

## A REINTERPRETATION OF RILEY REACTIVE EQUILIBRIUM AND THE PERFECT MARKET GENERALIZED REACTION EQUILIBRIUM IN A SCREENING MODEL

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### 1. INTRODUCTION

Recently, much progress has been achieved in the literature of asymmetric information. But, one of the big issues in this literature seems to be to find a reasonable equilibrium concept in a model of a competitive market. It is now fairly well known that a screening model has a problem of potential non-existence of a Nash Equilibrium in pure strategies while a signalling model has that of plethora of Nash Equilibria.<sup>1</sup>

As for screening models, many alternative equilibrium concepts have been proposed since the Rothschild and Stiglitz's (1976) potential non-existence theorem<sup>2</sup>, where a Nash Equilibrium in pure strategies, if exists, consists of the Pareto efficient separating contracts. They may be classified into two groups. One group belong to reaction equilibria, which are constructed by allowing players other than a defector to react under the same environment as in Rothschild and Stiglitz (1976). The other group are constructed by modifying the environment itself in one way or another.<sup>3</sup> A reaction equilibrium can be viewed as a non-cooperative equilibrium

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<sup>1</sup>In a market with asymmetric information, there are two parties: the informed and the uninformed. By the terminology of Stiglitz and Weiss (1984), the action of the informed of offering terms of trade to the other party is called signalling, while that of the uninformed is called screening. In a framework of a market game, signalling has been modelled by an informed-agents-move-first game and screening by an uninformed-agents-move-first game.

<sup>2</sup>A possible breakdown of a market due to asymmetric information was first recognized by Akerlof (1971), whose "lemon principle" is now well known.

<sup>3</sup>Grossman (1979) and Hellwig (1987) introduce an institutional factor (application procedure) into the basic framework, forming a three-stage game. Crocker and Snow (1985) considered a regulation of the government in the form of a balanced lumpsum tax-cum-subsidy. Clapp (1985) extends the basic model to a spatial dimension. Riley (1985) examines other behavioral rules to obtain a Nash Equilibrium.

On the other hand, Rosenthal and Weiss (1984) and Dasgupta and Maskin (1986) show that a Nash Equilibrium does exist in mixed strategies when it does not in pure strategies.

in which a defecting player can not be better off by his defection when some other players are allowed to react to the defection. What equilibrium emerges depends on the nature of restrictions on the space of reaction strategies imposed by the equilibrium concept. For example, reactors are restricted to choose between alternatives of holding or withdrawing their initial offers in the Wilson (1977) Anticipatory Equilibrium, which coincides to the Nash Equilibrium when the latter does exist and which equals the Pareto efficient pooling contract otherwise; only two, more precisely all but one, of the potential reactors are allowed to move one by one, independent of other potential reactors, in the Riley (1979a) Reactive Equilibrium that is asserted to be the Pareto efficient separating contracts.

None of these concepts, however, seems to have succeeded in establishing itself as a generally accepted one in the literature, though the Riley Reactive Equilibrium has one strong point in that it does exist, while neither Nash nor Wilson Equilibrium does, in a model with a continuum of types of the informed agents. This paper is an attempt to search for another reasonable equilibrium concept in a simple screening model of two types.

The first of our main results is that the Riley Reactive Equilibrium is not unique in a simple model of two types. The concept of Riley Reactive Equilibrium was originally built up to implement the schedule of Pareto efficient separating contracts in a weakly competitive market with asymmetric information and with a continuum of types. It is shown in this paper that not only the pair of Pareto efficient separating contracts (Riley's original Reactive Equilibrium) but also some Pareto inefficient pooling contracts which give positive profits to the uninformed are Riley Reactive Equilibria.

Besides this problem, the concept of Riley Reactive Equilibrium involves several unreasonable requirements in its structure. Correcting them, we define a Perfect Market Modified Reactive Equilibrium. Modifications are (a) all of the potential reactors that are neglected in the Riley Reactive Equilibrium are allowed, if they want, to react identically with the second reactor, (b) free entry is assumed, (c) the criterion by which a defector (or a new entrant) eventually decides to defect (or enter) is set by his payoff at the final, not an intermediate, stage of the whole reaction process, and (d) Market Modified Reactive Equilibria are refined by a kind of perfection based on tremble test on the part of the uninformed. With these modifications, a Perfect Market Modified Reactive Equilibrium seems to be more in line with the spirit of Riley, but is more reasonable equilibrium, than the original Riley Reactive Equilibrium. Then, it is shown that a unique Perfect Market Modified Reactive Equilibrium is the pair of Pareto efficient separating contracts, i.e., the asserted Riley Reactive Equilibrium. This is done in Section 3.

In Section 4, we extend the Perfect Market Modified Reactive Equilibrium further by introducing strategic aspects among all reactors and propose a Perfect Market Generalized Reaction Equilibrium. In a Generalized Reaction Equilibrium, a defector who can profitably defect unilaterally can not be better off when other

players choose their generalized reactions against the defector, the generalized reactions being recursively defined by a Generalized Reaction Equilibrium of the subgame played by the remaining potential reactors. The third of our main results is that a unique Perfect Market Generalized Reaction Equilibrium coincides with the Perfect Market Modified Reactive Equilibrium, and hence with the original Riley Reactive Equilibrium.

These results are surprising. They provide the Riley Reactive Equilibrium with solid theoretical foundations: in an equilibrium, information is conveyed to the uninformed in the market. They present a striking contrast to Hellwig's (1987) recent work showing that his Stable Sequential Nash Equilibrium coincides with the Wilson Equilibrium and the Grossman (1977) Disassembling Equilibrium: in terms of our model of two types, these equilibria switch from the pair of Pareto efficient separating contracts to the Pareto efficient pooling contract when Nash Equilibrium in pure strategies does not exist.<sup>4</sup>

In the next Section, we describe the economy to be studied, which is now fairly standard in the literature of screening and/or signalling, introduce several notations, and explain a Nash Equilibrium as a benchmark of our study. Sections 3 and 4 are the main part of this paper as described above. We conclude in Section 5.

## 2. MODEL

Consider Hellwig's (1987) model of a credit market.<sup>5</sup> There are two types ( $i = H, L$ ) of borrowers (firms). Each borrower of type  $i$  has a risky project whose return is  $R_i$  with probability  $P_i$  and zero otherwise. It is assumed that  $R_H > R_L$ ,  $P_H < P_L$ , and  $P_H R_H < P_L R_L$ . The project is financed by a bank loan  $Z$  of a fixed amount. So type  $L$  borrowers have a lower risk of default, and will be called a low risk (or  $L$ -) type. Similarly, a type  $H$  is a high risk type. The proportion of type  $H$  agents is denoted by  $\delta$ . It is a crucial parameter that determines whether a Nash Equilibrium exists or not. Each borrower has an end-of-period endowment  $W < Z$ .

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<sup>4</sup>Grossman Equilibrium and Hellwig Equilibrium can be briefly explained as follows. At stage 1, the uninformed agents offer contracts to the informed; at stage 2, the informed choose the best among the offers available; at stage 3, the uninformed reject unprofitable applications. At an example for a credit market, which is taken up at Section 2 below, the uninformed are banks, and the informed are firms which apply for loans. Grossman (1979) assumes that, given stage 3,  $H$ -types would not choose among the offers in a myopically optimal way, which may reveal their true types, but would mimic the behavior of  $L$ -types. Hellwig Stable Sequential Equilibrium is one of Nash Equilibria for the 3-stage game. Since the set of Nash Equilibrium is large, Hellwig refines it by the sequential rationality of Kreps and Wilson (1982) and by the stability of Kohlberg and Mertens (1986) to get a unique equilibrium. See, footnote 9 below for these equilibrium contracts in our basic model of Section 2.

<sup>5</sup>Hellwig's model, which is based on Bester (1987), is typical of those discussed in the literature, which is a two-type self-selection model. Models of insurance markets in Rothschild and Stiglitz (1976) and Wilson (1977), that of labor markets in Spence (1973), and that of product markets in Riley (1979b) have the same structure.

A loan contract is denoted by  $\gamma = (C, r)$  where  $C \leq W$  is collateral requirement and  $r$  is the amount of repayment when the project is successful.

Firms are assumed to be risk averse and to have an identical utility function  $u(\cdot)$ . The expected utility  $V_i(\gamma)$  of a firm  $i$  with a contract  $\gamma$ , and the expected profits  $\pi^i(\gamma)$  of a bank from a loan contract  $\gamma$  with a type  $i$  firm are, respectively.

$$\begin{aligned} V_i(\gamma) &= P_i u(W + R_i - r) + (1 - P_i) u(W - C), \\ \pi^i(\gamma) &= P_i r + (1 - P_i) C - (1 + \rho)Z, \end{aligned} \quad (i = H, L)$$

where  $\rho$  is a fixed cost of funds to banks. We assume that banks face a perfectly elastic supply of funds at a rate  $\rho$ . When a contract  $\gamma$  is purchased by both L- and H-type agents, a bank which is assumed to be risk neutral, earns

$$\pi(\gamma) = \delta \pi^H(\gamma) + (1 - \delta) \pi^L(\gamma)$$

We develop here the slope characteristics of utility and profit functions, which will be used in the pictorial proofs of our results. In  $C$ - $r$  space, the indifference curve  $V_i$  of type  $i$  are strictly concave and downward sloping because

$$\left. \frac{dr}{dC} \right|_{V_i} = - \frac{1 - P_i}{P_i} \frac{u'(W_f)}{u'(W_s)} < 0, \text{ and}$$

$$\left. \frac{dr^2}{dC^2} \right|_{V_i} = - \frac{1 - P_i}{P_i} \frac{u'(W_f) u''(W_s) [dr/dC|_{V_i} - u''(W_f) u'(W_s)]}{[u'(W_s)]^2} < 0,$$

where  $W_f = W - C$  and  $W_s = W + R_i - r$ . At a point  $(C, r)$ , an H-type's indifference curve is steeper than that of an L-type's since  $R_H > R_L$ . Thus, the so called single-crossing property holds for our model.

The locus of banks' zero-profit contracts for type  $i$  is given by  $\pi^i(\gamma) = 0$  or

$$(2.1) \quad r = -[(1 - P_i)/P_i]C + (1 + \rho)Z/P_i \quad (i = H, L)$$

Since  $\gamma$  is assumed to be less than  $R_i$ ,  $u'(W_f) > u'(W_s)$ . Thus,  $i$ -type's indifference curves are steeper than banks' zero profit line for type  $i$ . For analytical convenience, we assume that L-type's indifference curves are steeper than banks' zero profit line for pooling contracts:

$$(2.2) \quad - \frac{1 - P_L}{P_L} \frac{u'(W)}{u'(W + R_L - \gamma P(\delta))} < - \frac{1 - \bar{p}(\delta)}{\bar{p}(\delta)}$$

The left-hand side of (2.2) is the slope of an L-type's indifference curve evaluated at the Pareto efficient pooling contract  $\gamma P(\delta)$ , where

$$(2.3) \gamma P(\delta) = (0, \gamma P(\delta)), \gamma P(\delta) = (1 + \rho)Z / \bar{p}(\delta), \text{ and } \bar{p}(\delta) = \delta P_H + (1 - \delta)P_L$$

The contract  $\gamma P(\delta)$  is the C-axis intercept in C- $\gamma$  plane of the line  $\pi(\gamma) = 0$ . So the left hand side of (2.2) is the maximum of the slopes of the given indifference curve and the right hand side is the slope of the line  $\pi(\gamma) = 0$ .

An important assumption in this paper is an informational asymmetry: each firm knows its true type while banks a priori can not identify the true type of an individual firm but know the characteristics of the two types including the value of  $\delta$ , which are public information. Thus firms are called the informed and banks the uninformed. But, there arises no problem of moral hazard in this model because the size of project of loan is exogenously fixed.

There are many potential borrowers and many banks in the market. Let  $M$ ,  $N_i$ , and  $N$  be the set of banks, of type  $i$  borrowers, and of all borrowers, respectively. The subscript  $q$  denotes a typical bank, while the subscript  $i$  refers to type  $i$  ( $= H, L$ ) borrowers. And, let  $|M| = m$ ,  $|N_i| = n_i$ , and  $|N| = n = n_H + n_L$ , where  $|X|$  denotes the number of elements in the set  $X$ .

We consider a two-stage game in which the uninformed move first. Let a strategy  $x_q$  of an uninformed agent  $q$  be an offer of a pair of contracts  $x_q = (\gamma_q^H, \gamma_q^L)$ ,  $\gamma_q^i$  being designed for  $i$ -type agents. Offering a contract  $\gamma_q^i$  means the commitment that bank  $q$  will accept loan applications for it, whatever type of firms and however many firms may apply for it. Let  $x^M = U_{i,q} \{ \gamma_q^i \}$  be the set of offered contracts. Let an informed agent  $i$ 's strategy  $x_i$  be his purchase of a contract,  $f(i, x^M)$ , when offered  $x^M$ . Let  $x_M = (x_q: q \in M)$ ,  $x_N = (x_j: j \in N)$ , and  $x = (x_M, x_N)$  be  $m$ -,  $n$ - and  $(m + n)$ -vectors, respectively.

Let  $Y_j$  be the set of feasible strategies  $x_j$ ,  $j = q, i, M, N$ . We assume that each contract offered by a bank must yield nonnegative expected profits to the bank. By this we assume away cross-subsidization among contracts offered by a single bank.<sup>6</sup> This assumption represents a kind of banks' individual rationality. For a contract to be individually rational to the firms, it must give them no less expected utility than  $u(W)$ , the utility of a firm with no loan contract. Then the set  $Y_q$  of feasible strategies of a typical bank  $q$  is

$$Y_q = \{ x_q = (\gamma_q^H, \gamma_q^L): (i) \pi^i(\gamma_q^i) \geq 0, \forall i(\gamma_q^i) \geq u(W), i = H, L, v^H(\gamma_q^H) \geq V^H(\gamma_q^L), V^L(\gamma_q^L) \geq V^L(\gamma_q^H) \text{ and } C \leq W, \text{ or } (ii) \gamma_q^H = \gamma_q^L = \gamma_q, \pi(\gamma_q) \geq 0, \forall i(\gamma_q) \geq u(W), i = H, L, \text{ and } C \leq W) \}.$$

<sup>6</sup>Miyazaki (1977) introduces the cross-subsidization among types by the uninformed agents. That is, the uninformed offer a portfolio of contracts in which some contracts earn positive profits and others negative ones so that the portfolio breaks even as a whole; they subsidize the informed agents who purchase the latter contracts. Miyazaki emphasizes this cross-subsidization as a characteristic of the internal labor markets. So does Cave (1984) for the insurance markets. In this paper the traditional assumption of no cross-subsidization is adopted for analytical simplicity.

Parts (i) and (ii) in  $Y_q$  define feasible separating and pooling contracts, respectively. Note that  $V^i(\gamma^i) = u(W)$  implies that  $\gamma^i = (C^i, r^i) = (0, R_i)$ . Since firms move after banks' offer,  $Y_i$  is given by  $x^M$ .

Given a combination  $x$  of strategies, the payoff to a type  $i$  agent is

$$\tilde{H}_i(x) = V^i(x_i) = V^i(f(i, x^M))$$

It is independent of the behavior of the other informed agents. Thus, the informed are assumed to always choose their strategies in order to maximize their expected utility. That is,

$$(2.4) \quad x_i = f(i, x^M) \in \operatorname{argmax}_{\gamma \in x^M} V^i(\gamma), \quad (i = H, L)$$

where the following tie-breaking rules are commonly adopted: (a) when more than one bank offers an identical contract, borrowers who apply for it choose randomly among those banks and (b) when a borrower is indifferent between two different contracts ( $\gamma_q^H$  and  $\gamma_q^L$ ) offered by bank  $q$ , he chooses the one which is more profitable to the bank. The assumption (2.4) is termed the informed agents' Optimal Reaction Assumption by Stiglitz and Weiss (1984).<sup>7</sup>

Given a combination  $x$  of strategies, a bank  $q$ 's payoff  $\tilde{H}_q(x)$  depends on how many firms choose  $q$ 's offer. Let  $\tilde{H}_q^i(x)$  be bank  $q$ 's payoff from contracts with type  $i$  firms. Then, formally we have

$$\begin{aligned} \tilde{H}_q^i(x) &= \begin{cases} (n_i/n_o) \pi^i(\gamma_q^i), & \text{if } \gamma_q^i = f(i, x^M) \text{ and } \gamma_l^i = \gamma_q^i \text{ for all } l \in M_o^i \subset M \\ 0, & \text{if } \gamma_q^i \neq f(i, x^M), \end{cases} \\ \tilde{H}_q(x) &= \tilde{H}_q^H(x) + \tilde{H}_q^L(x), \end{aligned}$$

where  $n_o^i = /M_o^i/$ ,  $M_o^i$  being the set of banks which attract type  $i$  borrowers.

Given the Optimal Reaction Assumption of (2.4), our two-stage game can be reduced to a simpler one-stage game. If the informed (second-movers) are allowed to always choose their strategies optimally, then the informed become totally passive players in a two-stage game. That is, given the strategies of the first-movers, there is no possibility of strategic actions for the second moves: the strategies of the uninformed agents alone are sufficient to determine the payoffs to all the players. Thus we may remove the passive players together with the stage 2, and obtain the identical results from the reduced game. And the payoff of a bank of  $q \in M$  may be succinctly written as

<sup>7</sup>In Riley (1979a), this assumption, together with zero profit condition for each loan contract offered by a bank and purchased by each type of borrower, constitutes the condition for informational consistency.

$$(2.5) H_q(x_M) = \tilde{H}_q(x) = \tilde{H}_q(x_M, \{f(i, x_M)\}_{i \in N})$$

Though this reduced game is played by the uninformed only, their payoffs are determined by the optimal reactions of the informed. As a benchmark, the first equilibrium concept employed for this game is a Nash Equilibrium.

**NASH EQUILIBRIUM (NE):** A combination  $x_M^0$  of banks' strategies is a Nash Equilibrium if, for any  $q \in M$  and  $x_q \in Y_q$ ,  $H_q(x_M^0) > H_q(x_q, x_{M/q}^0)$ , where  $(M/q)$  denotes the set of banks other than bank  $q$ .

Since  $x_M^0 = ((\gamma_q^{HO}, \gamma_q^{LO}), q \in M)$  and each bank offers an indential offer in an equilibrium, the equilibrium can be identified simply by the equilibrium contract  $(\gamma^{HO}, \gamma^{LO})$ .

For future reference, we introduce several notations. First, we define a contract  $\gamma^L(\gamma)$  for a given contract  $\gamma$  by

$$(2.6) V^H(\gamma^L(\gamma)) = V^H(\gamma) \text{ and } \pi^L(\gamma^L(\gamma)) = 0$$

A new contract  $\gamma^L(\gamma)$  is an L-type contract which is incentive compatible for H-types given a contract  $\gamma$  and which breaks even when purchased by L-types. Second, given a contract  $\gamma$  define

$$(2.7) \begin{aligned} SH(\gamma) &\equiv \{\gamma': V^H(\gamma') > V^H(\gamma), \pi^H(\gamma') > 0\} \\ SL(\gamma) &\equiv \{\gamma': V^L(\gamma') > V^L(\gamma), V^H(\gamma') \leq V^H(\gamma), \pi^L(\gamma') \geq 0\} \\ SP(\gamma) &\equiv \{\gamma': V^L(\gamma') > V^L(\gamma), V^H(\gamma') > V^H(\gamma), \pi(\gamma') > 0\} \\ sP(\gamma) &\equiv \{\gamma': V^L(\gamma') \geq V^L(\gamma), V^H(\gamma') \geq V^H(\gamma), \pi(\gamma') \geq 0\} \\ B_\varepsilon(\gamma) &= \{\gamma': \|\gamma' - \gamma\| < \varepsilon\} \end{aligned}$$

If a defector  $j$  offers  $x_j^H = \gamma_j^H \in SH(\gamma)$ ,  $x_j^L = \gamma_j^L \in SL(\gamma)$ , and  $x_j^P = \gamma_j^P \in SP(\gamma)$ , then he can profitably attract only H-types, L-types, and all types of customers, respectively.  $B_\varepsilon(\gamma)$  is an arbitrarily small neighborhood of a contract  $\gamma$  in  $C$ -r space.

Third, denote by  $(\gamma^{H*}, \gamma^{L*})$  the pair of Pareto efficient separating contracts, where

$$(2.8) \gamma^{H*} = (CH^*, \gamma^{H*}), CH^* = 0, \gamma^{H*} = (1 + )Z/P_H, \text{ and } \gamma^{L*} = \gamma^L(\gamma^{H*})$$

Since  $\gamma^{H*}$  is the first best for H-types under our assumption of no crosssubsidization, there is no pair of contracts other than  $(\gamma^{H*}, \gamma^{L*})$  which is incentive compatible and which can given both H- and L-types higher expected utilities. This implies that  $(\gamma^{H*}, \gamma^{L*})$  is the pair of Pareto efficient separating contracts.

Finally, define  $\gamma(t)$ ,  $t = 1, \dots, m-1$ , by

$$(2.9) V^L(\gamma(t)) = V^L(\gamma^L(\gamma(t-1))) \text{ and } \pi(\gamma(t)) = 0, \text{ if } SP(\gamma^L(\gamma(t-1))) \neq \emptyset,$$

and  $\delta_t, t=0, \dots, m-1$  ( $\delta_0 > \delta_1 > \dots > \delta_{m-1}$ ), by

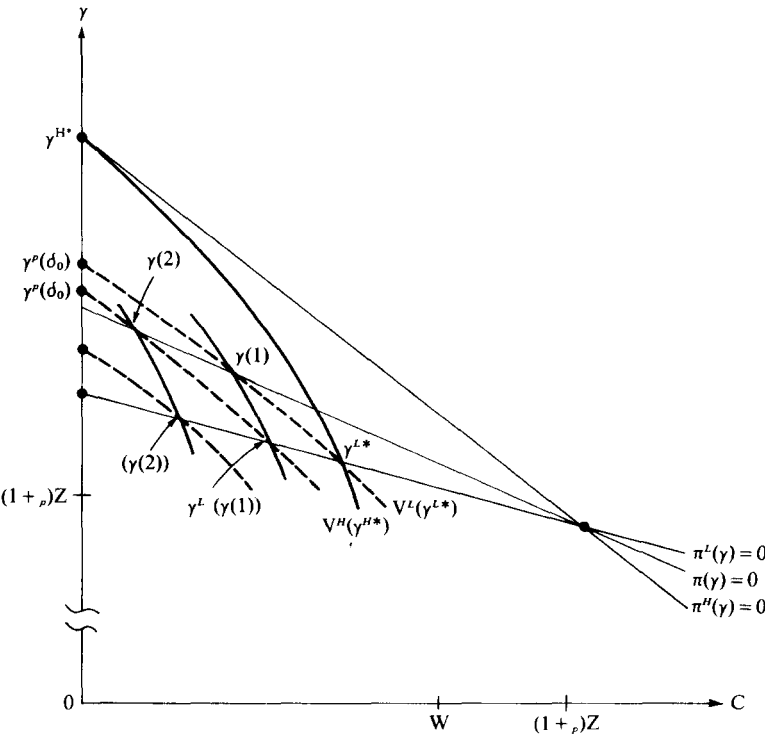
(2.10)  $V^L(\gamma^P(\delta_t)) = V^L(\gamma^L(\gamma(t)))$ ,

where  $\gamma^L(\gamma(0))$  is set to  $\gamma^{L*}$  and  $\gamma^P(\delta)$  is defined by (2.3).

Some of these notations are illustrated in Figure 1. The straight lines denoted by  $\pi^H(\gamma) = 0$ ,  $\pi^L(\gamma) = 0$ , and  $\pi(\gamma) = 0$ , are the locus of contracts which give banks zero expected profits if purchased by the firms of typ H, L and both (algebraically equation (2.1) with  $P_i$  replaced by  $P_H$ ,  $P_L$ , and  $\bar{p}$ , respectively). The contracts below these lines incur negative profits to banks. At the cross-section of two lines  $\pi^H(\gamma) = 0$  and  $\pi^L(\gamma) = 0$ ,  $\gamma = C = (1 + \rho)Z$ . The type i's indifference curve passing through a contract  $\gamma$  is denoted by  $\tilde{V}^i(\gamma)$ . L-type's indifference curves denoted by dotted curves are flatter than H-type's. The lower leftward curves represent higher utility levels. In figure 1,  $\delta$  is assumed to be  $\delta \in (\delta_2, \delta_1)$ .

From the above definitions (2.9) and (2.10) immediately follows:

LEMMA 1:  $SP(\gamma^L(\gamma(t))) = \phi$  if  $\delta \geq \delta_t, t = 0, \dots, m-1$ .



[Figure 1] Non-existence of a Nash Equilibrium :  
the case of  $\delta \in (\delta_2, \delta_1)$

PROOF: The proof is facilitated by using Figure 1.<sup>8</sup> Consider first the case of  $t=0$ . If  $\delta=\delta_0$ , then  $V^L(\gamma^P(\delta_0))=V^L(\gamma^L(\gamma(0)))=V^L(\gamma^{L*})$  from (2.9) and (2.10). Then, an L-type indifference curve  $\bar{V}^L(\gamma^{L*})$  will intersect the line  $\pi(\gamma)=0$  at a contract  $\gamma^P(\gamma_0)$  by (2.2), and hence  $SP(\gamma^{L*})=\phi$ . As  $\delta$  rises above  $\delta_0$ , the line  $\pi(\gamma)=0$  turns clockwise, while an indifference curve  $\bar{V}^L(\gamma^{L*})$  remains unchanged. Thus,  $SP(\gamma^L(\gamma(0)))=SP(\gamma^{L*})=\phi$  for  $\delta\geq\delta_0$ .

If  $\delta<\delta_0$ ,  $SP(\gamma^L(\gamma(0)))\neq\phi$  and  $\gamma(1)$  is defined by (2.9). But, since  $V^L(\gamma^P(\delta_1))=V^L(\gamma^L(\gamma(1)))$  from (2.10),  $SP(\gamma^L(\gamma(1)))=\phi$  for  $\delta\geq\delta_1$  by the same argument as above. By construction, this argument can be extended up to  $t=m-1$ . In Figure 1,  $SP(\gamma^L(\gamma(2)))=\phi$  since  $\delta>\delta_2$ . Q.E.D.

Now we present the potential non-existence theorem originally due to Rothschild and Stiglitz (1976).

**THEOREM 1:** *The pair  $(\gamma^{H*}, \gamma^{L*})$  of Pareto efficient separating contracts is a unique NE if  $\delta>\delta_0$ , and there is no NE in pure strategies otherwise..*

PROOF: See Appendix.

Theorem 1 is illustrated in Figure 1. If  $\delta>\delta_0$  so that the line  $\pi(\gamma)=0$  does not intersect an L-type's indifference curve  $\bar{V}^L(\gamma^{L*})$ , then  $(\gamma^{H*}, \gamma^{L*})$  is a NE. Figure 1 shows a case of  $\delta<\delta_0$ . In this case,  $\gamma_j^P\in SP(\gamma^{L*})$ , if offered, could attract both H- and L- types away from  $(\gamma^{H*}, \gamma^{L*})$ . This means that  $(\gamma^{H*}, \gamma^{L*})$  is not a NE.<sup>9</sup> Intuitively, when the proportion of H-type is small, a pooling through which L-type borrowers subsidize H-types is preferred by L-types to a separation which can be obtained by paying for incentive compatibility.

### 3. A REINTERPRETATION OF RILEY REACTIVE EQUILIBRIUM

Consider a Riley Reactive Equilibrium in our basic model of Section 2. When a NE in pure strategies does not exist, the would-be NE, that is, the pair  $(\gamma^{H*}, \gamma^{L*})$  of Pareto efficient separating contracts has been asserted as the Riley Reac-

<sup>8</sup>The proofs of Lemmas and Theorems in this paper are done geometrically. Such an approach is common in this literature though it is sometimes tedious.

<sup>9</sup>If  $\delta<\delta_0$  such that Nash Equilibrium does not exist, then  $\gamma^P(\gamma)$  defined by (2.3) is a Wilson Anticipatory Equilibrium since a defector, if offers  $\gamma_j^P\in SP(\gamma^{L*})$ , becomes worse off when all other potential reactors withdraw their losing contracts  $\gamma^P(\gamma)$  and hence all of H-types also apply to the defector.  $\gamma^P(\gamma)$  is also a Grossman Disassembling Equilibrium since all H-types, if offered  $\gamma_j^P$  by a defector, does not choose the new myopically preferred contract and hence the defector does not have an incentive to defect. This is so because, conjecturing that if there is any firm that chooses  $\gamma_j^P$  in the presence of  $\gamma^P(\gamma)$ , it is of H-type, the uninformed will reject H-type's applications. Finally, it can also be shown that  $\gamma^P(\gamma)$  is a Hellwig Stable Sequential Equilibrium. But the argument is rather complicated to be presented here. Interested readers should consult Hellwig (1987).

tive Equilibrium contract.<sup>10</sup> Formally, a Riley Reactive Equilibrium is defined as follows:

**RIELY REACTIVE EQUILIBRIUM (RRE)** [Riley (1979a)]: A combination  $x_M^0$  of strategies is a Riley Reactive Equilibrium either (a) if

$$(3.1a) \quad H_j(x_j, x_{M/j}^0) \leq H_j(x_M^0)$$

for any  $j \in M$  and  $x_j \in Y_j$  or (b) if, for any  $j \in M$  and  $x_j^l \in Y_j$  such that

$$(3.1b) \quad H_j(x_j^l, x_{M/j}^0) > H_j(x_M^0),$$

there exists a reactor  $k$  and  $x_k^l \in Y_k$  for which

$$(3.1c) \quad H_k(x_j^l, x_{M/j}^0) \leq H_k(x_M^0).$$

$$(3.1d) \quad H_k(x_j^l, x_k^l, x_{M/j, k}^0) > H_k(x_j^l, x_{M/j}^0),$$

$$(3.1e) \quad H_j(x_j^l, x_k^l, x_{M/j, k}^0) < H_j(x_M^0), \text{ and}$$

$$(3.1f) \quad H_k(x_j^l, x_k^l, x_l^l, x_{M/j, k, l}^0) \geq H_k(x_j^l, x_{M/j}^0)$$

for another reactor  $l \in (M/j, k)$  and for all  $x_j^l \in Y_1$

The part (a) defines nothing but a NE in pure strategies. It is clearly implied by part (b), but is not explicitly stated in Riley's original definition. Since Riley considered a model with a continuum of types and, as he proved, there is no NE in pure strategies in such a model, he may omit the part (a). In our model of two-type, however, it would be better to be explicitly stated. The part (b) requires that, given a defection  $x_j^l$  profitable before any reaction, there be a reactor  $k$  with  $x_k^l$  who has an incentive to react—in the sense that he is not better off by the defector [(3.1c)] and that he becomes strictly better off with his own reaction [(3.1d)] and that his reaction is riskless or safe from another reactor's attack [(3.1f)]—and who can ruin the defector [(3.1e)].

The RRE was constructed to show that, in a model with a continuum of types, the schedule of Pareto efficient separating contracts can be implemented in a (weakly) competitive market. In our model of two types, it corresponds to a pair of contracts  $(\gamma^{H^*}, \gamma^{L^*})$ . But, there are several problems with the concept of RRE. After discussing them, we will propose a more reasonable reactive equilibrium which, being basically in line with Riley's, may be viewed as a reinterpretation of RRE.

The first problem with the RRE is that the condition (3.1e) for a defector's ruin is required to hold at an intermediate stage in the process of reactions. Therefore, there may be RRE contracts in which for some  $x_k^l \in Y_1$  both (3.1e) and (3.1f) hold but the condition (3.1e) evaluated after  $l$ 's reaction, i.e.,

$$(3.1\acute{e}) \quad H_j(x_j^l, x_k^l, x_l^l, x_{M/j, k, l}^0) < H_j(x_M^0)$$

does not hold. In such a case, the defector might be better regarded as having

<sup>10</sup>See Riley (1979b) for a model of two types, and Riley (1979a), Besanko and Thakor (1987), and Engers and Fernandez (1987) for a model of a continuum of types.

an incentive to defect. For it is not (3.1e) but (3.1é) that matters to the defector by our assumption (2.4). Thus the contracts at hand would rather not be counted as a reasonable reactive equilibrium. The following Lemma 2 shows that this problem arises for the asserted RRE.

**LEMMA 2:** *A pair  $(\gamma^H, \gamma^L)$  of contracts is an RRE. But it does not satisfy for all  $x_j \in Y_I$  if  $\gamma < \gamma_I$ .*

**PROOF:** See Appendix.

The second problem with the RRE is that some contracts which give banks positive expected profits pass the test conditions defining the RRE, as the following Theorem 2 shows.<sup>11</sup>

**THEOREM 2:** *Any pooling contract  $\gamma \in S^p$  ( $\gamma^L$ ) is a RRE.*

**PROOF:** See Appendix.

The intuition behind this Theorem is simple: with an initial contract yielding a positive profit, a defector can make a larger profit at the expense of other uninformed agents, who are then left with zero profit; a reactor can design a contract which absorbs the defector's positive profits and which can not be made to earn a negative profit by a second reactor.

The second problem may be partially alleviated by introducing the assumption of free entry. Without formally modelling free entry, we simply assume that free entry amounts to the existence of outside players who have zero (expected) profits initially, that is,  $H_j(x_M^0) \equiv 0$  when  $j$  refers to a new entrant whatever  $x_M^0$  may actually be. Thus, when such players enter our game and earn non-negative expected profits after all potential reactors' moves, they are assumed to have an incentive to enter. A Reactive Equilibrium against which no entrant has an incentive to enter may be called a *Market Reactive Equilibrium*.<sup>12</sup>

The third problems is that, though the ex ante and ex post incentives for the first reactor  $k$  to move given a defection  $x_j^1$  are taken into account, those for the

<sup>11</sup>The reader may also prove that a pair  $(\gamma^H, \gamma^L)$  of separating contracts which give banks positive expected profits, i.e.,  $(\gamma^H, \gamma^L) \in \{(\gamma^H, \gamma^L): \gamma^L = \gamma^L(\gamma^H), \pi^H(\gamma^H, \gamma^L) \in Y_q\}$ , is also an RRE. See Nho (1988, p. 14) for a proof.

The fact that those contracts which give banks positive profits are RRE's but not NE's is not peculiar to a market with asymmetric information. For a certainty world, the same result follows if a bank's strategy is a contract  $\gamma = (C, r)$ , i.e., terms of trade being similar to price and quantity.

<sup>12</sup>In a signalling model in which there usually are many Nash Equilibria, some researchers adopt the condition of zero profits (or informational consistency) as one criterion of refinements. For example, see Waldman (1984) and Nalebuff and Scharfstein (1987). Waldman calls the resulting equilibrium a Market-Nash Equilibrium. The justification for imposing a zero profit condition is that free entry implies zero profits in an equilibrium. But, it is not shown how free entry enforces a zero profit within their models of a market game. Within the model of this paper, it does not as the following Theorems 3 and 5 show.

remaining players  $(M/j, k)$  are ignored. In particular, the final payoffs of the neglected players  $(M/j, k, l)$  may be negative, worse off than those obtainable from dropping their initial offers. In such a case, a more reasonable reactive equilibrium would allow them to react. To do this, we must consider a sequence of reactions,  $x_k^l, x_l^l, \dots, x_m^l$ . How to model these reactions by all potential reactors in a most general way is not an easy task. Familiar reactive equilibrium concepts such as Wilson Anticipatory Equilibrium and RRE may be viewed as ad hoc truncations of a more general reaction space. In Wilson's reaction rules, all potential reactors are restricted to identically withdraw losing contracts: feasible strategies for reactions are either to withdraw, or to hold, the initial contracts. In Riley's, restricted is not the space of reactive strategies but the space of reactors: only two members ( $k$  and  $l$ ) among potential reactors  $(M/j)$  are allowed to react.

Extending Riley-type reaction rules to all others—piece-wise realizations of both profitability and safeness by the successive reactors—seems to be very complicated. Moreover, it could not encompass strategic aspects among reactors in a meaningful manner. Instead, as a first attempt, we will assume that the neglected players  $(M/j, k, l)$  in an RRE take their strategies identical to the second reactor  $l$ 's reaction. By this simplifying assumption, it is at least guaranteed that no non-reacting player suffers from the losses due to non-reaction. An alternative specification, under which strategic aspects among reactors as a whole are taken into account, will be discussed in the next Section.

Basing upon the above discussions, we define Market Modified Reactive Equilibrium (Market-MRE) as follows:

**MARKET MODIFIED REACTIVE EQUILIBRIUM (MARKET-MRE):** A combination  $x_M^0$  of strategies is a Market-MRE either (a) if

$$(3.2a) \quad H_j(x_j, x_{M/j}^0) \leq H_j(x_M^0)$$

for any  $j \in M$  and  $x_j \in Y_j$ , or (b) if, for any  $j \in M$  and  $x_j^l \in Y_j$  such that

$$(3.2b) \quad H_j(x_j^l, x_{M/j}^0) > H_j(x_M^0),$$

there exists a reactor  $k$  and  $x_k^l \in Y_k$  for which

$$(3.2c) \quad H_k(x_j^l, x_{M/j}^0) \leq H_k(x_M^0),$$

$$(3.2d) \quad H_k(x_j^l, x_k^l, x_{M/j,k}^0) > H_k(x_j^l, x_{M/j}^0),$$

and there are reactions  $x_{M/j,k}^l = (\{x_l^l\}_{l \in (M/j,k)})$ ,  $x_l^l$  being identical for all  $l \in (M/j, k)$ , such that

$$(3.2c') \quad H_l(x_j^l, x_k^l, x_{M/j,k}^0) \leq H_l(x_M^0), \text{ all } l \in (M/j, k)$$

$$(3.2d') \quad H_l(x_j^l) > H_l(x_j^l, x_k^l, x_{M/j,k}^0), \text{ all } l \in (M/j, k)$$

$$(3.2e') \quad H_j(x_M^0) < H_j(x_M^0)$$

$$(3.2f') \quad H_k(x_M^0) \geq H_k(x_j^l, x_{M/j}^0),$$

where  $x_M^l = (x_j^l, x_k^l, x_{M/j,k}^l)$ . And whenever  $j$  refers to a new entrant,

$$(3.2g) \quad H_j(x_M^0) = 0.$$

Comparing the definition of Market-MRE with that of RRE, we know that (3.2a-

d) are identical to (3.1a-d) and (3.2c'-d') that are newly added in order to represent the incentives for all the remaining players  $(M/j, k)$  to react. (3.2e'-f') are versions of (3.1e-f) evaluated after all players have reacted. Note that the conditions (3.2d'-f') need to be satisfied not for all  $x_{M/j, k} \in Y_{M/j, k}$  but for at least one  $x_{M/j, k}^l \in Y_{M/j, k}$  since the incentives for players  $(M/j, k)$  to react are taken into account by (3.2c'-d'). Finally, (3.2g) represents the assumption of free entry.

Now we can prove that

**THEOREM 3:** *The set of Market-MRE is  $\{(\gamma^{H*}, \gamma^{L*})\} \cup \bar{s}^P(\gamma^{L*})$ , where  $\bar{s}^P(\gamma^{L*})$  defined by (2.7) is empty if  $\delta > \delta_0$*

**PROOF:** See Appendix.

A problem with our Market-MRE is its non-uniqueness: the set  $\bar{s}^P(\gamma^{L*})$  is still an infinite set. This problem presents a striking contrast to that of potential non-existence of Nash Equilibrium in our model. One might think of various refinements of the Market-MRE's. One might refine the concept of the Market-MRE by introducing an error either on the part of the uninformed in reacting or on the part of the informed. In the former case, we require an equilibrium not to be unraveled if at least one of the uninformed is assumed to fail in reacting. In the latter case, at least one of the informed agents is assumed to choose randomly between the two indifferent alternatives, violating the part (b) of tie-breaking rules in (2.4). We adopt the uninformed agents' error as a test of refinement and call the resulting Market-MRE a *Perfect Market-MRE* (PM-MRE) by analogy with a Perfect Nash Equilibrium of Selten (1975).<sup>13</sup> Then we can prove

**THEOREM 4:** *A pair  $(\gamma^{H*}, \gamma^{L*})$  is a unique PM-MRE.*

**PROOF:** See Appendix.

Intuitively, Theorem 4 is obvious. Recall that the original RRE has been asserted to be  $(\gamma^{H*}, \gamma^{L*})$ . Since Riley confined his attention only to informationally consistent (i.e., zero profit) contracts, free entry was in part considered implicitly. And, the reaction rules in the concept of RRE already requires the existence of non-reacting players. Thus, the concept of the original RRE seems to be almost the same as that of our PM-MRE. From Theorem 4, we can interpret the original RRE  $(\gamma^{H*}, \gamma^{L*})$  as a PM-MRE.

Riley's (1979b, p. 307) arguments for the RRE and against the Wilson Anticipatory Equilibrium is that "the 'reactive equilibrium' relies on a threat not from the market participants as a whole but from only one other firm ['firm' corresponds

<sup>13</sup>The idea is from Selten's (1975) trembling-hand-perfection, though our use of tremble differs from Selten's. Cave (1984) adopts the informed agents' error as a test of refinement and show that a Wilson-type equilibrium is indeed an  $\epsilon$ -perfect equilibrium.

to 'bank' in our model] (*reactions by more than one firm only serve to strengthen the equilibrium*).” [The text in the bracket and emphasis added]. Thus, Riley (1985, p. 959) regards an RRE as “the least demanding of the alternative equilibrium concepts.” In this Section we made it clear that, though reactions by more than one bank may serve to strengthen the equilibrium as can be seen from our Modified Reactive Equilibrium, reactions by all of the potential reactors can not. In other words, the existence of at least one non-reactor is crucial to the uniqueness of an RRE. We also showed that the partial non-reaction embodied in the original RRE or in the criterion of perfectness of a PM-MRE can be viewed as not “the least demanding” but rather restrictive. For under the assumptions of our model there is little difficulty in players’ detecting other players’ defection and/or reactions. Moreover, their costs of non-reaction are very high because non-reactors have to serve all H-types and hence earn negative profits. However, the assumption of partial non-reaction is less myopic than that of total non-reaction in a NE concept.

One might prove that the RRE under free entry, which may be termed a Market-RRE, is also equivalent to the PM-MRE. One reason why this one-step refinement was not taken in this paper is that the first and second problems with the RRE discussed earlier will arise in a Market-RRE. The other is to highlight the fact that the assumption on the existence of non-reactors in an RRE, which is in turn translated into the perfectness in a PM-MRE, is not less demanding but rather restrictive.

#### 4. GENERALIZED REACTION EQUILIBRIUM

It is assumed in a Market-MRE that, given the first reactor  $k$ 's reaction against a defector, the others  $(M/j, k)$  counter-react against  $k$  identically, that is, they act as if they are a single player. Under this reaction rule, the first reactor  $k$  can move strategically in consideration of the effect which the subsequent players  $(M/j, k)$  as a whole may bring on his payoff, but each player in the set  $(M/j, k)$  can not. If each of them is allowed to react in a more general way, he must not that his payoff depends not only on his own strategy but also on the others', in particular, on the last player's.

In order to model this strategic aspect among reactors, we define a strategic or generalized reaction as follows. Initially, all or the uninformed announce their strategies  $x_M^0 = (x_1^0, \dots, x_m^0)$  simultaneously. If a player  $j \in M$  offers an alternative strategy  $x_j^1 \in Y_j$  which can unilaterally improve his payoff, then the others offer their reaction strategies  $x_{M/j}^1 = (x_1^1, \dots, x_{j-1}^1, x_{j+1}^1, \dots, x_m^1)$  simultaneously. Then such a reaction  $x_{M/j}^1$  is defined to be a Generalized Reaction (GR) of players  $(M/j)$  with respect to  $x_j^1$ , if any one player, say  $k \in (M/j)$ , can not improve his payoff upon  $H_k(x_M^1)$  by an alternative strategy  $x_k^1 \neq x_k^1$  either unilaterally or with the others' GR  $x_{M/j, k}^1$  with respect to  $x_k^1$ . We call it a generalized reaction because the space of reactions by players  $(M/j)$  is not restricted in any way except that it should

be feasible, i.e.,  $x_{M/j}^l \in Y_{M/j}$ .

A Generalized Reaction Equilibrium is formally defined as follows:

**GENERALIZED REACTION EQUILIBRIUM (GRE):** A set  $x_m^0$  of strategies is a GRE either (a) if

$$(4.1a) \quad H_j(x_M^0) \geq H_j(x_j, x_{M/j}^0)$$

for any  $j \in M$  and  $x_j \in Y_j$ , or (b) if for any  $x_j^l \in Y_j$  for which

$$(4.1b) \quad H_j(x_M^0) < H_j(x_j^l, x_{M/j}^0)$$

there exists a Generalized Reaction (GR)  $x_{M/j}^l$  of players  $(M/j)$  such that

$$(4.1c) \quad H_k(x_j^l, x_{M/j}^0) \leq H_k(x_j^l, x_{M/j}^l), \text{ all } k \in (M/j)$$

$$(4.1d) \quad H_k(x_j^l, x_{M/j}^l) \geq H_k(x_j^l, x_{M/j}^0), \text{ all } k \in (M/j)$$

$$(4.1e) \quad H_j(x_j^l, x_{M/j}^l) < H_j(x_j^l, x_M^0),$$

where the GR  $x_{M/j}^l$  is a GRE of the subgame played by the remaining players  $(M/j)$ .

Generalized Reactions are recursively defined as follows:

**GENERALIZED REACTIONS (GR):** Given the first round defection  $x_j^l$ , the first round GR by the players  $(M/j)$  is a set  $x_{M/j}^l$  of strategies such that either (a)

$$(4.2a) \quad H_k(x_j^l, x_{M/j}^l) \geq H_k(x_j^l, x_k, x_{M/j, k}^l)$$

for any  $k \in (M/j)$  and  $x_k \in Y_k$ , or (b) for any second round defector  $k$  and  $x_k^l \in Y_k$  for which

$$(4.2b) \quad H_k(x_j^l, x_{M/j}^l) < H_k(x_j^l, x_k^l, x_{M/j, k}^l)$$

there exists the second round GR  $x_{M/j, k}^l$  of players  $(M/j, k)$  such that

$$(4.2c) \quad H_l(x_j^l, x_k^l, x_{M/j, k}^l) \leq H_l(x_j^l, x_{M/j, k}^l), \text{ all } l \in (M/j, k)$$

$$(4.2d) \quad H_l(x_j^l, x_k^l, x_{M/j, k}^l) \geq H_l(x_j^l, x_{M/j, k}^l), \text{ all } l \in (M/j, k)$$

$$(4.2e) \quad H_k(x_j^l, x_k^l, x_{M/j, k}^l) < H^k(x_j^l, x_{M/j}^l).$$

Without loss of generality, let  $j = 1$  and  $k = 2$ . Then, in general, the  $t$ th round GR by players  $(M/1, \dots, t)$  with respect to the  $t$ th defection  $x_t^l$  is a set  $x_{M/1, \dots, t}^l$  of strategies such that either (a)

$$(4.3a) \quad H_{t+1}(\{x\}_{i=1}^t, x_{M/1, \dots, t}^l) \geq H_{t+1}(\{x\}_{i=1}^t, x_{t+1}^l, x_{M/1, \dots, t+1}^l)$$

for any  $x_{t+1}^l \in Y_{t+1}$ , or (b) for any  $x_{t+1}^l$ , such that

$$(4.3b) \quad H_{t+1}(\{x\}_{i=1}^t, x_{M/1, \dots, t}^l) < H_{t+1}(\{x\}_{i=1}^t, x_{t+1}^{l+1}, x_{M/1, \dots, t+1}^l)$$

there is the  $(t+1)$ th round GR  $x_{M/1, \dots, t+1}^{l+1}$  of players  $(M/1, \dots, t+1)$  such that for all  $q \in (M/1, \dots, t+1)$

$$(4.3c) \quad H_q(\{x\}_{i=1}^t, x_{t+1}^{l+1}, x_{M/1, \dots, t+1}^l) \leq H_q(\{x\}_{i=1}^t, x_{M/1, \dots, t+1}^l)$$

$$(4.3d) \quad H_q(\{x\}_{i=1}^t, x_{t+1}^{l+1}, x_{M/1, \dots, t+1}^l) \geq H_q(\{x\}_{i=1}^t, x_{M/1, \dots, t+1}^l)$$

$$(4.3e) \quad H_q(\{x\}_{i=1}^t, x_{t+1}^{l+1}, x_{M/1, \dots, t+1}^l) < H_{t+1}(\{x\}_{i=1}^t, x_{M/1, \dots, t}^l)$$

The conditions (4.1a-e) for GRE are the same as those for MRE except for the GR  $x_{M/1, \dots, t}^l$ . That the  $t$ th round GR  $x_{M/1, \dots, t}^l$  is a GRE for players  $(M/1, \dots, t)$  means that none of the players  $(M/1, \dots, t)$  can better react against the  $t$ th round

defector  $t$  if he knows that, after his alternative reaction, others will again choose their best. For example, suppose that  $x_M^O$  is a GRE. And also suppose that  $x_{M/j}^l$ , is a GR of players  $(M/j)$  and that, given a GR  $x_{M/j}^l$ , a player  $K$  can profitably defect by  $x_k^2$  [(4.2b)]. Then, when  $(M/j, k)$ , react with  $x_{M/j, k}^2$ ,  $k$ 's payoff will be strictly less than his payoff under his initial reaction  $x_k^l$  [(4.2e)]. Thus knowing this, he will not deviate from  $x_k^l$ . Then the first round defector  $j$  also knows that his payoff will be less than that under the initial contract  $x_j^O$  if players  $(M/j)$  react with  $x_{M/j}^l$ . Knowing this, he will not deviate, either. In this way,  $x_M^O$  can be regarded as a kind of reactive equilibrium.

The main result of this Section follows the next Lemma.

LEMMA 3: *A pair  $(\gamma^{H*}, \gamma^{L*})$  of contracts is a GRE if  $\delta \geq \delta_{m-1}$*

PROOF: See Appendix.

Lemmas 3 is also illustrated in Figure 1 for  $\delta \in (\delta_2, \delta_1)$ . In the Figure,  $s^P(\gamma^L(\gamma(2))) = \phi$ . Given a second round defection  $x_2^2$  and GR  $x_{M/1,2}^2$ , there is no way of profitable defection for any of players  $(M/1,2)$ . Lemma 3 is similar to Lemma 1. A difference is that only the first and second round reactions are possible in the latter while reactions are possible up to  $(m-1)$  th round in the former. One might also prove that any  $\gamma \in s^P(\gamma^{L*})$  is a GRE if  $\delta \geq \delta_{m-1}$ . On the other hand  $(\gamma^{H*}, \gamma^{L*})$ , is no longer a GRE if  $\delta < \delta_{m-1}$  by Lemma 3. In such a case, it can also be shown that GRE may not exist. Intuitively, the reason is that  $(\gamma^{H*}, \gamma^{L*})$ , or one of the subsequent round GR's appearing in the proof of Lemma 3, becomes the first round GR for those candidates for GRE, but that it can not be the GR for some subsequent subgame.

However, the introduction of free entry solves most of these problems. For the number of new entrants may be regarded as infinite. Thus, free entry has two roles in our model: one is to remove most of those contracts which give banks positive expected profits from the set of GRE's, and the other is to lift the restraint of  $\delta$  on the existence of GR. The problem of multiple GRE's can be coped with the criterion of perfection defined in the previous Section. Let's define Market-GRE and Perfect Market-GRE (PM-GRE) in the same way as we did for MRE. Then we can prove the main result of this Section:

THEOREM 5: *The set of market-GRE is equivalent to Market-MRE, and the set of PM-GRE is also equivalent to that of PM-MRE.*

PROOF: See Appendix.

It is quite interesting to see that a unique PM-GRE  $(\gamma^{H*}, \gamma^{L*})$  is equivalent to a unique PM-MRE. Theorem 5 together with Theorem 4 gives a theoretical support to the asserted original RRE in different ways.

## 5. CONCLUSION

We have examined unreasonable elements in the concept of an RRE. Its straightforward improvements resulted in our PM-MRE. It coincides to the pair of Pareto efficient separating contracts, which has been incorrectly asserted as a unique RRE. This result may be viewed as a reinterpretation of an RRE.

This paper has also attempted to establish, for a competitive screening model, a new equilibrium concept which is quite general in allowing reactions. Our Market-GRE emerges when every potential reactor takes any feasible strategy strategically and simultaneously. Refining the set of Market-GRE's by a tremble test on the part of the uninformed, as was done for Market-MRE's, we get a unique PM-GRE. Surprisingly, it coincides to the PM-MRE. This result provides the pair of Pareto efficient separating contracts with another theoretical support.

This paper has the following limitations. First, one might try to refine our Market-GRE's by other criteria than the kind of perfection adopted in this paper, since the latter is restrictive.

Second, and more importantly, our model is incomplete in the sense that an essentially dynamic process of defections and reactions is analyzed within a static one-period model. In other words, reaction rules are imposed exogenously as in the other reaction equilibria rather than derived endogenously. One might model this process by using a genuinely game-theoretic framework. One such approach may be found in the reaction function equilibrium.<sup>14</sup>

Third, in this paper we restricted our attention to a game in which the uninformed agents move first. It was argued by Stiglitz and Weiss (1984) that there is no problem of potential non-existence, but rather of multiple Nash Equilibria, for games in which the informed agents move first. Recently, many researchers have proposed sophisticated criteria of refinements of Nash Equilibria in signalling games.<sup>15</sup> However, it is not always clearcut whether a particular market with asymmetric information should be approached by a screening or a signalling model. The current bi-polarization of these two models seems to be one of the most important problems in this area. Should the second problem stated above be solved

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<sup>14</sup>Friedman (1983) discusses a reaction function equilibrium as a non cooperative equilibrium for an oligopolistic market with complete information. The equilibrium strategy consists of both the sequences of contracts offered by each bank at each round (or period) and reaction function for each bank. It is an equilibrium in the sense that there is no other reaction functions and no other sequences of contracts given the other players' strategies that can improve any bank's payoff. However, according to Friedman (1983, p. 121), it is not yet known under what conditions this reaction function equilibrium does exist.

<sup>15</sup>See, Kreps and Wilson (1982) and Cho and Kreps (1987) for refinements of Nash Equilibria in a signalling model.

successfully, one might try to integrate these two models.<sup>16</sup>

Fourth, our results is based on a model of two types. Note that one strong argument for the RRE is its existence, coupled with the non-existence of NE in pure strategies and of Wilson Equilibrium, in a model with a continuum of types as was proved by Riley (1979a). So it is essential to extend our results to a continuum model. But, remember that a NE does not exist in our model only for small values of the proportion of H-types. So if the feature of a model with a continuum of types of could be characterized mathematically by small (infinitesimally small) values of the proportions of higher risk types, it is natural to conjecture that our results may be extended to a model of a continuum of types. For the assumption of free entry can cope with this problem, as is implied by Theorem 5. However, this is yet to be proved formally.

## APPENDIX

**THEOREM 1:** *The pair  $(\gamma^{H*}, \gamma^{L*})$  of Pareto efficient separating contracts is a unique NE if  $\delta \geq \delta_0$ , and there is no NE in pure strategies otherwise, where  $\gamma^{H*} = (\gamma^{H*}, CH^*)$ ,  $\gamma^{H*} = (1 + )Z/P_H$ ,  $CH^* = 0$ , and  $\gamma^{L*} = \gamma^L(\gamma^{H*})$ .*

**PROOF:** (i) By Lemma 1, there is no way of profitable defection from  $(\gamma^{H*}, \gamma^{L*})$  if  $\delta \geq \delta_0$ . Thus it is a NE in pure strategies in such a case. If  $\delta < \delta_0$ ,  $s^P(\delta^{L*}) \neq \phi$ . Let  $x_j^1 = \gamma_j^1 \in s^P(\gamma^{L*})$  be a defector's offer. Then he can earn strictly positive expected profits by attracting all types. So,  $(\gamma^{H*}, \gamma^{L*})$  is not a NE in pure strategies if  $\delta < \delta_0$ .

(ii) Next, we prove that any other contracts are not NE. For a pair  $(\gamma^H, \gamma^L) \in Y_q$  of separating contracts, let a defector  $j$ 's offer be  $x_j^1 = (\gamma_j^1, \gamma^L(\gamma_j^1))$ , where  $\gamma_j^1 \in s^H(\gamma^H) \cap B_\epsilon(\gamma^H)$ , if  $\gamma^H \neq \gamma^{H*}$ , and  $x_j^1 = (\gamma^{H*}, \gamma_j^1) \in Y_q$ , where  $\gamma_j^1 \in s^L(\gamma^L) \cap B_\epsilon(\gamma^L)$ , if  $\gamma^{H*}$  and  $\gamma^L \neq \gamma^{L*}$ . For a pooling contract  $\gamma \in Y_q$ , let  $x_j^1 = \gamma_j^1 \in s^L(\gamma) \cap B_\epsilon(\gamma)$ . By offering these contracts, the defector can earn positive profits larger than his initial profit. For he can attract all types or only L-types in the market with only infinitesimally smaller margin. This proves that these contracts are not NE. Q.E.D.

**LEMMA 2:** *A pair  $(\gamma^{H*}, \gamma^{L*})$  of contracts is a Riley Reactive Equilibrium. But it does not satisfy (3.1e) for all  $x_j^1 \in Y_l$  if  $\delta < \delta_l$ .*

**PROOF:** (i) The first part of the Lemma is now well known. If  $\delta \geq \delta_0$ , (3.1a) holds because  $s^P(\gamma^{L*}) = \phi$  by Lemma 1. If  $\delta < \delta_0$ , there is only one way of profitable

<sup>16</sup>As can be seen from Hellwig (1987) as well as Section 3 and 4 in this paper, the problem of multiple equilibria may arise in a screening model. Moreover, the last mover is the uninformed (banks) in a Hellwig's 3-stage screening game. Thus, when some informed agents (loan applicants) are rejected at the third stage, they have no chance to reapply. Such an environment affects the strategic behavior of the informed in an important way. Therefore, it may be conjectured that not only who move first but also who move last is important to the nature of an equilibrium.

defection,  $x_j^l = \gamma_j^l \in s^P(\gamma^{L*})$ . By  $\gamma_j^l$ , a defector  $j$  can attract all types to earn positive profitst [(3.1b)], while the others (M/j) earn zero profits [(3.1c) holds with equality]. Then, a reactor  $k$  must find his reaction strategy  $x_k^l$  not from  $s^P(\gamma_j^l)$  but from  $s^L(\gamma_j^l)$  in order to ensure the condition (3.1f). For if  $x_k^l = \gamma_k^l \in s^P(\gamma_j^l)$  is taken,  $x_k^l \in s^L(\gamma_k^l)$  will invalidate (3.1f). Taking  $\gamma_k^l \in s^L(\gamma_j^l)$ , the reactor  $k$  attracts only L-types away from the defector  $j$ . By definition of  $s^L(\gamma_j^l)$ , the reactor  $k$  earns positive profits [(3.1d)]. Being left with only H-types, the defector  $j$  earns negative profits[(3.1e)].

Given  $\gamma_k^l$ , there are at most two ways of profitable reaction open to another reactor  $l$ :  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$ , and  $\gamma_l^l \in s^P(\gamma_k^l)$  if  $s^P(\gamma_k^l) \neq \emptyset$ . Whichever is taken, the reactor  $k$  just loses his customers [(3.1f) holds with equality]. Thus  $(\gamma^{H*}, \gamma^{L*})$  is an RRE.

(ii) The final payoff of a defector  $j$  will be again negative if  $\gamma_l^l$  is taken [(3.1é)]. But it will be zero if  $\gamma_l^l$  is taken, violating (3.1é). The sufficient condition for  $\gamma_l^l \in s^P(\gamma_k^l)$  to be available to the reactor  $l$  is just  $\delta < \delta_1$ . To see this, note that if  $s^P(\gamma_k^l) \neq \emptyset$  for  $\gamma_j^l \in s^P(\gamma^{L*}) \cap B_c(\gamma(1))$  and  $\gamma_k^l \in s^L(\gamma_j^l) \cap B_c(\gamma^L(\gamma(1)))$ , then  $s^P(\gamma_k^l) \neq \emptyset$  for all  $\gamma_k^l \in s^L(\gamma_j^l)$ . In the limit, such choices of  $\gamma_j^l$  and  $\gamma_k^l$  are  $\gamma(1)$  and  $\gamma^L(\gamma(1))$ , respectively. By Lemma 1,  $s^P(\gamma^L(\gamma(1))) \neq \emptyset$  if  $\delta < \delta_1$ . This case is also illustrated in Figure 1. If  $\delta \geq \delta_1$ , then an indifference curve  $\bar{V}^L(\gamma^L(\gamma(1)))$  lies below the line  $\pi(\gamma) = 0$  or intersects it only at  $\gamma^P(\delta)$  by our assumption (2.2), and hence  $s^P(\gamma_k^l) = \emptyset$  for some  $\gamma_k^l \in s^L(\gamma_j^l)$  given  $\gamma_j^l \in s^P(\gamma^{L*}) \cap B_c(\gamma(1))$ . Since (3.1é) must hold for all  $x_k^l \in Y_1$ ,  $(\gamma^{H*}, \gamma^{L*})$  turns out not to satisfy it for all  $x_k^l \in Y_1$  in case of  $\delta < \delta_1$ . Q.E.D.

**THEOREM 2:** Any pooling contract  $\gamma \in s^P(\gamma^{L*})$  is an RRE.

**PROOF:** Note that, given a contract  $\gamma \in s^P(\gamma^{L*})$  and  $x_M^Q = ((\gamma)_{q \in M})$ ,  $H_q(x_M^Q) > 0$  for any  $q \in M$ . Then, there are two ways of profitable defection:  $x_j^l = \gamma_j^l \in s^P(\gamma)$  and  $x_j^l = \gamma_j^l \in s^L(\gamma)$ .

Taking  $\gamma_j^l \in s^P(\gamma) \cap B_c(\gamma)$ , a defector  $j$  can earn even larger profits than  $H_j(x_M^Q)$  by attracting all types in the whole market [(3.1b)]; the others (M/j) earn zero profit [(3.1c)]. Then a reactor  $k$  can earn positive profits by offering  $x_k^l = \gamma_k^l \in s^L(\gamma_j^l)$  and attracting only L-types away from the defector [(3.1d)]. Being left with H-types, the defector earns negative profits [(3.1e)]. Another reactor  $l$  may offer either  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$  or  $x_l^l = \gamma_l^l \in s^P(\gamma_k^l)$  if  $s^P(\gamma_k^l) \neq \emptyset$ . Whichever is taken, the first reactor  $k$  just loses his customers and hence his payoff becomes zero [(3.1f)] holds with equality.

In case of  $\gamma_j^l$ , similar argument applies. With  $\gamma_j^l \in s^L(\gamma) \cap B_c(\gamma)$ , defector  $j$  collects only L-types, leaving H-types to the others [(3.1b) and (3.1c)]. Let  $x_k^l = \gamma_k^l \in s^L(\gamma_j^l)$ . Then L-types move from the defector to a reactor  $k$  [(3.1d) and (3.1e)]. Another reactor  $l$ 's options and their effects are the same as above [(3.1f)]. This completes the proof that  $\gamma \in s^P(\gamma^{L*})$  is indeed an RRE. Q.E.D.

**THEOREM 3:** The set of Market-MRE is  $\{(\gamma^{H*}, \gamma^{L*})\} \cup \bar{s}^P(\gamma^{L*})$ , where  $\bar{s}^P(\gamma^{L*})$  is empty if  $\delta > \delta_0$ .

PROOF: (i) It is first proved that a contract  $\gamma \in \{(\gamma^{H*}, \gamma^{L*})\} \cup \bar{s}^P(\gamma^{L*})$  is a Market-MRE. To do this, we must check whether such a  $\gamma$  satisfies (3.2a) or (3.2b-f'), using (3.2g) if necessary.

(i-1) It can be easily shown by letting  $x_j^l = \gamma_j^l$  as in Lemma 2 that  $(\gamma^{H*}, \gamma^{L*})$  is a Market-MRE. The conditions (3.2a)-(3.2d) hold by Lemma 2. Given  $\gamma_j^p$  and  $\gamma_k^l$ , the others serve none [(3.2c') with equality]. Once they offer  $\gamma_j^l$  identically, they earn positive profits as a reactor 1 does in Lemma 2 [(3.2d')]. Then defector  $j$  attracts only H-types [(3.2e')] and reactor  $k$  none [(3.2f')].

(i-2) Consider a contract  $\gamma \in \bar{s}^P(\gamma^{L*})$ . If  $\delta > \delta_0$ , it is clear from the definitions, (2.7), (2.9) and (2.10) that  $\bar{s}^P(\gamma^{L*})$  is empty. Otherwise, given an initial contract  $\gamma \in \bar{s}^P(\gamma^{L*})$ , there are only two ways of profitable entrance:  $x_j^l = \gamma_j^l \in s^L(\gamma)$  and  $x_j^l = \gamma_j^p \in s^P(\gamma)$ .

In case of  $x_j^l = \gamma_j^l$ , let  $x_k^l = \gamma_k^l \in s^L(\gamma_j^l)$  and  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$ . An entrant  $j$  can attract all L-types by  $\gamma_j^l$ , earning positive profits [(3.2b)]; the others (M/j) make losses by serving H-types with the initial contract  $\gamma$  [(3.2c)]. When  $\gamma_k^l$  is offered, L-types move from the entrant  $j$  to the first reactor  $k$ : the latter earns positive profits [(3.2d)], and the others (M/j,  $k$ ) do again negative profit [(3.2c')]. Once all banks (M/j,  $k$ ) offer  $\gamma_j^l$ , they jointly attract all L-types away from the first reactor  $k$  [(3.2d')]. Then, since the initial contract  $\gamma$  is no longer offered, H-types apply to the entrant's offer  $\gamma_j^l$  [(3.2é)]. And the reactor  $k$  just loses his customers [(3.2f') holds with equality].

In case of  $x_j^l = \gamma_j^p$ , let  $x_k^l = \gamma_k^l \in s^L(\gamma_j^p)$  and  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$ . An entrant  $j$  can attract all types by  $\gamma_j^p$  [(3.2b)] while the others (M/j) lose customers [(3.2c) holds with equality]. As  $\gamma_k^l$  and  $\gamma_l^l$  are offered, the reactors  $k$  and (M/j,  $k$ ) attract L-types successively, [(3.2d), (3.2d'), and (3.2f')] while the entrant  $j$  is left with only H-types [(3.2é)]. The reactor  $k$  eventually loses customers. This completes the proof that  $\gamma \in \bar{s}^P(\gamma^{L*})$  is a Market-MRE.

(ii) Next, we show that the contracts other than those discussed above are not Market-MRE. To do this, it suffices to show that there exists at least one profitable way of defection or entrance  $x_j^l$  [(3.2b) holds] but the defector or entrant does not become worse off [(3.2é) does not hold] when  $x_k^l$  and  $x_l^l$  are offered with incentives to react.

For a pooling contract  $\gamma \in \bar{s}^P(\gamma^{L*})$  which gives banks non-negative profits, suppose that a new entrant  $j$  enters with  $x_j^l = (\gamma^{H*}, \gamma_j^l)$ , where  $\gamma_j^l \in (\gamma^l: V^H(\gamma^{H*}) = V^H(\gamma^l), V^L(\gamma^l) > V^L(\gamma), \pi^L(\gamma^l) > 0) \neq \emptyset$ . Then the entrant earns positive profits [(3.2b) holds by (3.2g)]. When  $x_k^l = \gamma_k^l \in s^L(\gamma_j^l)$  and  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$  are taken, [(3.2d), (3.2d') and (3.2f')], the entrant  $j$  ends up with H-types. However, the entrant earns zero profit because H-types choose  $\gamma^{H*}$  rather than  $\gamma_j^l$  by the tie-breaking rule (b) in (2.4). This violates the condition (3.2é) with (3.2g) applied.

For a pair  $(\gamma^H, \gamma^L) = (\gamma^{H*}, \gamma^{L*})$  of separating contracts which give banks non-negative expected profits, let a new entrant  $j$  offer the same  $x_j^l$  as defined in the proof of Theorem 1. Then the entrant  $j$  earns positive profit [(3.2b) holds by (3.2g)].

When  $x_k^l$  and  $x_l^l$  are taken sequentially in the same manner as  $x_j^l$ , the entrant attracts no type and the same argument as the above paragraph applies. Q.E.D.

**THEOREM 4:** *A pair  $(\gamma^{H*}, \gamma^{L*})$  of contracts is a unique Perfect Market-MRE.*

**PROOF:** Suppose that a subset of players  $(M/j, k)$  fail in reacting. Without loss of generality, assume that only one player  $m$  does so. To show that  $(\gamma^{H*}, \gamma^{L*})$  is a Perfect Market-MRE, let's go back to the proof of Lemma 2. If  $\delta \geq \delta_0$ , (3.2a) holds. If  $\delta \leq \delta_0$ , the only way of profitable defection is to offer  $x_j^l = \gamma_j^l \in s^P(\gamma^{L*})$ . Let  $x_k^l = \gamma_k^l \in s^L(\gamma_j^l)$  as in Lemma 2, and  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$  identical for all  $l \in (M/j, k, m)$ . Since the defector  $j$  attracts all types by  $\gamma_j^l$  and then is left with only H-types as  $x_k^l$  and  $x_l^l$  are offered, it is clear that all conditions (3.2b)-(3.2f') are met.

To show that  $\gamma \in s^P(\gamma^{L*})$  is not a PM-MRE, we reinterpret the part (i-2) in proof of Theorem 3 above. When players  $(M/j, k, m)$  react by offering  $x_l^l = \gamma_l^l \in s^L(\gamma_k^l)$  identically, player  $m$  still offers  $\gamma$ . Thus, H-types choose  $\gamma$  among the offers made, and hence the entrant  $j$  is left with no customer. This implies (3.2é) that is violated. Q.E.D.

**LEMMA 3:** *A pair  $(\gamma^{H*}, \gamma^{L*})$  of contract is a GRE if  $\delta \geq \delta_{m-1}$ .*

**PROOF:** (i) If  $\delta \geq \delta_0$ ,  $s^P(\gamma^L(\gamma(0))) = \phi$  by Lemma 1, and hence there is no way of profitable defection. [(4.1a)] Thus is a GRE.

(ii) If  $\delta < \delta_0$ ,  $s^P(\gamma^{L*}) \neq \phi$ . The only way of profitable defection is to offer  $x_j^l \in s^P(\gamma^{L*})$ . By offering such a  $x_j^l$ , a defector  $j$  can attract all types with positive profits [(4.1b-c)]. It will be shown in part (iii) below that, given such a  $x_j^l$ , the GR  $x_{M/j}^l$  of players  $(M/j)$  is  $\gamma^L(x_j^l)$  if  $\delta \geq \delta_{m-1}$ . Then, the defector  $j$  will be left with only H-types to earn negative profits [(4.1e)] and the others  $(M/j)$  will earn zero profits [4.1d)]. Thus,  $(\gamma^{H*}, \gamma^{L*})$  is also a GRE if  $\delta_{m-1} \leq \delta < \delta_0$ .

(iii) Now we show that  $\gamma^L(x_j^l)$  is a GR of players  $(M/j)$  if  $\delta \geq \delta_{m-1}$  (iii-1) Given  $x_j^l$ ,  $\gamma^L(x_j^l)$  satisfies if  $s^P(\gamma^L(x_j^l)) = \phi$ . For this condition to be satisfied for every choice of  $x_j^l \in s^P(\gamma^{L*})$ , it must hold in the limit for  $x_j^l = \gamma(1)$ . By Lemma 1,  $s^P(\gamma^L(\gamma(1))) = \phi$  if  $\delta \geq \delta_1$ . Thus,  $\gamma^L(x_j^l)$  is the GR if  $\delta \geq \delta_1$ .

(iii-2) If  $\delta < \delta_1$ , then  $s^P(\gamma^L(\gamma(1))) \neq \phi$ . And, a reactor  $k$  can profitably defect by offering  $x_k^l \in s^P(\gamma^L(\gamma(1)))$  against  $x_{M/j}^l = \gamma^L(x_j^l)$ . The other banks are left with no customer [(4.2c)]. It will be shown soon that, given  $x_k^l$  and  $x_{M/j}^l$ , the GR  $x_{M/j, k}^l$  of the players  $(M/j, k)$  is  $\gamma^L(x_k^l)$ , if  $\delta \geq \delta_{m-1}$ . When this GR is taken, the secondary defector  $k$  is left with only H-types to earn negative profits [(4.2e)] and  $(M/j, k)$  players earn zero profits [4.2d)]. Thus,  $\gamma^L(x_j^l)$  is a GR of players  $(M/j)$  if  $\gamma^L(x_k^l)$  is a GR of  $(M/j, k)$ .

(iv) Using the same argument as (iii-1), it can be easily shown that  $\gamma^L(x_k^l)$  is a GR of players  $(M/j, k)$ , if  $\delta \geq \delta_2$ . Repeating the arguments (iii) again and again, We finally get to the conclusion that, given the  $(m-1)$ th round defection  $x_{m-1}^l$  in the limit,  $\gamma^L(\gamma(m-1))$  is a GR of the player  $(M/1, \dots, m-1)$ , that is, the last player  $m$ , if  $\delta \geq \delta_{m-1}$ . This completes the proof that  $\gamma^L(x_j^l)$  is a GR of players  $(M/j)$ , and

hence that  $(\gamma^H, \gamma^L)$  is a GRE if  $\delta_{m-1}$ .

(V) If  $\delta < \delta_{m-1}$ , take successive defections  $x_t^l$  by  $t = 1, \dots, m-1$  around  $\gamma(t)$ . Then the last player  $m$  can profitably defect from his GR  $\gamma^L(\gamma(m-1))$  by offering  $x_m^m \in S^P(\gamma^L(\gamma(m-1)))$ . This implies that  $(\gamma^H, \gamma^L)$  is not a GRE if  $\delta < \delta_{m-1}$ . Q.E.D.

**THEOREM 5:** *The set of market-GRE is equivalent to that of Market-MRE, and the set of Perfect Market-GRE is also equivalent to that of Perfect Market-MRE.*

**PROOF:** (i) It is first proved that  $(\gamma^H, \gamma^L)$  is both a Market-GRE and a PM-GRE. (i-1) By Lemma 3,  $(\gamma^H, \gamma^L)$  is not a GRE if  $\delta < \delta_{m-1}$ , because the last player  $m$  can profitably counter-react against the  $(m-1)$ th round defection  $x_{m-1}^m$  by  $\gamma_m \in S^P(\gamma^L(\gamma(m-1))) \neq \emptyset$ . However, as many new entrants as can ensure  $S^P(\gamma^L(\gamma(t))) = \emptyset$ ,  $t \geq m$ , can enter profitably under free entry. Therefore,  $(\gamma^H, \gamma^L)$  is a Market-GRE.

(i-2) Even if a player in the set  $(M/j)$ , say  $m$ , does not react but offer the initial contract  $(\gamma^H, \gamma^L)$ , he does not affect the payoff profile of others. For he does not attract any customer given  $x_M^l$ . Thus,  $(\gamma^H, \gamma^L)$  is also a PM-GRE.

(ii) Now we prove that a  $\gamma \in S^P(\gamma^L)$  is a Market-GRE but not a PM-GRE. (ii-1) For such a contract, there are two ways of profitable defection:  $x_j^l = \gamma_j^l \in S^P(\gamma)$  and  $x_j^l = \gamma_j^l \in S^L(\gamma)$ . In case of  $\gamma_j^l$ ,  $\gamma^L(\gamma_j^l)$  is the GR  $x_{M/j}^l$  of players  $(M/j)$ . For  $\gamma^L(\gamma_j^l)$  corresponds to the GR of a relevant subset of players  $(M/j)$ , which appears in the proof of Lemma 3 and in the discussion (i-1) above. In case of  $\gamma_j^l$ ,  $\gamma^L(\gamma_j^l)$  is a GR  $x_{M/j}^l$  in the same reason.

Offering  $x_j^l$  close to the initial contract  $\gamma$ , a defector can earn even larger profits than  $H_j(x_M^Q)$ ,  $x_M^Q = (\{\gamma_q\}_{q \in M})$  [(4.1b)]; the payoffs of others  $(M/j)$  are zero if  $x_j^l = \gamma_j^l$ , and negative if  $x_j^l = \gamma_j^l$ , i.e., the defector attracts only L-types [(4.1c)]. Given  $x_{M/j}^l$ , the defector earns negative profits by serving H-types [(4.1 e)] while the others zero profit [(4.1d)]. This proves that  $\gamma \in S^P(\gamma^L)$  is a Market-GRE.

(ii-2) To see  $\gamma \in S^P(\gamma^L)$  is not a PM-GRE, let a new entrant  $j$  take  $x_j^l = \gamma_j^l$ . Given  $x_j^l$  and the corresponding GR  $x_{M/j}^l$ , a player  $m$ , if he does not react but hold the initial  $\gamma$ , has to serve H-types: he earns zero profit because no type applies to him [(4.1e)] is violated).

(iii) Next, it can be shown that a pooling contract  $\gamma \in S^P(\gamma^L)$ , or a pair  $(\gamma^H, \gamma^L) \neq (\gamma^H, \gamma^L)$  of separating contracts, which gives banks positive expected profits, is not a Market-GRE. Let a new entrant  $j$  offer the same  $x_j^l$  as defined in the proof of Theorem 1. Then it is easy to see that the GR  $x_{M/j}^l$  of players  $(M/j)$  is  $(\gamma^H, \gamma^L)$ . So the entrant  $j$  ends up with zero profit [(4.1e)] is violated. Q.E.D.

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