills. The modern education requires more costs than the traditional training by a constant value, c, or the modern education charges constant costs, c, on each trainee but the traditional training does not change any costs except a certain time period of training. The skills can be obtained only when workers are young enough, and each sector uses sector-specific skill or human capital which is not diversifiable in our setup. We realize that the mobility from the traditional sector to the modern sector may be so rare, if we define the modern sector strictly as we did above. And we realize it as a fact that an educated young worker is reluctant to settle down in the traditional sector, even though he is not employed, and searches for a vacancy and gets a job in the modern sector as time passes.

On the other hand, the economy is dynamic. First of all, capital is increasing and the economy is growing. Technology may be improved. Furthermore, economic agents have a lifetime horizon of decision making.

However, we adopt simplifying assumptions that population, number of firms, land and technology are fixed that the economy is a small open economy with both products tradable. People live for T+1 periods. In the last period all workers will be employed and during the first T periods some workers may not be employed. Again for simplicity we set T=1 and assume that at the second period labor market is cleared at the equilibrium wage rate.

We use notations as follows; the first subscript denotes time, t=0,1, present and future and the second subscript denotes sector, j=0,1,2, unemployment (or home production) sector, modern sector subscripts will be used; U for urban area and R for rural area, M for the modern sectors and

T for the traditional sectors.

Now we give more explicit descriptions of the dynamic dual econmy as explained above.

Labor Endowment:

$$l = l_U + l_R \tag{1}$$

Assumption 1

(a) worker-consumers are born homogeneously and measured continuously and normalized with $l=1$

(b) worker-consumers move between urban area (U) and rural area (R) without any costs,

(c) each consumer-worker has one unit of time to work in any of the sectors j=0,1,2 each period.

Under this asumption, we can consider that $l_U=0$ and that only the modern sector exists in the urban area and the skilled workers are equivalent to the migrants. (Obviously the equivalence does not hold if it is positive or if the informal sector exists in the urban area.)

Modern Production Function:

$$X_{t1} = f(l_{t1},\ k_{t1}),\ t=0,1. \tag{2}$$

Assumption 2

(a) f is smooth with $f'>0$, $f''<0$ in l and $f^k(=\frac{\partial f}{\partial k})>0$;

(b) $f' \to \infty$ as $l_{t1} \to 0$, $f' \to 0$ as $l_{t1} \to 1$ with given k_{t1};

(c) capital level is increasing through equation (3) below,

(d) each homogeneous firm maximizes profits.

$$k_{11} = (l-\delta)k_{01} + f(l_{01}, k_{01}) - w_{01}.l_{01}. \tag{3}$$

This assumption says that k_{01} is historically given and is depreciated with rate δ, and that all surplus in the modern sector is invested in the modern sector like a classical assumption. We may think that a capitialist's utility function depends on the investment, equivalent to the whole surplus.

Traditional Production Function:

$$x_{t2}=g(l_{t2},k_{t2}),\ \ t=0,1. \tag{4}$$

Assumption 3

(a) g is smooth with $g'>0$, $g''<0$ in l;

(b) $g'\rightarrow\infty$ as $l_{t2}\rightarrow 0, g'\rightarrow 0$ as $l_{t2}\rightarrow 1$ with given k_{t2};

(c) k_{t2} is constant for each t, if not explicitly specified;

(d) each homogeneous self-owned "firm" by a family maximizes its output.

Under this assumption k_{t2} will be surpressed such as:

$x_{t2}=g(l_{t2})$ and $x_{02}=x_{12}$ if $l_{02}=l_{12}$.

Price Determination:

Assumption 4

(a) $p_{t1}=p_{t2}=1,\ t=0,1$;

(b) $w_{01}=\bar{w}o$, but w_{11} is free,

(c) $w_{t2}=g(l_{t2})/l_{t2},\ t=0,1.$

This assumption is from the structure of the economy. The economy is a small open economy where both products are tradables with given normalized international prices and the modern sector has a binding minimum wage at the first period but the future wage has no restriction, but reflecting the implicit assumption that old workers at the second period earn higher wages than the minimum wage by the seniority rule. Moreover, in the traditional sector owned by a group of workers (a family), income is distributed evenly among production participants. However, once a worker leaves and migrates, then he will lose the right to share the family production.

B. A Migration Decision Function

We are going to assume, similarly to HT (1970), that the choice of modern skill and the decision of migration is based on expected income, but, departing from HT, agents are concerned about lifetime income.

We will proceed to consider a typical worker-consumer's choice problem, migration decision function, and labor market equilibrium condition in succession.

Worker-Consumer Indirect Utility Function:

$$v_{ij}\ (w_{0i},\ w_{ij}\ ;\ p)=w_{0i}+(w_{0j}-c_j),\ i=0,1,2 \text{ and } j=1,2, \tag{5}$$

where

w_{tj}=wage income (=total income)at time t from working in j sector, with $w_{00}=0$;

c_j=training costs to get the skill or human capital required in j sector and to be returned at t=l, with $c_j=c$ for $j=0,1$ and $c_j=0$ for $j=2$;

$p=(p_{01},\ p_{02},\ p_{11},\ p_{12})$, a unit price vector.

This indirect utility function is derived from solving the usual intertemporal consumer choice problem (5′) under Assumption 5 below.

$$\begin{aligned}(w_{0i},\ w_{ij}\ ;\ p)&=v_i(w_{0i}\ ;\ p_0)+v_j\ (w_{1j}-c_j\ ;\ p_1)\\ &=\max\ [U(C_{0i})+U(C_{1j})] \text{ over } \{(C_{0i},\ C_{1j})\ p_0\cdot C_{0i}=w_{0i} \text{ and}\\ &\quad p_1\cdot C_{1j}=w_{1j}-c_j\}\end{aligned} \tag{5'}$$

where

C_{0i}=consumption bundle at t=0,

C_{1j}=consumption bundle at t=1,

$p_0=(p_{01},\ p_{02})$, unit price vector, at t=0;

$p_1=(p_{11},\ p_{12})$, unit price vector, at t=1.

Assumption 5

(a) more general intertemporal utility function, $U(C_{0i},\ C_{1j})$, is additively seperable for future,

(b) the utility function each period is continuous and strictly quasi-concave,

(c) the indirect utility function each period is linear in the first argument, disposable income, with coefficient one,

(d)$U(w_{00})=0$, where $w_{00}=0$.

A Migration Decision Function:

Assumption 6:

A worker-consumer maximizes the expected income during lifetime net o training costs, defined in (5), and the choice of modern skill and migration continues as long as the expected income in the modern sector is larger than the income in the traditional sector.

Here we assume the same type of a migration decision function as HT (1970) such that expected income, a special form of expected utility function, is the wage times the probability to be employed, based on the random job selection process. But, departing from them agents are concernd about lifetime income or lifetime utility.

The number of youngsters enrolling in modern training institutions will increase as long as expected utility in modern sector is higher than that in the traditional sector. And thus the balance in the expected utility between the two sectors is one of the labor market equilibrium condition, like HT (1970). The unemployment rate, π, has the same role as in their model to equate the expected utility across sectors, but it is not the only factor, departing from them, since the future wage rate in the modern sector, w_{11}, is flexible here.

Labor Market Equilibrium Conditions:

$$v_M = v_T \tag{6}$$

where

$$V_M = w_{01} \cdot (l - \pi) + (w_{11} - c) \text{ with } \pi_{00}/(l_{01} + l_{00}),$$

$$V_T = w_{02} + w_{12} = g(l_{02}, k_{02})/l_{02} + g(l_{12}, k_{12})/l_{12} = 2g(l_{02})/l_{02}.$$

(a) $l = l_{00} + l_{01} + l_{02}$,

(b) $l_{11} = l_{01} + l_{00}$ and $l_{12} = l_{02}$. (7)

2. The Existence of a Competitive Equilibrium

An ever-lasting competitive firm in the modern sector is assumed to maximize total profits of two periods. All the profit each period is used as investment to increase the capital stock of the next period. Similarly the capitalist, owner of the competitive firm, may be thougt to maximize his utility function of profits such as $U(p_0, p_1)$or$(p_0, p_1) = r \cdot (p_0 + p_1)$where p_t denotes profits at t and r is a constant.

In either case, the optimization problem of the competitive firm in the modern sector is as follows:

$$\text{Max } p\,(l_{01}, l_{11}\,;\, w_{01}, w_{11}) = f(l_{01}, k_{01}) - w_{01}.l_{01} + f(l_{11}, k_{11}) - w_{11}.l_{11} \quad (8)$$

subject to

(a) $1 \geqq l_{11} \geqq l_{01} \geqq 0.$

The program (8) is modified into program (8′) below, because all non—negativity constraonts are not binding under the assumptions on the production functions (A.2.b, A.3.b) except $l_{oo} \geqq l_{11} - l_{01} \geqq 0$. But we know that $l_{11} = l_{01} + l_{00}$ at equilibrium. Thus we reformulate the optimization problem in terms of $l_{01}$and l_{00}instead of $l_{01}$and l_{11}as follows:

$$\text{Max } p(l_{01}, l_{00}\,;\, w_{01}, w_{11}) = f(l_{01}, k_{01}) - w_{01}.l_{01} + f(l_{01} + l_{00}, k_{11}) - w_{11}.(l_{01} + l_{00}) \quad (8')$$

subject to

(a) $l_{oo} \geqq 0.$

Lemma 1:

If Assumption 2 holds, with given k_{01} and k_{11}, there is a unique solution, $(1_{01}{}^{0}, 1_{00}{}^{0})$, to the program (8′), for any nonnegative parameters w_{01} and w_{11}. proof. The maximand, the profit function, is continuous and the constraint set is compact. A solution to (8′) exists, since the continuous image of the compact set is compact. (See Willard, 1970, 119, for example). The solution, $(1^{0}{}_{01}, 1^{0}{}_{00})$, is unique, since the profit function is strictly a concave function of 1_{01} and 1_{00} with given $1 < 01$ and k_{11} · QED.

Now we can characterize the solution, by using the first-order necessary conditions as follows: simplifying the notation by using f_t for $f(1_{t1}, k_{t1})$ and evaluating all at optimum,

(a) $f_0' - w_{01} + f_1^{k} \cdot (f_0' - w_{01}) + f_1' - w_{11} = 0,$

(b) $(f_1' - w_{11}) \cdot \dfrac{\partial l_{11}}{\partial l_{00}} = 0$, if $l^{0}_{00} > 0$, i.e., if $l^{0}_{11} > l^{0}_{01}$,

$(f_1' - w_{11}) = 0$, if $l^{0}_{00} = 0$, i.e., if $l^{0}_{11} = l^{0}_{01}$. (9)

By simplifying (9), we have the usual profit maximization conditions as (9′);

(a) $f'(1_{01}\,;\, K_{01}) - w_{01} = 0$

(b) $f'(l_{01} + l_{00}, k_{11}) - w_{11} = 0$ (9′)

And the solution, (l^{0}_{10}, l^{0}_{00}), is a unique differentiable function of w_{01}

and w_{11}, since the maximand is continuous, the constraint set can be adjusted to be an open set, and the full rank condition is met due to the strict concavity, by applying the implicit function theorem (Rudin 1976, 224—5, for example). Furthermore, an equilibrium unemployment can exist as following lemma.

Lemma 2 :

Under the same conditions as Lemma 1, the optimum unemployment at t $= 0$, $l_{00}{}^0$, is positive if $f_i'(l_{01}{}^0, k_{11}^0) - w_{11} \neq 0$.

Proof. Consider (9.b) $f_i'(l_{11}^0, k_{11}^0) w_{11} = 0$ only if $l_{11}^0 = l_{01}^0$.

Thus $l_{00}{}^0 = l_{11}{}^0 - l_{01}{}^0 > 0$, otherwise. QED.

In this simple developing economy, homogeneity of the firms and the worker—consumers enables us to think that each firm in the modern sector composes a small island or a replica and the number of islands is large and workers can move among islands without costs. In the latter sense the firms are competitive. By the homogeneity, the "competitive" equilibrium in one of the islands represents the whole economy's equilibrium. Thus we will arrive at the competitive equilibrium with unemployment of the young and educated by using the lemmas above.

Theorem 1.

Under Assumption 1 ~ 6, there is a unique competitive equilibrium of the dynamic dual economy, $(p_t{}^0, w_t{}^0, c_t{}^0, I_t{}^0, X_t{}^0, l_t{}^0)$ $t = 0,1$. The optimum unemployment of the young and educated workers at $t = 0$ exists if $f_1'(l_{01}{}^0, k_{11}{}^0) - w_{11} \neq 0$.

Proof. The positive output price vector $(p_t{}^0)t = 0,1$ is given by (A.4.a). Each demand price of the output market is constant, and each supply price function is continuous and monotone from zero to infinite. Thus there exists a unique output vector $(x_t{}^0)t = 0,1$ clearing the markets by the intermediate value theorem. (See Rudin 1974, 93, for example).

In the labor market, one knows that V_T is a continuous and monotone function of l_{02} from infinite to zero by Assumption 3 and V_M is a continuous and monotone function of l_{02} from zero to infinite by Assumption 2. Any positive vector of w_{01} (fixed) and w_{11} determines the vector of l

$_{01}$ and $l_{11} = l_{01} + l_{00}$ uniquely by Lemma 1 and thus V_M and anu $w_{02} = w_{12}$ determines $l_{02} = l_{12}$ uniquely by Assumption 3 and then V_T. Therefore there exists a unique value V to equate V_M and V_T again by the intermediate value theorem and a unique labor market equilibrium labor allocation and wage rate vector $(l_t^0, w_t^0)t = 0,1$.

And the utility maximizing consumption vector $(C_t^0)t = 0,1$ is uniquely determined with given wage income by (A.5.b), i.e., the utility function is continuous and strictly quasi-concave, and the utility maximizing investment of firm owner $(I_t^0)t = 0,1$ is determined by the profits fulfilled.

Finally, $l_{00}^0 > 0$ under the assumption by Lemma 2. QED.

Even if the existence of the equilibrium unemployment is well-established since HT (1970), so far no explicit explanations are given to the empirical findings that the unemployed workers are among the young and educated as we do here. This result also will be one way, by introducing education costs, to cope with another empirical fact that the actual unemployment rate is less than that predicted by the HT model which is relying on the simple wage differentials between sectors.

Ⅲ. Policy Implications

1. The State of a Social Optimum

A. A Social Welfare Function

A certain measure of national product is rather loosely referred to as a social welfare function instead of to a planning criterion. However, national product may be a proxy of social welfare, or it is under some conditions. As discussed before, the income of a worker-consumer is the indirect utility function and the profit is considered a measure of the utility level of the capitalist. Then the national product is a measure of social welfare as shown in equation (10).

$$W = V + P \tag{10}$$

where

W = a social welfare function, depending on labor allocations and wage rates,

$V = V_M = V_T$, as shown in (6),

P = profit, as shown in (8).

This social welfare function is a measure of economic performance and may be a goal of the government or economic planners. This function may also be thought of as the maximand of economic planners in a planned economy or a command economy. This is a strictly concave function of labor allocation with given capital levels under the condition that the production functions are strictly concave.

B. The Existence of a Social Optimum

Suppose a government or a planning committee is deeply intervening in the economy, where the traditional sector is operated by the community and the modern sector is controlled by the government with each sector using sector-specific human capital. However, the government cannot control the migration itself and the ongoing minimum wage. Then the task of the government is to solve such a optimization problem as to maximize the social welfare function (10) subject to the labor endowment constraint (8.a); the migration equilibrium condition (6), the capital accumulation process (3), and the minimum wage rate in Assumption 4, with respect to l_{01}, l_{11} and w_{11} as follows:

$$\text{Max } w(l_{01}, l_{11}; w_{01}, w_{11}) = f(l_{01}, k_{01}) + f(l_{11}, k_{11}) + g(l_{02}, k_{02}) + g(l_{12}, k_{12}) - c \cdot l_{11} \tag{11}$$

subject to

(a) $l \geqq l_{11} \geqq l_{01} \geqq 0, l - l_{11} \geqq l_{02} \geqq l_{12} \geqq 0,$

(b) $V_M = V_T,$

(c) $k_{11} = (1 - \delta)k_{01} + f(l_{01}, k_{01}) - w_{01}.l_{01}, k_{02} = k_{12},$

(d) $w_{01} = \bar{w}_{01}.$

This program (11) can be transformed into (11′) below, for just the same reason as program (8) is transformed into (8′)before.

$$\text{Max } w(l_{01}, l_{00}, \bar{w}_{01}, w_{11}) = f(l_{01}, k_{01}) + (l_{01} + l_{00}, k_{11}) + 2g(l - l_{01} - l_{00}) - c\,(l_{01} + l_{00}) \tag{11′}$$

subject to

(a) $l_{00} \geqq 0,$

(b) $V_M = V_T,$

(c) $k_{11} = (1 - \delta)k_{01} + f(l_{01}, k_{01}) - w_{01} \,.\, l_{01}.$

Here the decisions on the employment level and wage rate are separable. Thus we first solve the optimization problem without migration equilibrium constraint and next set the wage rate to balance between sectors. Then we get the following lemma.

Lemma 3 :

If Assumption 2 and 3 hold, with given k_{01} and k_{11}, there is a unique solution, $(l_{01}{}^*, l_{00}{}^*, w_{11}{}^*)$, to the program (11′).

Proof. The maximand, the social welfare function, is continuous and the constraint set is compact. A solution to (11′)exists, since the continuous image of the compact set is compact, as in Lemma 1, and since we find $w_{11}{}^*$ to make (11′b)hold. The solution, $(l_{01}{}^*, l_{00}{}^*, w_{11}{}^*)$, is unique, since the maximand is strictly concave and V_M is a monotone function of w_{11}, applying the intermediate value theorem. QED.

Now consider the first-order necessary condition and the constraints to characterize the solution, supressing the capital accumulation process, as follows ;

(a) $f_0'+f_1'+f_1^k \cdot (f_0'-w_{01})-2g'-c=0$,

(b) $(f_1'-2g'-c) \cdot \dfrac{\partial l_{11}}{\partial l_{00}}=0$, if $l_{00}^*>0$, i.e.,

$l_{11}^*>l_{01}^*$,$(f_1'-2'g-c)=$, if $l_{00}^*=0$, i.e., if $l_{11}^*=l_{01}^*$

$$\text{(c) } w_{01}\cdot\frac{l_{01}}{l_{01}+l_{00}}+w_{11}-c=2g/(1-l_{01}-l_{00}). \tag{12}$$

By simplifying (12), at optimum, we have the following optimum conditions ;

(a) $f_0'+f_1^k \cdot (f_0'-w_{01})=0$,

(b) $f_1'-2g'-c=0$,

$$\text{(c) } w_{01}\cdot\frac{l_{01}}{l_{01}+l_{00}}+w_{11}-c-2g/(1-l_{01}-l_{00})=0. \tag{12'}$$

And by using the implicit function theorem, we get the optimum solution, $(l_{01}^*, l_{00}^*, W_{11}^*)$, which is differentiable functions, since the maximand is continuous, the constraint set can be adjusted to be an open set and the full rank condition is met here. Furthermore, a social optimum unemployment can exist as the following lemma.

Lemma 4 :

Under the same condition as Lemma 3, the social optimum unemployment at t $= 0$, l^*_{00}, is positive if

$f'_1(l^*_{01}, k^*_{11}) - 2g'(1-l^*_{01}) - c \neq 0$.

Proof : Consider (12,b) where $f'_1(l^*_{01}, k^*_{11})-2g'(1-l^*_{01})-c=0$ only if $l^*_{01}=l^*_{11}$. Thus $l^*_{00}=l^*_{11}-l^*_{01}>0$, otherwise. QED.

Finally, using the above two lemmas, we prove the existence of a unique social optimum state and the supporting prices.

Theorem 2 :

Under Assumption 1~6 there exists a unique social optimum allocation vector of output and labor, $(l^*_{tj}, C^*_{tj}, X^*_{tj}, I^*_t)$ t $=$ 0,1 and j $=$ 0,1,2, and a unique vector of prices and wages to support the social optimum allocation, $(p^*_{tj}, w^*_{tj},)$t $=$ 0,1, and j $=$ 1,2. And some level of unemployment exists at the social optimum, if $f'_1(l^*_{01}, k^*_{11})-2g(1-l^*_{01})-c\neq 0$.

Proof : By Lemma 3 a unique labor allocation vector, (l^*_{tj}), is determined, which in turn determines the labor income vector, (w^*_{tj}), and thus the consumption vector, (C^*_{tj}). The labor allocation vector also determines the output vector, (X^*_{tj}), and profits and thus the investment vector, (l^*_t). And the price vector, (P^*_{tj}), is given. It is trivial to show uniqueness of the allocations and the supporting prices. Finally, some level of unemployment exists, $l^*_{00} > 0$, at the social optimum, if the additional condition holds, by Lemma 4. QED.

This theorem shows that some level of industrial reserve army exists even in the command developing economy if the migration and the minimum wage are not controllable under quite weak conditions on the dynamic dual economy with sector-specific human capital such as relatively small modern sector and relatively large traditional sector.

2. Comparisons between the Competitive Equilibrium and the Social Optimum

A. Welfare Comparison

The social welfare function is the sum of the indirect utility functions of workers and capitalists: The worker's utility comes from the lifetime consumption determined by the wage income and the capitalist's utility comes from the lifetime investment determined by the net surplus under the internationally given prices. We are interested, first of all, in the question which economic system between the competitive market system and the command planning system can give higher welfare level of the economic agents, with the given economy. Under the command economy, planners maximize the social welfare function directly and give rise to the profit level indirectly as a component of the social welfare, while capitalists maximize profits directly and give rise to the utility level of workers indirectly, under the market economy.

Welfare comparison is rather easy in this case. By Theorem 1 and 2 the profit maximization allocation and the social welfar maximization allocation are both unique and each allocation is different bliss solution. The following result is simple but is not shown in the literature before.

Lemma 5:

If both the competitive equilibrium and the social optimum exist uniquely, then $P^0 > P^*$, $W^0 < W^*$, and $V^0 < V^*$.

Proof. If both exist uniquely then they are different allocation due to the different first-order necessary conditions. Furthermore, they are bliss solutions. Thus $P^0 > P$ for all and especially $P^0 > P^*$, and similarly $W^* > W^0$. Finally $V^0 < V^*$ comes from $W^0 < W^*$, since if it does not hold then, giving contradition,

$$W^0 = P^0 + V^0 > P^* + V^* = W^*. \text{ QED.}$$

B. Migration and Employment Comparison

Rapid urbanization and relatively high unemploymejnt are observed in most developing countries. One interesting and important question is whether the modern sector creates enough jobs and the urbanization is too rapid. It is true that many including Kelly and Williamson (1982) believe that developing countries are overurbanized. Some optimists view urban growth as the natural outcome of economic development as the central mechanism by which the average living standard and labor productivity are raised. Now we have this optimism as follows:

Theorem 3 :

If both the competitive equilibrium and the social optimum exist, then $l^0_{01} < l^*_{01}$, $l^0_{11} < l^*_{11}$, and $1 - l^0_{02} < 1 - l^*_{02}$.

Proof. Comparing the first-order conditions for the competitive equilibrium and the social optimum, (9′) and (12′), that is,

$f'_0(l^0_{01}, k_{01}) - w_{01} = 0$ and

$f'_0(l_{01}, k_{01}) + f^k_1 \cdot (f'_0\ (l^*_{01}, k_{01}) - w_{01}) = 0$,

one can conclude $l^0_{01} < l^*_{11}$, since $f^k_1 > 0$. And since $V^0 < V^*$ by

Lemma 5, $\dfrac{g(1 - l^0_{11})}{(1 - l^0_{11})} < \dfrac{g(1 - l^*_{11})}{(1 - l^*_{11})}$, onc can conclude $l^0_{11} < l^*_{11}$ and $1 - l^0_{02} < 1 - l^*_{02}$. QED.

From this result we can conclude that the job creation in the modern sector under the market system is suboptimal and that the urbanization is not too rapid. Also we can conclude that more job creation and more migration will increase social welfare of the economy. On the other hand, we note that the capital accumulation under the market system is more rapid than the social optimum level due to the increased profits.

C. The Degree of Suboptimum

The first-order necessary conditions for the competitive equilibrium and the social optimum, (9') and (12'), can be rearranged as follows, using the migration equilibrium condition (6) :

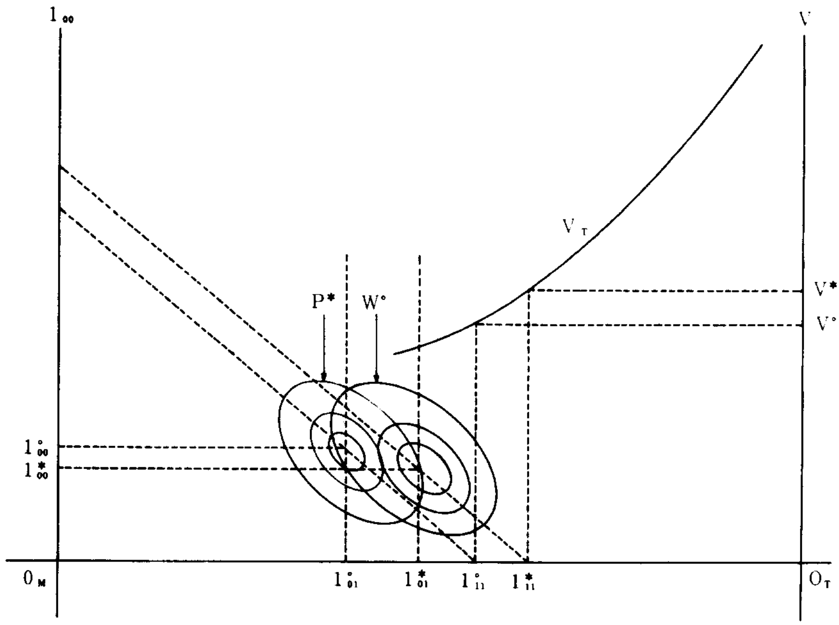
$$\text{(a)}\ f_0'(l^0_{01}, k_{01}) - f'(l^*_{01}, k_{01}) = \frac{1}{f_1 k(l^*_{11}, k^*_{11}) + 1} \cdot w_{01},$$

$$\text{(b)}\ f_1'(l^0_{11}, k^0_{11}) - f_1'(l^*_{11}, k^*_{11}) = 2 \cdot \frac{g(l^0_{02})}{l^0_{02}} - g'(l^*_{02}) - w^0_{01}\frac{l^0_{01}}{l^0_{11}}. \tag{13}$$

Both are known positive.

In a loose manner, we suppose that the loss in profits under the social optimum is neglectable and thus k^0_{11} and k^*_{11} are the same. Then the degree of suboptimum of the competitive equilibrium, 1^0_{01}, increases as capital is relatively less suboptimum (i.e., $f_1{}^k$ is small) and as the minimum wage rate is relatively high. On the other hand, for the migration, the gap between l^0_{11} and l^*_{11} increases as the traditional sector shows more labor

Figure 1 Illustration of the Competitive Equilibrium and the Social Optimum



surplus and the modern sector shows more rapid expansion.

Now we illustrate the labor allocation to contrast the competitive equilibrium and the social optimum in a diagram. Rearranging the first-order conditions for competitive equilibrium,(9), and the first-order condition for social optimum,(12), we have foot-ball shaped iso-profit curves and social indifference curves, examining (9″) and (12″).

$$\frac{d\,l_{00}}{d\,l_{01}} = -\frac{P_1}{P_2} = -\left(1 + \frac{(f_0' - w_{01}).(1 + f_1^k)}{f_1' - w_{11}}\right), \tag{9''}$$

$$\frac{d\,l_{00}}{d\,l_{01}} = -\frac{W_1}{W_2} = -\left(1 + \frac{f_0' + f_1.(f_0' - w_{01})}{f_1' - 2g' - c}\right). \tag{12''}$$

In the diagram, the competitive equilibrium (l^0_{01}, l^0_{00})and the social optimum (l^*_{01}, l^*_{00}) are depicted, and the inequalities between them are illustrated :

$l^0_{02} < l^*_{01}$, $l^0_{01} + l^0_{00} < l^*_{01} + l^*_{00}$, $P^0 > P^*$ but $W^0 < W^*$, $V^0 < V^*$

3. Policy Implications of the Social Optimum

A. Shadow Wage Rate

The usual approach to the question of the optimum industrial employment and related economic development in the dual developing economy is to equate the social marginal product of labor with social opportunity cost. The social marginal product at the optimum employment level is called as the shadow wage rate in the literature such as Dixit (1971).

The social welfare function given in (14) is similar to Stiglitz (1982) and can be regarded as a special case of Dixit (1971), extened in the dynamic setup. Stiglitz (1982) defines the shadow wage rate as the effect on the private output hiring one more worker in the public sector. In this definition, the shadow wage equals the social opportunity cost, but the social opportunity cost of labor is different from the private opportunity cost which is the supply price of labor if labor markets are distorted.

The usual conclusion of the shadow wage in the developing countries as in Dasgupta et. al. (1972) or Little and Mirrlees (1968) is that the social opportunity cost is quite low compared to the private opportunity cost or the supply price of labor due to the labor market distortions. Some literature such as Stiglitz (1982) find many cases where the shadow wage rate is even negative. These findings are not unusual even in the static framework.

The main instrument of the government for affecting employment in the private sector is a wage subsidy, while the government planners could instruct government agencies to evaluate projects within the government sector according to the shadow prices. The government could prescribe the wage to be paid in the government-related sectors. It would be difficult but feasible for the goverment to control wages within the private sector, as Stiglitz (1982) points out.

The shadow wage in the dynamic framework may be defined in two ways; the shadow wage for a period and the shadow wage for a whole lifetime. Since we concentrate on the first period and there is unemployment in the first period, the one-period shadow wage will be discussed first. And also the lifetime shadow 'income' will be discussed next. For this purpose, we modify the social welfare function (11) into (11″) to include the government employment (l_{0g}, l_{1g}), and get

$$W = f(l_{01}, k_{01}) + f(l_{11}, k_{11}) + 2g(l_{02}) - c.(l_{11} + l_{1g}) \tag{11″}$$

where

(a) $l_{11} = l_{01} + l_{00} + l_{0g}$ and $l_{1g} = 0$, for one-period hiring,

(b) $l_{11} = l_{01} + l_{00}$ and $l_{1g} = l_{0g}$, for life-time hiring.

First we consider the shadow wage rate for the case of one-period hiring and define it as w^s_{01}, using the relationship above and evaluating at $l_{0g} =$ 0,

$$w^s_{01} = -\frac{\partial W}{\partial l_{0g}} = -(f_1' - 2g' - c) \cdot \frac{\partial l_{11}}{\partial l_{0g}}. \tag{14}$$

knowing that $(f_1' - 2g' - c) = 0$ at the social optimum, the one-period shadow wage rate is identically zero regardless of the value of $\frac{\partial l_{11}}{\partial l_{0g}}$. If we would evaluate the one-period shadow wage at the competitive equilibrium then we would have a negative one-period shadow wage, since.

$$-(f_1' - 2g' - c) < 0, \text{ for } l^0_{11} < l^*_{11}, \frac{\partial l_{11}}{\partial l_{og}} > 0.$$

The latter comes from the resualt that the Todaro paradox does not occur in this case.

Now consider the shadow wage rate for the case of the life-time hiring and define it as $w._1{}^s$, also using the given relationship above and evaluating at $l_{og} = 0$,

$$\underset{1}{\overset{S}{w}} = -\frac{\partial w}{\partial l_{og}} = -(f_1' - 2g' - c) \cdot \frac{dl_{00}}{dl_{og}} + (2g' + c). \tag{14′}$$

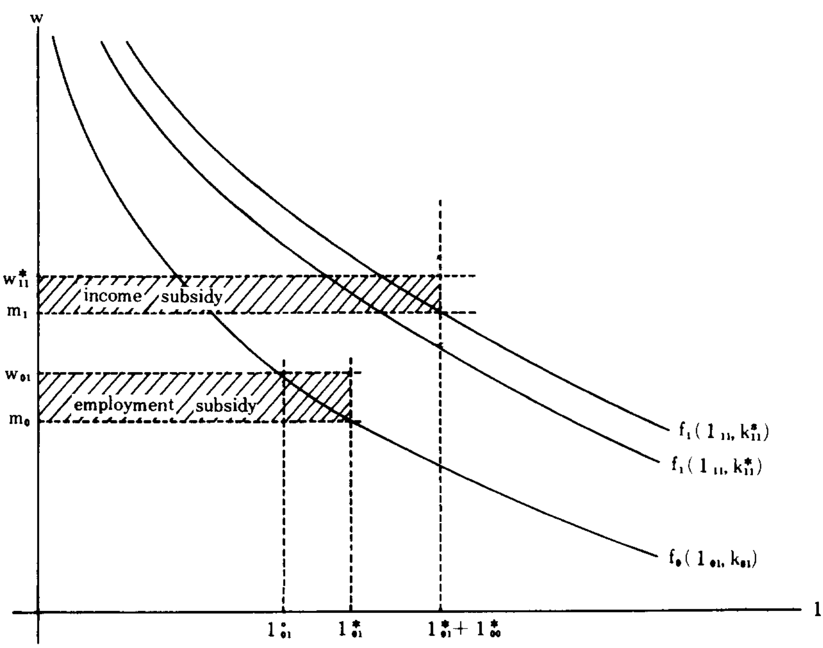
As might be expected, at the social optimum where $(f_1' - 2g' - c)$ is zero, the shadow wage of hiring a skilled worker during the life-time is $(2g' + c)$, the social opportunity cost of the worker.

B. Optimum Wage Poicy

It is generally agreed that an employment subsidy in the modern sector corrects the lagbor market distortions such as the minimum wage to some extent and improves the economic allocations and performance, following the initial suggestion by HT (1970). In this dynamic setup, the desirability of the employment subsidy is confirmed. Since the employment level of the competitive equilibrium in the modern sector falls short of the social optimum, such measures as employment subsidy will be desirable to increase the employment and to arrive at the social optimum level. However, the employment subsidy in not sufficient to improve the competitive allocation to the social optimum allocation. And the subsidy may change the social optimum allocation itself. we suppose the initial social optimum allocation is still desirable, even though there is some change in the process of capital accumulation. However, the one-time employment subsidy is not sufficient to improve the competitive equilibrium allocation of labor to the social optimum allocation. An extra measure to tax or subsidize is necessary for this purpose.

To be more precise, we put this optimum wage policy as a proposition :

Figure 2. Illustration of the Wage Policy



Proposition 1 :

Under the same assumptions as Theorem 1 and 2, to improve the competitive equilibrium allocation of labor to the social optimum allocation, a set of optimum wage policies are necessary as follows : To subsidize the employment to the capitalist with rate of $s = (w_{01} - m_0)$, where $m_0 = f_0'(l_{01}^*, k_{01})$, at $t = 0$ and to tax (or subsidize) on the wage income with the amount of $\tau = m_1 - w_{11}$, where $m_1 = f_1'(l_{11}^*, k_{11})^S$ with $k_{11}^s = (1 - \delta)k_{01} + f_0(l_{01}^*, k_{01}) - (w_{01} - s). l_{01}^*$, at $t = 1$.

This result is obvious from the diagram, using the results of Theorem 1−3, where $l_{01}^0 < l_{01}^*$, $l_{11}^0 < l_{11}^*$, and $k_{11}^* < k_{11}^0 < k_{11}^s$ and thus the labor demand curves are in the order as shown in the diagram. The difference between w_{01} and m_0, $(w_{01} - m_0)$, is the employment subsidy per worker and the subsidy rate is $(w_{01} - m_0)/m_0$ at $t = 0$. The difference between m_1 and w_{11}^*, $(m_1 - w_{11}^*)$, is the income tax per worker and tax rate is $(m_1 - w_{11}^*)/m_1$ at $t = 1$.

This optimum wage policy may deteriorate the income distribution, which we shall not consider explicitly, and may not be so realistic in the developing country. In practice governments may influence resource allocation in many other ways which have somewhat similar effects. The main methods are trade policies.

C. Optimum Trade Policy

Protective trade policy has a long history in the economic development literature. However, trade policy is rarely discussed in the HT type literature with some exceptions like Corden and Findlay (1975), which is reluctant to recommend to use protective trade policy.

Almost the same argument as the optimum wage policy holds for the optimum trade policy, and the latter policy may be a good substitute of the former because it is more realistic and easier to handle for the government.

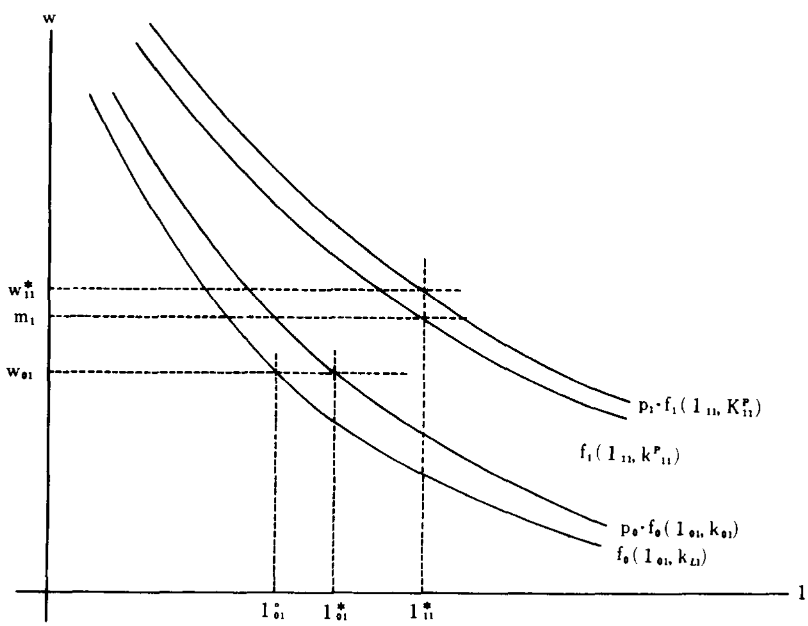
As before, we suppose the initial social optimum allocation of labor is still desirable and the government uses the trade policy only to correct the labor market distortions such as the minimum wage which, we assume, depends on the price of the agricultural output, and to improve resource allocation. Hovever, the one-time tariff or import restriction is not sufficient to improve the competitive equilibrium allocation of labor to the social optimum, and an extra measure to continue some tariff (or subsidy) on the imports of industrial product is necessary.

To be more precise, we put this optimum trade policy as another proposition.

Proposition :

Under the same assumptions as Theorem 1 and 2, to improve the competitive equilibrium allocation of labor to the social optimum allocation, a set of optimum trade policies are necessary as follows : To impose tariff on the import of industrial output with the rate of ($p_0 - 1$)such that $w_{01} = p_0 \cdot m_0$, where $m_0 = f_0'(l^*_{01}, k_{01})$ as before, at $t = 0$ and to impose a tariff (or to subsidize) on the import with the rate of ($p_1 - 1$) such that $w^*_{11} = p_1 \cdot m_1$, where $m_1 = f_1'(l^*_{11}, k^p_{11})$ with $k^p_{11} = (1 - \delta) k_{01} + p_0 \cdot f_0(l^*_{01}, k_{01}) - w_{01} \cdot l^*_{01}$, at $t = 1$.

Figure 3. Illustration of the Optimum Policy



This result is obvious from the diagram, using the results of Theorem 1 ~3 and knowing $k_{11}^{*} < k_{11}^{0} < k_{11}^{p}$. By imposing tariffs on the imports of the industrial output this will shift the labor demand curves to the right. This protective industrial trade policy, the degree of which depends on the labor market distortions and the stage of economic development, will improve resource allocation of the economy. Thus this result may provide a rationale to defend the developing country against the recent pressure by the advanced countries to adopt more liberalized trade policies, the argument of which is based on the static analysis that price distortions by tariffs or import restrictions hurt the efficiency of the economy under the condition that markets are not distorted in other ways.

IV. Concluding Remarks

A typical developing economy contains some modern elements among traditional sectors, and the reallocation of resources from the traditional sector to the modern sector is not so smooth as the neoclassical HT model considers. The recently developing economy is dualistic, where each sector has signficantly different technology, distribution system, and labor markets from the other. It is inevitable to recognize the dualistic aspects of the developing economy. The modern sector often uses capital-intensive and imported technology, frequently relies on overseas resources, usually has corporate ownership and chases profits, and especially requires formally acqûires skills, while the traditional sector is on the opposite side. Also, a dynamic framework seems to be necessary to properly deal with such dynamic phenomena as migration, job creation, and economic development. The direction to analyze these dynamic phenomena in the dynamic setup is natural and desirable to follow the long tradition of economic development literature.

Recent empirical studies have revealed that the unemployment level predicted by the HT model is too high, based on the homogeneity of workers and the simple wage differences between sectors, and most unemployed workers are found to be young and educated, and that the earnings in the urban traditional sector are no less than those in the other sectors and the mobility of worker between the modern sector and the urban traditional sector is quite low.

Realizing these empirical findings as well as the dynamic dual structure of the typical developing economy, we tried to build a more realistic model to combine the HT type models and to analyze the issues with dynamic properties within a dynamic and general equilibrium framework. Almost inevitably we used several simplifying assumptions to handle the issues more explicitly. The developing economy in this model is a small open economy to which the output prices are given exogeneously, and the economy is composed of dual sectors, a modern sector and a traditional sector, and the traditional sector is composed of a rural traditional sector and an urban traditional sector. Each of the dual sectors requires different skills of workers and the skills are obtained in the different training procedure and with different costs. Especially the modern sector uses skilled labor and the skilled labor is obtained in the formal and modern training institutions. It also uses capital which is growing as time pases. The traditional sector uses different labor trained in the traditional manner such as apprenticeship. But the traditional sector may use land and capital, which are determined exogeneously like HT (1970) where capital is fixed in both sectors.

Following HT, we were able to show the existence of the equilibrium unemployment and could show the possibility of the Todaro paradox explicitly, depending on the development stage and the degree of labor market distortions of the economy. The unemployment here is among the young and educated workers, which could not be explained in their model. And we found that the pattern of job creation and its expectation of workers are important factors to determine migration and unemployment.

Explicitly considering the social optimum, we were able to deal with such development issues as shadow wage rate, optimum wage policy and optimum trade policy in the dynamic setup, which seems to be more explicit and more realistic than the HT type models. Furthermore, we found that the employment expansion and the urbanization level of the typical developing economy is not excessive, contrary to the common belief that the developing economy is overurbanized, compared with the development stage. Also we showed that some protective industrial trade policy, the degrees of which depends on the development stage and the labor market distortions, would improve the resource allocation.

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