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DEMAND FOR RISKY ASSET UNDER REGULATION

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Negative association between liquidity or the velocity of money and the real demand for risky asset under a partially regulated environment has been demonstrated. These relationships are, however, shown to be different depending on the types of the restrictions and the sources of excess credit. The crucial links derive from the transactions demand for money, which can be a direct function of the controlled parameters of regulation.

I. INTRODUCTION

Financial markets are subject to regulations and restrictions at a few different levels. One set of regulations pertains to those related with market microstructures that include, for example, margin requirements, the limits on the daily price changes, bound on the price discount for a new stock issue. They tend to affect the movement of asset prices and thereby investor behaviour. Another set regulates the structure or the management of financial institutions. Examples are interest rate regulations, audit and supervision of financial firms on the matters ranging from capital adequacy to lending practices, legal restrictions on new entry or the area of business. These regulations are important for the perfomance of financial firms or the operational efficiency of financial market, and thus have been analyzed in such a context.

However, except for those restrictions on yields or interest rates or on the liquidity, none of these regulations are likely to change the investor behaviour, namely the demand for risky asset, in a fundamental way. Interest rates and credit policy have been traditionally treated as macroeconomic policy tools and their effectiveness have been debated along the perspective of monetary policies. However, regardless of which monetary policy is to be undertaken, the regulations intended mainly for a 'smooth' working of financial markets are essentially those on interest rates and credit supply. The margin requirement and deposit rate ceiling are good examples. Despite the trend to a rapid derregulation in the financial

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markets worldwide, to which Korea is no exception, some form of these regulations are likely to survive. Consider, for example, the regulated deposit rate but with market determined lending rate. The role of such a regulation to determine the investor behaviour, the demand for risky asset in particular, under a partially regulated environment needs to be explained.

Using a simple portfolio model based on the transactions demand for money, this paper presents an analysis for the demand for risky asset in a market with partially regulated interest rate and restricted credit availability. Our approach is not to explicitly introduce a detailed regulatory process into the model, but to evaluate the effect of the changing liquidity it brings about through monetary system. Thus it is more in line with a monetary-economics interpretation of the effect on asset demand. Models using similar approaches have been studied elsewhere, LeRoy [1985] and Kimbrough [1986] among others. However, the source of economic disturbances and the treatment of liquidity or real balance differ significantly in our paper. Next section develops the basic model and derives some useful results. Section III analyzes the effect of regulated credit on asset demand. Section IV examines the case of partially restricted interest rates. Section V concludes the paper.

II. LIQUIDITY, VELOCITY OF MONEY AND ASSET DEMAND

The basic framework for the present analysis is similar to Kimbrough [1986] or Jaang [1988]. Consider an Economy with many, identical, infinitely-lived individuals. Each of them starts the current period with an endowment of k and M, capital and money respectively, and use them for purchasing consumption good and shares of risky asset denoted by c and s respectively. There is no distinction between capital and consumption good and therefore her competitively perceived wealth, w, is given by $w = k + M/p$ with p being the money price of the commodity. Money is used to provide liquidity. Its use arises due to the fact that nonmoney transactions waste real resources[1] and may be, sometimes prohibitively, costly. This cost depends upon the amount of liquidity or real balance she holds. Denote by μ the amount of real balance per each real unit of transaction. Thus the higher μ is, the endogenuously determined, 'real liquidity' will be lower. The real resource cost of transaction per each real unit of transaction is assumed to be given by function $T(\mu)$ with the following properties

[1]In Kimbrough [1986] the use of the real resources takes the form of time spent on transacting. However, it can be easily shown that such a specification can be easily transformed to the present form of transactions cost function.

$$T(\mu) > 0,\ T'(\mu) \leqslant 0,\ T''(\mu) \geqslant 0 \text{ for all } \mu, \text{ and}$$
$$T(0) = \infty,\ \lim_{\mu \to \mu^0} T(\mu) = 0 \text{ for some } \mu^0$$

The individual can also borrow or obtain a credit, denoted by a in the goods unit, from financial institution. Assuming nonsatiation, we can write her budget constraint as

$$(1)\ w = (s + c)[1 + T(\mu) + \mu] - a$$

Money stock in the economy increases through a transfer payment in the next period to gM where g can be interpreted as one plus growth rate of money. Each unit of risky asset earns a random real return r, while a nominally riskfree interest rate of n must be paid to the lender in the next period. Its real rate of return is $n\delta$, where δ is relative purchasing power, i.e. the inverse of one plus rate of inflation. On the other hand, the real value of each unit of money balance becomes $g\delta$. Using (1), her real wealth in the next period can be written as

$$\begin{aligned}(2)\ w_1 = {} & s[r + \mu\delta g - (1 + T(\mu) + \mu)n\delta] \\ & + c[\mu\delta g - (1 + T(\mu) + \mu)n\delta] + wn\delta\end{aligned}$$

Her economic problem is assumed to be given as

$$(3)\ \underset{c,s,\mu}{\text{Maximize}}\ c^{\theta}/\theta + \beta\phi E w_1^{\theta}/\theta \text{ subject to (1) and (2)}$$

where E is her expectations operator, and θ, β, ϕ are the parameters of her utility function. If we assume risk aversion, $1 - \theta$ is the coefficient of constant relative risk aversion[2] and thus $\theta < 1$. Time preference is indicated by β, $\beta < 1$, while ϕ is a constant that summarizes future consumption-investment opportunities. It depends only on the joint distribution of r and g beyond the next period under the assumption that they are independently and identically distributed over periods. Furthermore, the real and the monetary risks, r and g, themselves are independently distributed. As such, the optimization problem corresponds to the multiperiod consumption-portfolio choice problem considered elsewhere.[3] Equilibrium is

[2]To be more precise, the risk aversion here should be understood as the concavity of the power utility function in the form of c^{θ}/θ in each period, which requires that $\theta < 1$. An infinite horizon problem with such a utility function can be shown to be reduced (see Samuelson [1969]) to the present form. While a risk aversion in the multiperiod context is not well-defined in such a context, it is nevertheless useful, however, to interprete $1 - \theta$ term as the parameter of constant relative risk aversion. This is clear from the fact that $-V''w/V' = 1 - \theta$ where $V(w) = w_1^{\theta}/\theta$.

[3]See Samuelson [1969].

characterized by the following set of the first order conditions

(4) $c^{\theta-1} - \beta\phi E w_1^{\theta-1} r = 0$
(5) $E w_1^{\theta-1}[r + \mu\delta g - (1 + T(\mu) + \mu)n\delta] = 0$
(6) $E w_1^{\theta-1}\delta[g - (1 + T'(\mu)n] = 0$

and of market clearing conditions

(7) $(c+s)(1+T(\mu)) = k,\ M/p = (c+s)\mu - H/p,\ a = M/p$

The three equations in (7) can be combined to give

(8) $p = v(M+H)/k$, with $v \equiv (1+T(\mu))/\mu$,

which implies that the velocity of money is monotonically decreasing function of μ. Intutively, this is not surprising since μ can be considered as an inverse of real liquidity. The gross output or asset in the next period, k_1, is assumed to be generated by[4] $k_1 = sr$, which, recalling that the distribution of (r,g) and therefore μ also, is stationary,[5] can be used to derive

(9) $\delta = (r/g)(s/k)$
(10) $w_1 = sr\varphi$, where $\varphi \equiv 1 + \mu + (n/g)[\mu M(1+T')^{-1}(M+H)^{-1} - T - \mu]$

The description of equilibrium is now complete with equations (4)-(10), which can be analyzed to provide us with informations about demands for risky asset, consumption, liquidity, credit demand as well as prices like p, n and δ. Equation (6), coupled with (9) and (10), can be rearranged to derive nominal interest rate as

(11) $n = (1+T'(\mu))^{-1}[E(1/g) + \mathrm{Cov}(\varphi^{\theta-1}/E\varphi^{\theta-1},\ 1/g)]^{-1}$

Equation (11) indicates that in equilibrium nominal interest rate consists of three components. The first term in (11) accounts for the opportunity cost of holding nominal bond instead of holding real balance as it reduces real transactions cost by $T'(\mu)$ units. Since the real balance, and hence the real liquidity, is determined endogenuously in the model, nominal interest is determined endogenuously as well. Thus in our model it is market determined unlike in other monetary models[6] where

[4]Saving is assumed to be always in equilibrium with investment and therefore denoted simply as s, which is in line with our main concern, namely financial market equilibrium.

[5]This is because (r,g) is identically, independently distributed and μ is independent of initial wealth. The latter can be shown in the same way as Jaang [1988].

[6]See, for example, Kimbrough [1986]. This will be the case in our model only if μ goes to infinity or the marginal transactions cost can be ignored.

its role is limited to compensate for the loss of value in the nominal asset due to the increase in the money stock in the next period. This is explained by the first expectation term in the bracket in equation (11). Finally, a risk premium for the monetary risk, the covariance term in the equation, is also required, which is positive due to the risk aversion. Because of the risk premium term nominal interest cannot be solved explicitly in general. However, if the correlation between φ and g term is negligibly small,[7] it can be explicitly solved from equation (11) as

$$(12)\ n = [(1+T'(\mu))E(1/g)]^{-1}$$

Note that nominal interest accounts only for the effect of money growth only if the money growth is nonrandom and the transaction cost is zero. It can be shown that, using equations (7)-(11), equation (5) simplifies to

$$(13)\ k+s[\mu-(1+T(\mu)+\mu)/(1+T'(\mu))] = 0$$

Similarly, equation (4) can be simplified to

$$(14)\ k-s(1+T(\mu))[1+(\beta\phi Er^{\theta})^{1/(\theta-1)}(E\varphi^{\theta-1})^{1/(\theta-1)}] = 0$$

To evaluate the behaviours of the economic variables in equilibrium, it is sufficient to analyze three equations (11), (13) and (14). In doing so complications arise due mainly to the existence of the randomness in the monetary risk term as is clear from (11). Since our main concern here is the effects of the controllable regulatory parameters, in the discussion to follow only the results for the case of nonrandom g will be reported. It should be noted, however, that similar results hold for the more general case of random monetary risk as well.[8] Equations (11) and (14) can now be simplified to

$$(11)'\ n = g/(1+T(\mu)')$$
$$(14)'\ k-s(1+T(\mu))(1+(\beta\phi Er^{\theta})^{1/(\theta-1)}[1+\mu+\mu(1+T(\mu)')^{-2}$$
$$(1+H/M)^{-1} - (T(\mu)+\mu)(1+T(\mu)')^{-1}]\} = 0$$

The comparative statics with equations (13) and (14)′ would provide us information about the responses of asset demand, s, and real liquidity represented by μ to changes in the system parameters. An inspection of (13) and (14)′ shows that, except for k, all the exogenuous parameters appear only in (14)′. Using this, it can be shown (see appendix 1) that s and μ always move in the same direction.

[7]This will be so if risk aversion is not too great and (or) the deviation of money growth from its mean is not too great. The nonrandom money growth is an obvious example.

[8]See Jaang [1990].

It follows that

$$(15)\ \text{sign}\ (ds/dx) = \text{sign}\ (d\mu/dx) = -\text{sign}\ (dv/dx) = -\text{sign}\ (dn/dx),\ \text{where}\ x \in \{\beta,\phi,\theta,M,H\}$$

The positive association between velocity and nominal interest, the last equality in (15), is consistent with usual observation. Any change in the parameter causing a higher nominal interest makes nominal bonds more attractive, thereby reducing the demand for risky asset[9] as well as keeping less real balance (alternatively, requiring more real liquidity to complete a transaction). As an example, consider the effect of risk aversion on the asset demand. Unless a risk-adjusted, time-adjusted expected real return on stock, $\beta\phi Er^{\theta}$ term, is very small, the higher risk aversion means the lower demand for risky asset (the lower real stock price) as well as the higher velocity in the following sense (see appendix 1 and 2).

$\partial s/\partial(1-\theta) < 0$ and $\partial v/\partial(1-\theta) < 0$
provided that $(\theta-1)E(\log r) < \log(\beta\phi Er^{\theta})$

III. REGULATED CREDIT SUPPLY

There are two ways by which the liquidity in the economy might be directly affected. One is an outright change in money stock, a change in g in the model. Such a situation has been extensively studied, mostly in the context of monetary policy. The liquidity can also be affected whenever financial institution extends credit to economic units without an offsetting increase in their liabilities. This, in effect, can be accomplished, for example, through central banks' lending to them. It redistributes liquidity not only among the economic units but also between periods. Such a nonzero credit position can occur within a period regardless of government's monetary policy taken. Furthermore, the intraperiod credit change is possible without an interperiod change in money stock. A clear example is the case of a purchase of risky shares on margin. Thus a change in overall credit can be as much a consequence of fluctuations in financial market condition as a result of monetary policy.

However, such a fluctuation in the market is seldom left to the market free of any intervetion. We will consider the two most visible types of the regulations on the overall credit position below. First, consider the case where $H = hM$, that is, the credit regulation calls for the allowable 'excess credit' being given as a con-

[9]This does not necessarily imply, however, that the demand in nominal term or nominal asset price will fall. It is because the goods price will rise at the same time velocity rises. Depending on the relative strength, it may well be that nominal demand would rise as velocity rises (see equation (8) and also the discussion in the next section).

stant fraction of existing stock of money. Then, it can be shown that (see appendix 3 for detail) μ should be positive functions of the excess credit. If follows, from (15), that

$$\partial s/\partial h > 0,\ \partial v/\partial h < 0 \text{ and } \partial n/\partial h < 0$$

Easing the control on the lending practices of financial institutions would unambiguously raise real asset demand while at the same time put a downward pressure on nominal interest rate and velocity. Since the goods price in the current period may actually fall, especially if the velocity gets too sluggish, the asset demand in nominal term may fall with velocity,[10] thus resulting in an apparently positive association between the two. An inspection of the coefficients for ds and $d\mu$ indicates that such a positive association is more likely if current stock price is higher.

Another way to control intraperiod excess credit is to tie it with the value of existing asset, for example, to let $H = qps$. For the portion of the credit regulated by such a formula, $1-q$ represents the margin ratio on a margin purchse of a risky share.[11] Following the same procedure, it can be shown that (see appendix 3)

$$\partial s/\partial q > 0,\ \partial v/\partial q < 0 \text{ and } \partial n/\partial q < 0,$$

which implies that a higher margin requirement would depress the demand and raise the velocity and nominal interest rate.

IV. REGULATED INTEREST RATES AND ASSET DEMAND

Restrictions on interest rates have long been used to control credit. In fact, monetary policies in the heydays of Keynsian macroeconomic policy were geared to control credit by using interest rate as a target variable. While its role has been diminished since early 1980s, it still remains as a powerful device to affect financial markets and thereby either to stimulate or to contract an economy. This seems to be true in many parts of the world despite the fact that financial derregulation has eliminated a good part of interest rate regulations. In Korea, the deposit rate is still under a strict control while lending rates are partially controlled by the monetary authority.

Controlled interest rates can be easily incorporated into the present analysis. First, suppose the individuals in the economy can purchase short-term bonds or make loans among themselves, by making deposits at financial institutions, at

[10] Note that $d(ps)/dh = sdp/dh + pds/dh = s[(T'\mu - 1 - T)(M+H)\mu^{-2}k^{-1}d\mu/dh + vM/k] + pds/dh$. The larger s is the relative magnitude of $d\mu/dh$ over ds/dh will larger (see appendix 1 and 2).

[11] While, in principle, nominal bonds can be purchased on margin, they are uncommon.

nominally risk-free rate of m. The initial wealth constraint, equation (1), now inclues the term $b - bR(\mu)T(\mu)$, where function $R(\mu)$ represents "monyness" of the deposit. The higher the value of $R(\mu)$ is, the deposit is more like credit, a in equation (1), and the lower, more like a risky asset. Reallistically, however, it is likely that[12]

$$0 < R(\mu) < 1/T(\mu) \text{ for all } \mu$$

The next period's wealth, equation (2), would contain the following term

$$b[m\delta - (1 - R(\mu)T(\mu))n\delta]$$

Except for the first order condition

$$(16) \quad Ew_1^{\theta-1}\delta[m - (1 + R(\mu)T(\mu))n] = 0,$$

the equilibrium would exactly look like the same as before.[13] Equation (16) indicates that deposit and credit are essentially the same asset except for their monyness, which is itself a nonrandom function of liquidity. Where there no restrictions on both m and n, the deposit rate must be lower than the borrowing rate as indicated by

$$(17) \quad m = (1 - R(\mu)T(\mu))n,$$

which is nothing but a no-arbitrage condition in loan markets.

If deposit rate is regulated such that m is a number, it is clear, from (11) and (17), that the borrowing rate, n, and liquidity are no longer independently determined. The latter is completely determined by g and the controlled rate, from which it can be shown that (see appendix 4 and 5)

$$\partial\mu/\partial m < 0 \text{ and } \partial\mu/\partial g > 0$$

Given (11) and (15), the demand for risky asset is determined completely by (14)′, from which it follows that (see appendix 4) $ds/d\mu > 0$ and

$$(18) \quad \partial s/\partial m < 0,\ \partial s/\partial g > 0 \text{ and } \partial v/\partial m > 0,\ \partial v/\partial g < 0$$

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[12]If $b - bRT = b(1 + T + \mu)$ or $R = -(\mu + T)/T$, the deposit would be exactly like a risky share. On the other hand, if $1 - RT > 0$, it would behave like a credit. In fact, the liquidity of a deposit is not likely to be a perfect substitute for either of the two.

[13]This is obvious because the market clearing condition for deposit market is $b = 0$, while all others are the same.

If, on the other hand, the borrowing rate is a controlled rate, a similar argument shows that (see appendix 5)

(19) $\partial s/\partial n > 0,\ \partial s/\partial g > 0$ and $\partial v/\partial g < 0,\ \partial v/\partial g < 0$

If both rates are controlled, liquidity is essentially determined by the ratio m/n and both (18) and (19) should hold.

V. CONCLUDING REMARKS

A negative association of real demand for risky asset with liquidity or with velocity of money derives from the fact that the former responds to economic disturbances in an opposite way from that for nominal assets. This result is similar in spirit to that of Mundel [1951] where an inflationary monetary disturbance would reduce real value of real balance which is turn stimulate real investment. However, the similarity is superficial. The negative association is much stronger in our paper in that it applies to a broader class of disturbances that includes the case of the parameters of risk aversion. The application of the result to intraperiod credit restriction and regulated interest rate studied in the present paper should be helpful to understand the performance of financial markets against such a regulatory parameters. The model's merit can also be found in the explicit consideration of transactions cost aspects of holding money balance within the model rather than implicitly theorizing the motive for holding real balance.

It would be interesting to extend the present analysis to accept assets with different risk characteristics or a nontrivial correlation of monetary uncertainty with that of real risk, which is left out for a future study.

REFERENCES

BERANEK, W., HUMPHREY, T.M. and TIMBERLAKE, R.H. (1986), "Fisher, Thornton and the Analysis of Inflation Premium", *Journal of Money, Credit and Banking,* 17, No. 3, pp. 371-377.

FAMA, E. (1981), "Stock Return, Real Activity and Money", *American Economic Review,* pp. 545-65.

GESKE, R. and ROLLE, R. (1983), "The Fiscal and Monetary Linkage between Stock Return and Inflation", *Journal of Finance,* 38, pp. 1-37.

JAANG, D.H. (1990), "Sources of Risk Premia in Financial Markets Revisited", Hallym Univ. Working paper.

________ (1988), "Financial Asset Pricing under Real Inflation", *Hallym University Journal,* Vol. 6, pp. 13-24.

KIMBROUGH, K.P. (1986), "Inflation, Employment and Welfare in the Presence of Transactions Costs", *Journal of Money, Credit and Banking,* 18, No. 2, pp. 127-140.

KOHN, M. (1981), "In Defence of Finance Constraint", *Economic Inquiry,* pp. 177-195.

LANDSKRONER, Y. and LIVIATAN, N. (1981), "Risk Premia and the Sources of Inflation", *Journal of Money, Credit and Banking,* 13, No. 2, pp. 205-14.

LEROY, S. (1985), "Nominal Prices and Interest Rates in General Equilibrium: Endowment Shock", *Journal of Business,* 57.

LUCAS, R. (1980), "Equilibrium in a Pure Currency Economy", *Economic Inquiry,* 18, pp. 203-20.

MUNDEL, R. (1951), "Inflation and Real Interest", *Journal of Political Economy,* pp. 93-116.

SAMUELSON, P.A. (1969), "Lifetime Portfolio Selection by Dynamic Stochastic Programming", *Review of Economics and Statistics,* pp. 239-46.

STULTZ, R.M. (1986), "Asset Pricing and Expected Inflation", *Journal of Finance,* 51, pp. 209-23.

SWEENEY, R.J. (1987), "Some Macro Implication of Risk", *Journal of Money, Credit and Banking,'* 19, pp. 222-34.

APPENDIX 1

Let $y \equiv 1+\mu+\mu(1+T')^{-2}(1+H/M)^{-1}-(T+\mu)(1+T')^{-1}$, $z \equiv (\beta\delta Er^{\theta})^{1/(\theta-1)}$ and rewrite equations (13) and (14)$'$ as

$$\text{(a1)}\quad k+s[\mu-(1+T+\mu)/(1+T')] = 0$$
$$\text{(a2)}\quad k-s(1+T)(1+yz)$$

Totally differentiating (a1) and (a2) with respect to s, μ and x, with $x \in \{\beta,\phi,\theta,H,M\}$, yields

$$\text{(a3)}\quad \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} ds \\ d\mu \end{bmatrix} = \begin{bmatrix} 0 \\ N \end{bmatrix} dx$$

where $A=(\mu T'-T-1)/(1+T') < 0$
$B=sT''(1+T+\mu)/(1+T') > 0$
$C=-(1+yz)/(1+T) < 0$
$D=-T's(1+vz)-s(dy/d\mu)$
$N=s(1+T)d(yz)/dx$

Letting $\Delta=AD-BC$, we have

$$\text{(a4)}\quad ds/dx=-\Delta^{-1}BN,\ d\mu/dx=\Delta^{-1}AN$$

While the exact sign of either D or Δ is ambiguous, the latter can be deduced under a plausible set of conditions that yield intuitively appealing results. First, consider the effect of time prefrence parameter β as a proxy for the inverse of real interest rate, that is x in (a3) is β. If a higher interest implies a higher velocity, that is

$dv/d\beta<0$, as is consistent with Tobin-Baumol model of money demand, the sign of $d\mu/d\beta$ must be positive. This is so because v is monotonically decreasing in μ. Since $dz/d\beta=(\theta-1)^{-1}(\beta\phi Er^{\theta})^{(2-\theta)/(\theta-1)}(\phi Er^{\theta})<0$ due to the risk aversion ($\theta<1$), $N=s(1+T)y\partial z/\partial\beta$ is negative. It follows that Δ must be positive, which is also consistent with the observation that a higher real interest rate (lower β) implies a lower stock price, that is $ds/dN=-\Delta^{-1}BN>0$. Furthermore, since both v and n are decreasing in μ, equation (a3) implies, regardless of sign of N, that

$$\text{sign}(ds/dx) = \text{sign}(d\mu/dx) = -\text{sign}(dv/dx) = -\text{sign}(dn/dx)$$

APPENDIX 2

The coefficient for $d\theta$ with (a2) can be derived as

$$syz(1+T)(\theta-1)^{-2}[-\log(\beta\phi Er^{\theta})+(\theta-1)E\log r] \equiv G,$$

from which it follows that

$$d\mu/d\theta=\Delta^{-1}AG > (=, <)\ 0 \text{ if } G < (=, >)\ 0$$

APPENDIX 3

Since $1/(1+H/M)=1/(1+h)$, a direct calculation yields the coefficient for $1/(1+h)$ with (a2) as $N=s\mu z(1+T)/(1+T')^{-2}>0$. It follows that

$$d\mu/d(1/(1+h))=\Delta^{-1}AN < 0 \text{ and } d\mu dh > 0$$

If $H=qs$, the coefficient C in appendix 1 needs to be replaced by $C'=(T'-T-\mu)/(1-T')+z\mu M^2(M+qs^{-2})/(1+T')$. Assuming that q is sufficiently large that $(1+H/M)^2>z\mu/(T'-T-\mu)$, $C'>0$ as before, and thus the sign of $\Delta'=AD-BC'$ also remains the same as that of Δ. The coefficient for dq $N=-s^2z\mu M(M+qs)^2/(1+T')<0$, from which it follows that the signs for ds/dq, $d\mu/dq$ are the same as those of ds/dh, $d\mu/dh$.

APPENDIX 4

Since $m=(1-RT)n=(1-RT)Eg/(1+T')$, calculating either for fixed m or n, we get

$$d\mu/dg = 1/(nT'') > 0,\ d\mu/dh = (1+T')/(nt'') > 0,$$
$$d\mu/dg = (1-RT)/(mT''+RT'g+R'Tg) > 0,$$
$$d\mu/dm = -(1+T')/(mT''+RT'g+R'Tg) < 0$$

APPENDIX 5

From (a2), $ds/d\mu$ can be computed as $Cds = -Dd\mu$, which implies that its sign must be positive. Therefore we have

$ds/dm < 0$, $ds/dg < 0$ for controlled m
$ds/dh > 0$, $ds/dg > 0$ for controlled n.